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Solving Trigonometric Equations - Lesson 4

Solving Quadratic Trinomial Trigonometric Equations

LI

- Solve trigonometric equations of the form :

$$a \sin^2 x + b \sin x + c = 0$$
$$a \cos^2 x + b \cos x + c = 0$$
$$a \tan^2 x + b \tan x + c = 0$$

for various ranges of x (in degrees or radians).

SC

- Factorising trinomials.
- Solve linear trig. equations.

Jan 10-12:11

Strategy

- If $f(x)$ is one of $\sin x$, $\cos x$ or $\tan x$, factorise the quadratic in $f(x)$ as a pair of brackets :

$$(L\,f(x) + M)\,(N\,f(x) + P) = 0$$

- Put each bracket equal to 0 and solve the resulting 2 linear trig. equations.

Nov 18-09:07

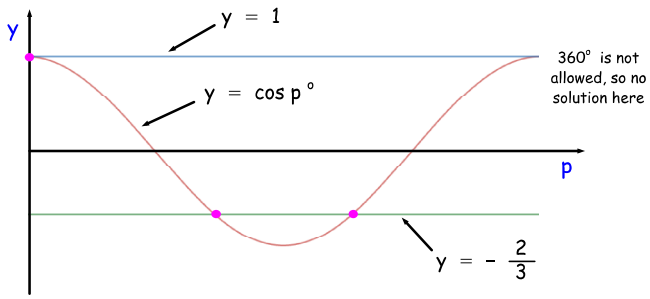
Example 1

Solve $3\cos^2 p^\circ - \cos p^\circ - 2 = 0$
$(0 \le p < 360)$.

$$3\cos^2 p^\circ - \cos p^\circ - 2 = 0$$

$$(3\cos p^\circ + 2)(\cos p^\circ - 1) = 0$$

$$\cos p^\circ = -\frac{2}{3},\ \cos p^\circ = 1$$



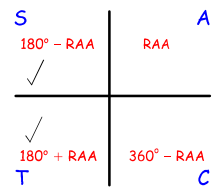
3 solutions expected

$\underline{\cos p^\circ = -\frac{2}{3}}$:

$$RAA = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\Rightarrow \quad \underline{RAA = 48.18\ldots^\circ}$$

cos is − ve



$\therefore \quad p^\circ = 180^\circ - 48.18\ldots^\circ,\ 180^\circ + 48.18\ldots^\circ$

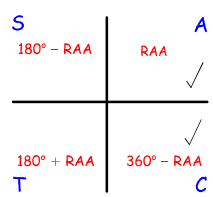
$\Rightarrow \quad \underline{p^\circ = 131.8^\circ,\ 228.2^\circ}$

$\underline{\cos p^\circ = 1}$:

$$RAA = \cos^{-1}(1)$$

$$\Rightarrow \quad \underline{RAA = 0^\circ}$$

cos is + ve



$\therefore \quad p^\circ = 0^\circ,\ 360^\circ - 0^\circ$

$\Rightarrow \quad \underline{p^\circ = 0^\circ}$ 360° is not allowed

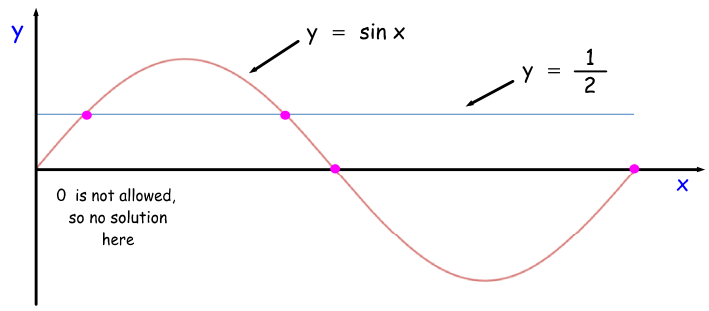
$$\Rightarrow \quad \boxed{p^\circ = 0^\circ,\ 131.8^\circ,\ 228.2^\circ}$$

Mar 3-16:58

Example 2 (non-calculator)

Solve $2\sin^2 x = \sin x$ $(0 < x \leq 2\pi)$.

$$2\sin^2 x = \sin x$$
$$2\sin^2 x - \sin x = 0$$
$$\sin x(2\sin x - 1) = 0$$
$$\sin x = 0, \sin x = \frac{1}{2}$$



4 solutions expected

$\underline{\sin x = 0:}$

$$RAA = \sin^{-1}(0)$$
$$\Rightarrow \quad \underline{RAA = 0}$$

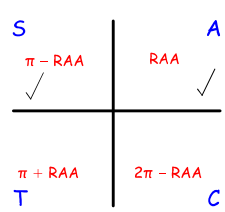
From the graph of $y = \sin x$, there are only 2 solutions :

$$\underline{x = \pi, 2\pi}$$

$\underline{\sin x = \frac{1}{2}:}$

$$RAA = \sin^{-1}\left(\frac{1}{2}\right)$$
$$\Rightarrow \quad \underline{RAA = \pi/6}$$

sin is + ve



$\therefore \quad x = \pi/6, \pi - \pi/6$

$\Rightarrow \quad \underline{x = \pi/6, 5\pi/6}$

$\therefore \quad \boxed{x = \pi/6, 5\pi/6, 2\pi}$

CfE Higher Maths

pg. 180 Ex. 8C Q 1 c, d, 2, 3, a, b

pg. 186 Ex. 8E Q 2, 3

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