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Exponentials and Logarithms - Lesson 4

Using Exponential and Logarithmic Equations in Context

LI

- Solve contextual questions involving exponentials and logarithms.

SC

- Logarithmic Rules.
- Using the log and ln buttons on the calculator correctly.

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Example 1

In an experiment, the number of cells left after t days is given by $C_t = C_0 e^{-kt}$, where C_0 is the initial number of cells.

(a) If the experiment began with 250 000 cells and half the cells died after 8 days, determine the value of k (to 3 s.f.).

(b) Calculate the time taken (to 1 d.p.) for the cells to reduce to 20 % of the initial population.

(a)

$$C_t = C_0 e^{-kt}$$

$$(C_0 = 250\,000, C_t = 125\,000, t = 8)$$

$$\therefore \quad 125\,000 = 250\,000 e^{-8k}$$

$$\Rightarrow \quad e^{-8k} = 0.5$$

$$\therefore \quad -8k = \ln(0.5)$$

$$\Rightarrow \quad k = (\ln 0.5) / (-8)$$

$$\Rightarrow \quad k = 0.086\,643\ldots$$

$$\Rightarrow \quad \boxed{k = 0.086\,6 \text{ (3 s.f.)}}$$

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(b)

$$C_t = C_0 \, e^{-0.086\,6\ldots t}$$

(20 % of 250 000 = 50 000)

$$\therefore \quad 50\,000 = 250\,000 \, e^{-0.086\,6\ldots t}$$

$$\Rightarrow \quad e^{-0.086\,6\ldots t} = 0.2$$

$$\therefore \quad -0.086\,6\ldots t = \ln(0.2)$$

$$\Rightarrow \quad t = (\ln 0.2) / (-0.086\,6\ldots)$$

$$\Rightarrow \quad \boxed{t = 18.6 \text{ days}}$$

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Example 2

The magnitude M of a 'marsquake' is given by $M = \log_{10}(I / I_0)$ where I is the intensity and I_0 is the intensity of a marsquake measuring 0.

A marsquake has a magnitude of 8.6 and a second marsquake is 173 times stronger than the first.

Find the magnitude of the second marsquake (1 d.p.).

$$M_F = \log_{10}(I_F / I_0), \quad M_S = \log_{10}(I_S / I_0)$$

$$(M_F = 8.6, I_S = 173\, I_F)$$

$$M_S = \log_{10}(I_S / I_0)$$

$$\therefore \quad M_S = \log_{10}(173\, I_F / I_0)$$

$$\Rightarrow \quad M_S = \log_{10} 173 + \log_{10}(I_F / I_0)$$

$$\Rightarrow \quad M_S = \log_{10} 173 + M_F$$

$$\Rightarrow \quad M_S = \log_{10} 173 + 8.6$$

$$\Rightarrow \quad M_S = \log_{10} 173 + 8.6$$

$$\Rightarrow \quad \boxed{M_S = 10.8}$$

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