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*Applications of Calculus - Lesson 5*

# Areas Between Curves

LI

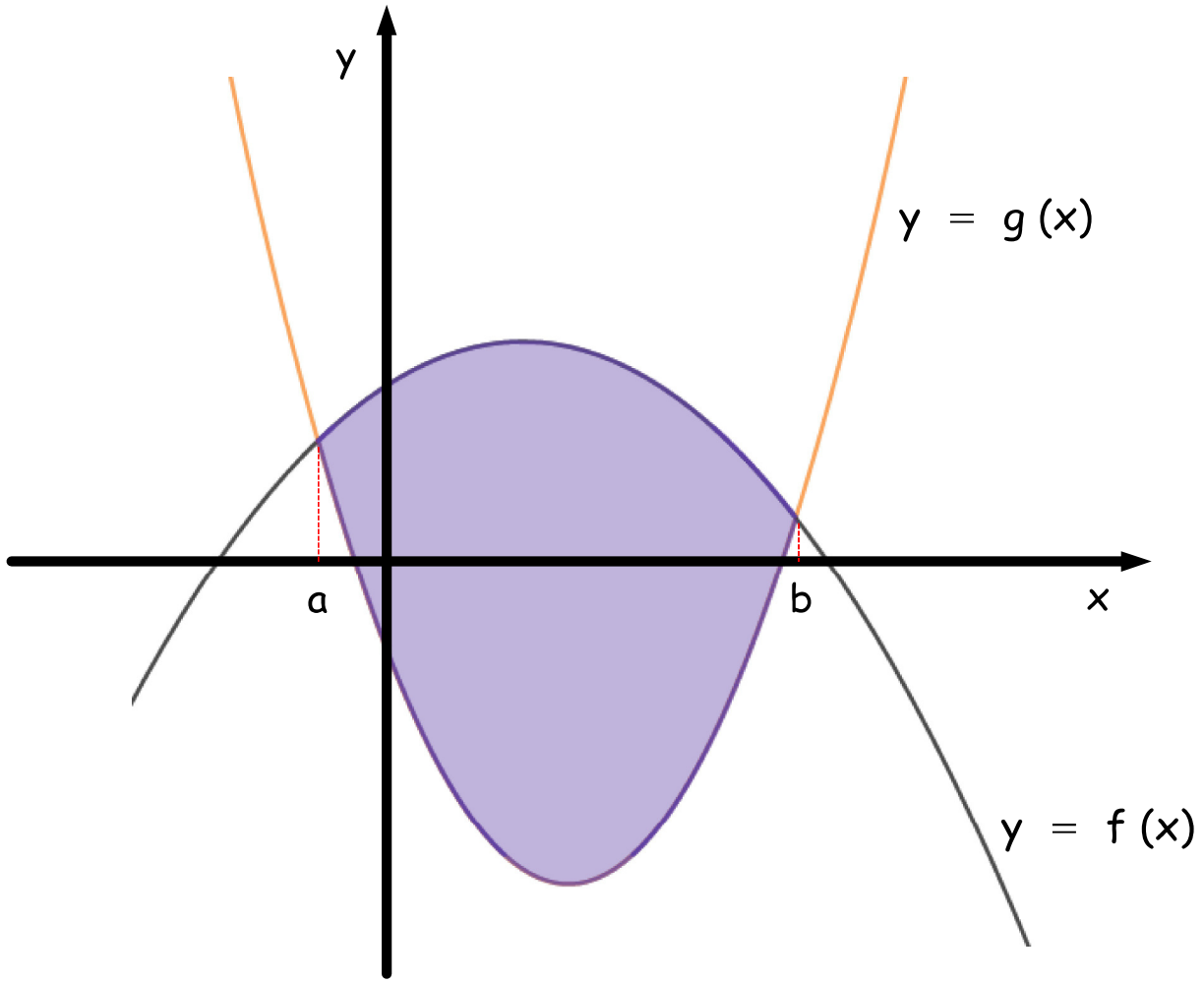
- Calculate the area between curves.

SC

- Definite Integration.

Jan 10-12:11

Consider the area $A$ bounded between two curves $y = f(x)$ and $y = g(x)$, which meet at the x - coordinates $x = a$ and $x = b$ :




The shaded area $A$ is given by :

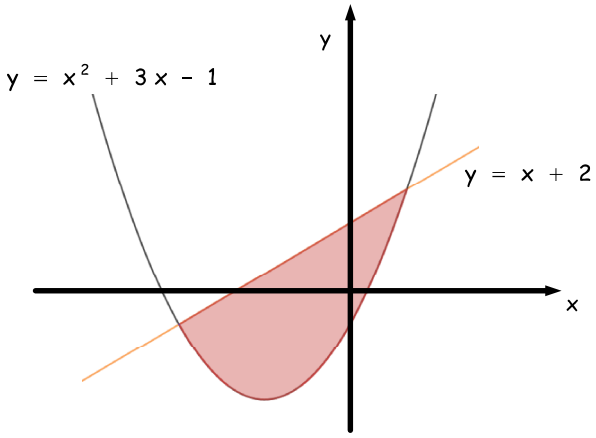
$$A = \int_a^b (f(x) - g(x))\ dx$$

('integral of top function minus bottom function')

Dec 11-19:39

Example 1 (Non-Calc)

Find the following shaded area :




We first need to find where the curves meet :

$$x^2 + 3x - 1 = x + 2$$

$$x^2 + 2x - 3 = 0$$

$$(x - 1)(x + 3) = 0$$

$$\underline{x = -3, 1}$$

$$A = \int_{-3}^{1} (x + 2 - (x^2 + 3x - 1))\, dx$$

$$= \int_{-3}^{1} (3 - 2x - x^2)\, dx$$

$$= \left[ 3x - x^2 - \frac{x^3}{3} \right]_{-3}^{1}$$

$$= \left( 3(1) - 1^2 - \frac{1^3}{3} \right)$$

$$- \left( 3(-3) - (-3)^2 - \frac{(-3)^3}{3} \right)$$

$$= \frac{-1 - 27}{3} + 3 - 1 + 9 + 9$$

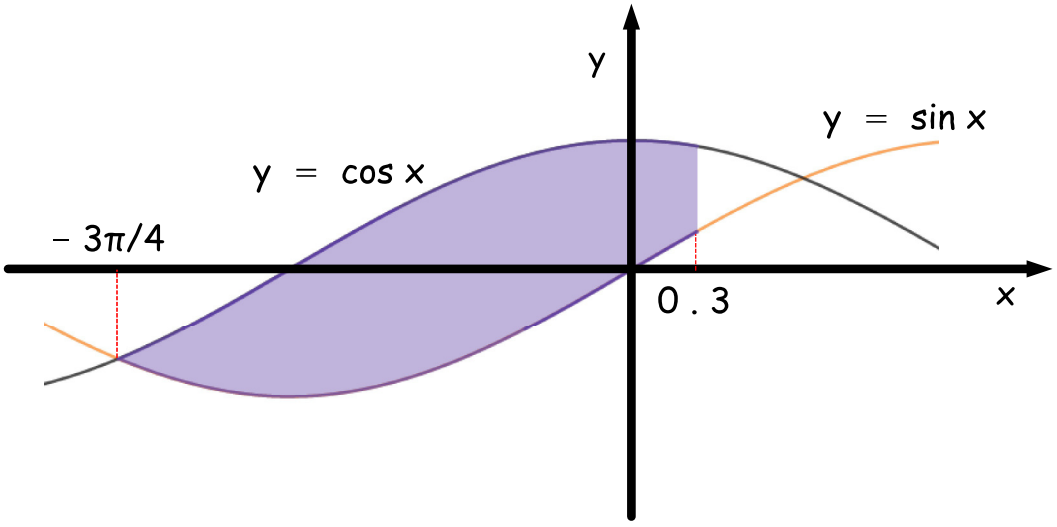
$$= 20 - \frac{28}{3}$$

$$= \boxed{\frac{32}{3} \text{ square units}}$$

Mar 1-14:16

Example 2 (Calc)

Find the area (to 4 s . f.) bounded by the curves $y = \sin x$, $y = \cos x$ and the lines $x = -3\pi/4$ and $x = 0.3$ :




$$A = \int_{-3\pi/4}^{0.3} (\cos x - \sin x)\, dx$$

$$= \Big[ \sin x + \cos x \Big]_{-3\pi/4}^{0.3}$$

$$= \Big( \sin(0.3) + \cos(0.3) \Big)$$

$$- \Big( \sin(-3/4) + \cos(-3/4) \Big)$$

$$= 2.665\,070\ldots$$

$$= \boxed{2.665 \text{ units}^2 \text{ (to 4 s . f.)}}$$

Mar 1-14:16

CfE Higher Maths

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Oct 29-09:14