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Exponentials and Logarithms - Lesson 4

Using Exponential and Logarithmic Equations in Context

**LT**
- Solve contextual questions involving exponentials and logarithms.

**SC**
- Logarithmic Rules.
- Using the log and ln buttons on the calculator correctly.
Example 1

In an experiment, the number of cells left after \( t \) days is given by \( C_t = C_0 \ e^{-kt} \), where \( C_0 \) is the initial number of cells.

(a) If the experiment began with 250 000 cells and half the cells died after 8 days, determine the value of \( k \) (to 3 s.f.).

(b) Calculate the time taken (to 1 d.p.) for the cells to reduce to 20% of the initial population.

(a)

\[
C_t = C_0 \ e^{-kt}
\]

\((C_0 = 250 000, C_t = 125 000, t = 8)\)

\[
\therefore 125 000 = 250 000 \ e^{-8k}
\]

\[
\Rightarrow e^{-8k} = 0.5
\]

\[
\therefore -8k = \ln(0.5)
\]

\[
\Rightarrow k = (\ln 0.5) / (-8)
\]

\[
\Rightarrow k = 0.086 643 \ldots
\]

\[
\Rightarrow k = 0.086 6 (3 \text{ s.f.})
\]
(b) 

\[ C_t = C_0 e^{-0.0866... t} \]

(20% of 250 000 = 50 000)

∴ 50 000 = 250 000 e^{-0.0866... t}

⇒ \[ e^{-0.0866... t} = 0.2 \]

∴ \[-0.0866... t = \ln(0.2)\]

⇒ \[ t = (\ln 0.2) / (-0.0866...) \]

⇒ \[ t = 18.6 \text{ days} \]
Example 2

The magnitude $M$ of a 'marsquake' is given by $M = \log_{10} \left( \frac{I}{I_0} \right)$ where $I$ is the intensity and $I_0$ is the intensity of a marsquake measuring 0.

A marsquake has a magnitude of 8.6 and a second marsquake is 173 times stronger than the first.

Find the magnitude of the second marsquake (1 d.p.).

$$M_F = \log_{10} \left( \frac{I_F}{I_0} \right), \quad M_S = \log_{10} \left( \frac{I_S}{I_0} \right)$$

\[ (M_F = 8.6, I_S = 173 I_F) \]

\[ \therefore \quad M_S = \log_{10} \left( \frac{I_S}{I_0} \right) \]

\[ \Rightarrow \quad M_S = \log_{10} 173 + \log_{10} \left( \frac{I_F}{I_0} \right) \]

\[ \Rightarrow \quad M_S = \log_{10} 173 + M_F \]

\[ \Rightarrow \quad M_S = \log_{10} 173 + 8.6 \]

\[ \Rightarrow \quad M_S = 10.8 \]
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