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Trigonometric Phenomena - Lesson 4

Proving Trigonometric Identities

LI

- Prove fancy trigonometric identities.

SC

- Addition Formulae.
- $\sin^2 x + \cos^2 x = 1.$
- $\tan x = \sin x \div \cos x.$

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A trigonometric identity is an equation that is true for every value of the variable(s) for which the equation makes sense

When proving trigonometric identities, generally look for the more complicated side and simplify it to reach the other side

2 Very Useful Identities

Pythagorean Identity

$$\boxed{\sin^2 A + \cos^2 A = 1}$$

Tan

$$\boxed{\tan A = \frac{\sin A}{\cos A}}$$

Example 1

Prove that $\cos(360 - x)^\circ = \cos x^\circ$.

$$\begin{aligned} \text{LHS} &= \cos(360 - x)^\circ \\ &= \cos 360^\circ \cos x^\circ + \sin 360^\circ \sin x^\circ \\ &= 1 \,.\, \cos x^\circ + 0 \,.\, \sin x^\circ \end{aligned}$$

from graphs

$$\begin{aligned} &= \cos x^\circ \\ &= \text{RHS} \end{aligned}$$

i.e. LHS = RHS

$$\therefore \boxed{\cos(360 - x)^\circ = \cos x^\circ}$$

Example 2

Prove that $\cos(30 - b)^\circ + \sin(120 + b)^\circ = \sqrt{3}\cos b^\circ$.

$$\begin{aligned}
\text{LHS} &= \cos(30 - b)^\circ + \sin(120 + b)^\circ \\
&= \cos 30^\circ \cos b^\circ + \sin 30^\circ \sin b^\circ \\
&\quad + (\sin 120^\circ \cos b^\circ + \cos 120^\circ \sin b^\circ) \\
&= \frac{\sqrt{3}}{2}\cos b^\circ + \frac{1}{2}\sin b^\circ \\
&\quad + \frac{\sqrt{3}}{2}\cos b^\circ - \frac{1}{2}\sin b^\circ \\
&= \sqrt{3}\cos b^\circ \\
&= \text{RHS}
\end{aligned}$$

i.e. LHS = RHS

$$\therefore \boxed{\cos(30 - b)^\circ + \sin(120 + b)^\circ = \sqrt{3}\cos b^\circ}$$

Example 3

Prove that $\dfrac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta} = \tan\alpha - \tan\beta.$

$$\text{LHS} = \frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta}$$

$$= \frac{(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}$$

$$= \tan\alpha - \tan\beta$$

$$= \text{RHS}$$

i. e. LHS = RHS

$$\therefore \boxed{\frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta} = \tan\alpha - \tan\beta}$$

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Example 4

Prove that $\cos 3N = 4\cos^3 N - 3\cos N$.

$$
\begin{aligned}
LHS &= \cos 3N \\
&= \cos(2N + N) \\
&= \cos 2N \cos N - \sin 2N \sin N \\
&= (2\cos^2 N - 1)\cos N - (2\sin N \cos N)\sin N \\
&= 2\cos^3 N - \cos N - 2\cos N \sin^2 N \\
&= 2\cos^3 N - \cos N - 2\cos N(1 - \cos^2 N) \\
&= 2\cos^3 N - \cos N - 2\cos N + 2\cos^3 N \\
&= 4\cos^3 N - 3\cos N \\
&= RHS
\end{aligned}
$$

i.e. LHS = RHS

$$\boxed{\therefore \cos 3N = 4\cos^3 N - 3\cos N}$$

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CfE Higher Maths

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