

Solving the simultaneous equations,

a x + b y = ec x + d y = f

gives,

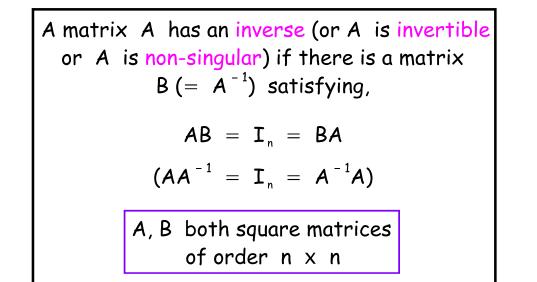
$$x = \frac{de - bf}{ad - bc} , \quad y = \frac{af - ce}{ad - bc}$$

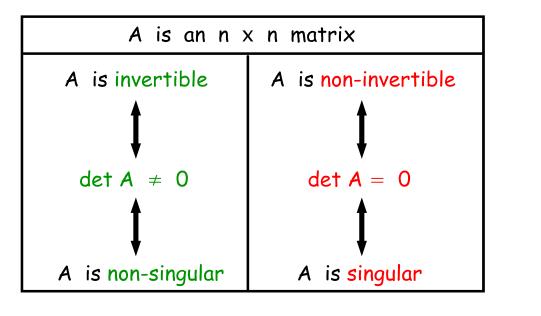
For solutions to exist, we must therefore have $a d - b c \neq 0$; if not, then no solutions.

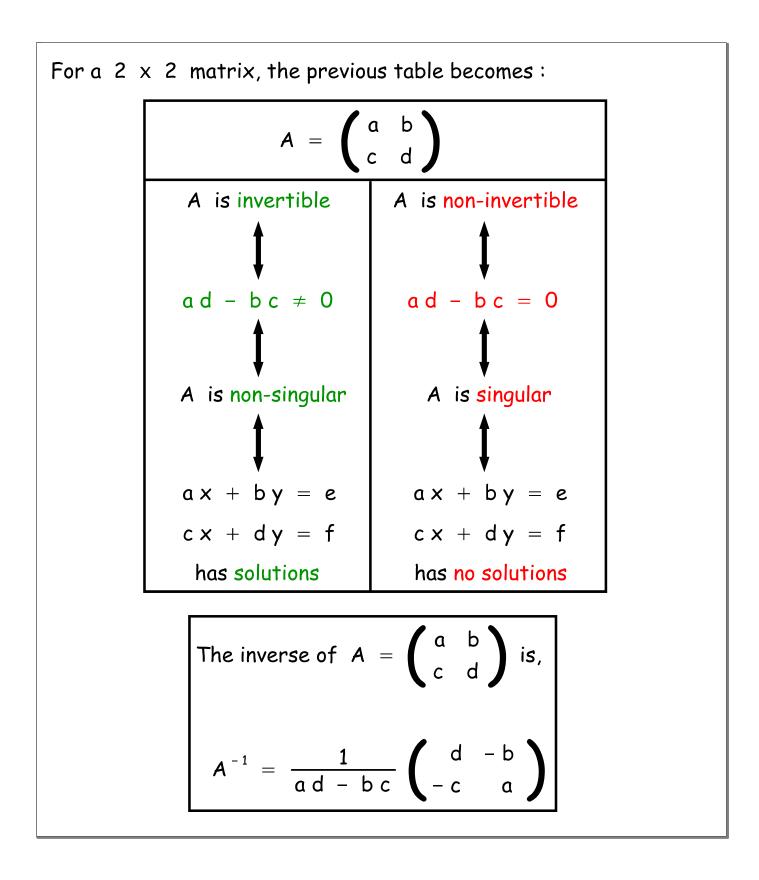
The above system of equations can be written as,

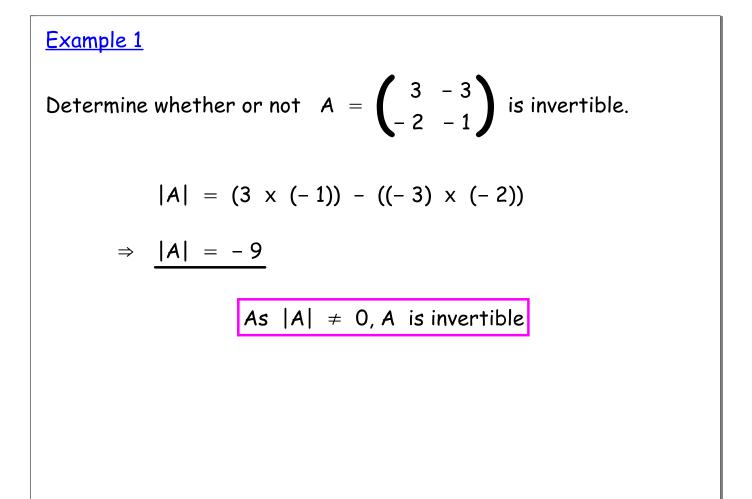
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e \\ f \end{pmatrix}$$

The determinant of $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is,
$$\det A = |A| = ad - bc$$

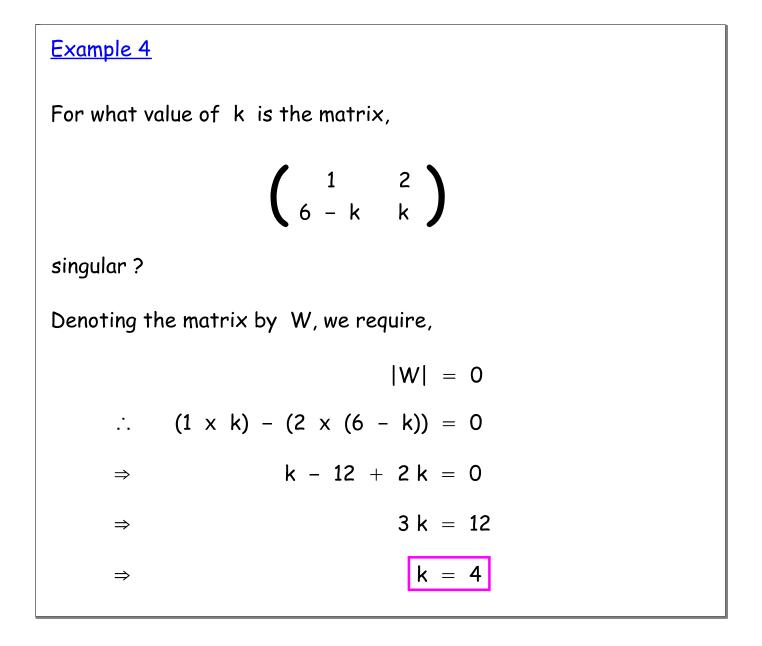








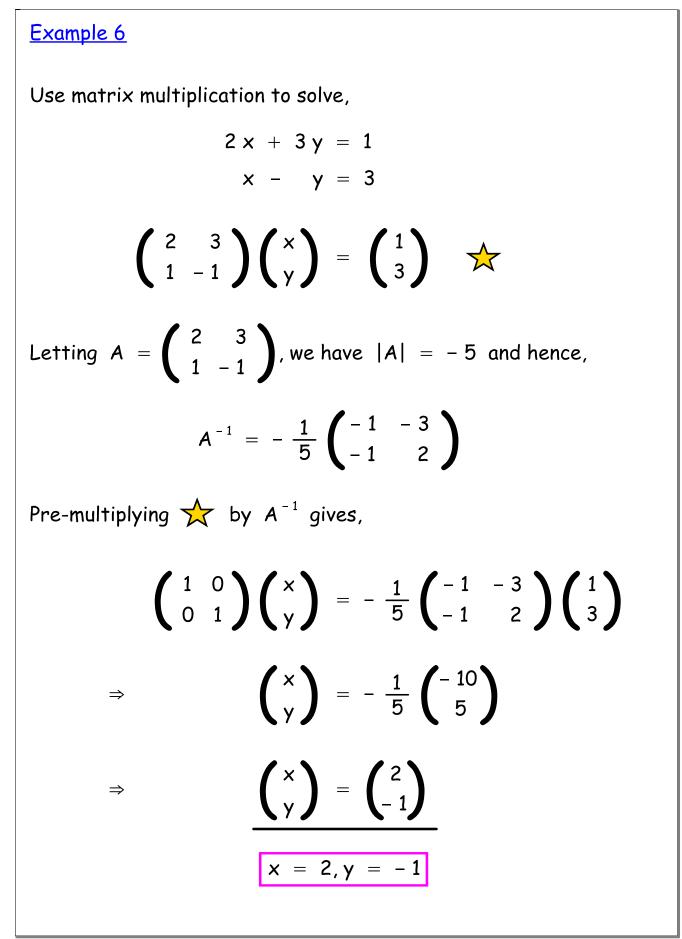
Example 2 Find the value(s) of k for which the following system of equations has no solutions. 4 x - 2 k y = -1 $\mathbf{k} \mathbf{x} - \mathbf{3} \mathbf{y} = \mathbf{0}$ Let $B = \begin{pmatrix} 4 & -2k \\ k & -3 \end{pmatrix}$. For no solutions, det B = 0. det B = 0 \therefore (4 x (-3)) - ((-2 k) x k) = 0 $-12 + 2k^2 = 0$ \Rightarrow $k^{2} = 6$ \Rightarrow $k = \pm \sqrt{6}$ \Rightarrow

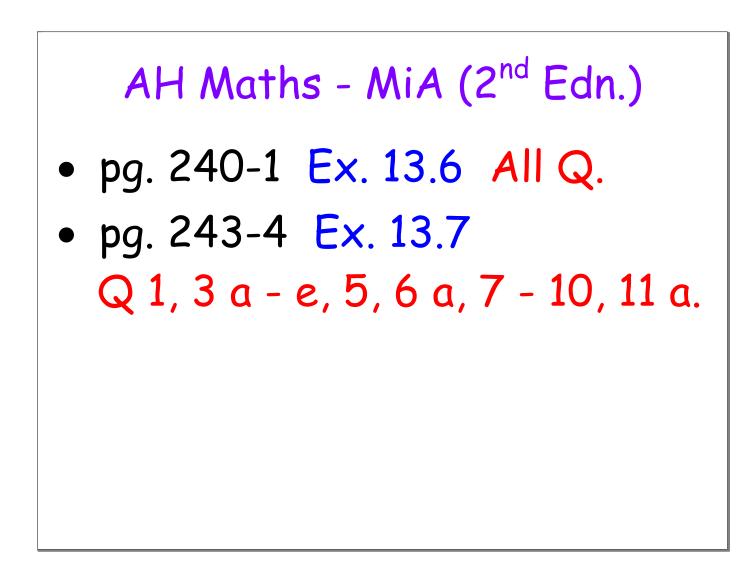


Example 5

A non-singular matrix F satisfies $F^2 = 3F - 2I$, where I is the identity matrix.

Show that
$$F^{-1} = \frac{3}{2}I - \frac{1}{2}F$$
.
 $F^2 = 3F - 2I$
 $\therefore \qquad F^{-1}F^2 = F^{-1}(3F - 2I)$
 $\Rightarrow \qquad F = 3F^{-1}F - 2F^{-1}I$
 $\Rightarrow \qquad F = 3I - 2F^{-1}$
 $\Rightarrow \qquad 2F^{-1} = 3I - F$
 $\Rightarrow \qquad F^{-1} = \frac{3}{2}I - \frac{1}{2}F$





Ex. 13.6											
1 0	Calculate the determinant of each matrix.										
a	$ \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix} $	$\mathbf{b} \begin{pmatrix} 12 & 3 \\ 8 & 2 \end{pmatrix}$		$ d \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} $							
e	$\begin{pmatrix} -4 & -5 \\ 2 & -1 \end{pmatrix}$	$ f \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} $	$g \begin{pmatrix} 7 & 1 \\ -2 & -1 \end{pmatrix}$	$\mathbf{h} \begin{pmatrix} \sin x & -\cos x \\ \cos x & \sin x \end{pmatrix}$							
2 I	2 Find <i>x</i> in each case.										
a	$\begin{vmatrix} 2 & 1 \\ 1 & x \end{vmatrix} = 7$	$\begin{vmatrix} 3 & -1 \\ 2x & 4 \end{vmatrix} = 26$	$\begin{vmatrix} 3x & 1 \\ 1 & x \end{vmatrix} = 47$	d $\begin{vmatrix} 2x & x \\ -3 & x \end{vmatrix} = 2$							
e	$\begin{vmatrix} x & x \\ 3 & x \end{vmatrix} = 10$	$f \begin{vmatrix} \sin x^{\circ} & -1 \\ 1 & 1 \end{vmatrix} = 0, 0$	$\leq x \leq 360$	$ g \begin{vmatrix} e^x & e \\ 1 & e^x \end{vmatrix} = 0 $							
	For each of these systems of equations i express the system of equations as a matrix equation of the form $AX = B$ where $X = \begin{pmatrix} x \\ y \end{pmatrix}$										
ii by considering det(A), determine whether the system has a solution.											
a	a x + y = 1 $3x + 2y = 4$	b 4x + y = 2 $8x + 2y = 3$	c 5x - 2y = 2 $10x - 4y = 1$	$ \begin{array}{l} d 5x - y = 1 \\ 3x - 2y = 0 \end{array} $							
4 5	4 Study each system of equations and say for what value(s) of <i>k</i> the system has no solution.										
	a 3x + ky = 3		c $6x - 2ky = -$	1 d $kx + (k - 4)y = 5$							

Ex. 13.7 **1** Find A^{-1} , the inverse of the matrix $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, and verify that $A^{-1}A = I = AA^{-1}$ 3 Find the inverse, where it exists, of each of these matrices. b $\begin{pmatrix} 5 & 4 \\ 4 & 3 \end{pmatrix}$ c $\begin{pmatrix} 9 & 6 \\ 3 & 2 \end{pmatrix}$ d $\begin{pmatrix} 4 & -2 \\ -7 & 3 \end{pmatrix}$ e $\begin{pmatrix} 9 & 2 \\ 13 & 3 \end{pmatrix}$ $\begin{pmatrix} 2 & 4 \\ 3 & c \end{pmatrix}$ **5** Given that $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 2 \\ 4 & 1 \end{pmatrix}$, show that a $A^{-1}B^{-1} = (BA)^{-1}$ b $(AB)^{-1} = B^{-1}A^{-1}$ c |A| |B| = |AB|6 a Given that A and B are square matrices, simplify ii $BAA^{-1}B^{-1}$ iii $B^{-1}A^{-1}AB$ iv $ABB^{-1}A^{-1}$ $i A^{-1}B^{-1}BA$ 7 Find the inverse of $\mathbf{b} \begin{pmatrix} 3x & 5x \\ 2x & 4x \end{pmatrix} \qquad \mathbf{c} \begin{pmatrix} 3t & t^3 \\ 2 & t^2 \end{pmatrix}$ a $\begin{pmatrix} a & -a \\ -a & 2a \end{pmatrix}$ $\begin{array}{ccc} a & \begin{pmatrix} a & -a \\ -a & 2a \end{pmatrix} & b \begin{pmatrix} 3x & 3x \\ 2x & 4x \end{pmatrix} & c \begin{pmatrix} 3t & t \\ 2 & t^2 \end{pmatrix} \\ d \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} & e \begin{pmatrix} 1-x & x \\ -x & 1+x \end{pmatrix} & f \begin{pmatrix} a & a+1 \\ a-1 & a \end{pmatrix} \end{array}$ 8 For what values of k are these matrices singular? b $\begin{pmatrix} 1 & 2k \\ 3 & 6 \end{pmatrix}$ c $\begin{pmatrix} 1-k & -1 \\ 3 & 1+k \end{pmatrix}$ d $\begin{pmatrix} 2+k & -6 \\ 4 & 3-k \end{pmatrix}$ a $\begin{pmatrix} 2 & 2 \\ 4 & b \end{pmatrix}$ **9** Show that these matrices are their own inverse (that is, $A^{-1} = A$). a $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ b $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ c $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ d $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ e $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ f $\begin{pmatrix} \sin\theta & \cos\theta\\ \cos\theta & -\sin\theta \end{pmatrix}$ g $\begin{pmatrix} \frac{4}{5} & \frac{3}{5}\\ \frac{3}{2} & -\frac{4}{2} \end{pmatrix}$ **10** $A = \begin{pmatrix} 2 & 3 \\ 7 & 11 \end{pmatrix}$ a Show that $A^2 = 13A - I$. b Hence show (without evaluating A^3 or A^{-1}) that ii $A^{-1} = 13I - A$ $i A^3 = 168A - 13I$ 11 For each of these systems of equations i express the system of equations in the form AX = Bii find the inverse of the matrix A iii hence solve the system for *x* and *y*. a 2x + y = 93x + 2y = 16

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Answers to AH Maths (MiA), pg. 240-1, Ex. 13.6											
					-11						
					-5						
2 a	4	b	7	с	± 4	d	-2, 0.5				
e	- 2, 5	f	270	g	$\frac{1}{2}$						
3 a	$\mathbf{i} \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$			ii $ A = -1 \Rightarrow \exists$ a solution							
b	$\mathbf{i} \begin{pmatrix} 4 & 1 \\ 8 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$			ii No solution							
с	$\mathbf{i} \begin{pmatrix} 5 & -2 \\ 10 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$			ii No solution							
d	$\mathbf{i} \begin{pmatrix} 5 & -1 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$			ii $ A = -7 \Rightarrow \exists$ a solution							
4 a	6	b	-5	с	±6	d	No k				

Answers to AH Maths (MiA), pg. 243-4, Ex. 13.7 $1 \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ **3 a** No inverse **b** $\begin{pmatrix} -3 & 4 \\ 4 & -5 \end{pmatrix}$ **c** No inverse d $-\frac{1}{2}\begin{pmatrix} 3 & 2 \\ 7 & 4 \end{pmatrix}$ e $\begin{pmatrix} 3 & -2 \\ -13 & 9 \end{pmatrix}$ 5 a Proof b Proof c Proof 6 a i ii iii iv I **b** $\frac{1}{2x}\begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$ 7 a $\frac{1}{a} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ c $\frac{1}{t^3}\begin{pmatrix} t^2 - t^3 \\ -2 & 3t \end{pmatrix}$ d $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ e $\begin{pmatrix} 1+x & -x \\ x & 1-x \end{pmatrix}$ f $\begin{pmatrix} a & -1-a \\ 1-a & a \end{pmatrix}$ **8 a** 4 **b** 1 **c** ±2 **d** -5,6 9 a-gProofs 10 a Proof b i ii Proofs 11 a i $\binom{2}{3} \binom{1}{2} \binom{x}{y} = \binom{9}{6}$ ii $\binom{2}{-3} \binom{2}{-3}$ iii (x, y) = (2, 5)