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Applications of Calculus - Lesson 5

Areas Between Curves

LI
- Calculate the area between curves.

SC
- Definite Integration.
Consider the area $A$ bounded between two curves $y = f(x)$ and $y = g(x)$, which meet at the $x$-coordinates $x = a$ and $x = b$:

![Graph showing the area between two curves](image)

The shaded area $A$ is given by:

$$A = \int_{a}^{b} (f(x) - g(x)) \, dx$$

('integral of top function minus bottom function')
Example 1 (Non-Calc)

Find the following shaded area:

\[ y = x^2 + 3x - 1 \]

\[ y = x + 2 \]

We first need to find where the curves meet:

\[ x^2 + 3x - 1 = x + 2 \]
\[ x^2 + 2x - 3 = 0 \]
\[ (x - 1)(x + 3) = 0 \]
\[ x = -3, 1 \]

\[ A = \int_{-3}^{1} (x + 2 - (x^2 + 3x - 1)) \, dx \]
\[ = \int_{-3}^{1} (3 - 2x - x^2) \, dx \]
\[ = \left[ 3x - x^2 - \frac{x^3}{3} \right]_{-3}^{1} \]
\[ = \left( 3(1) - 1^2 - \frac{1^3}{3} \right) - \left( 3(-3) - (-3)^2 - \frac{(-3)^3}{3} \right) \]
\[ = \frac{-1 - 27}{3} + 3 - 1 + 9 + 9 \]
\[ = 20 - \frac{28}{3} \]
\[ = \frac{32}{3} \text{ square units} \]
Example 2 (Calc)

Find the area (to 4 s.f.) bounded by the curves $y = \sin x$, $y = \cos x$ and the lines $x = -3\pi/4$ and $x = 0.3$:

\[
A = \int_{-3\pi/4}^{0.3} (\cos x - \sin x) \, dx
\]

\[
= \left[ \sin x + \cos x \right]_{-3\pi/4}^{0.3}
\]

\[
= \left( \sin (0.3) + \cos (0.3) \right) - \left( \sin (-3/4) + \cos (-3/4) \right)
\]

\[
= 2.665070 \ldots
\]

\[
= 2.665 \text{ units}^2 \text{ (to 4 s.f.)}
\]
CfE Higher Maths

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