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Linear and Parabolic Motion - Lesson 4

Relative Motion

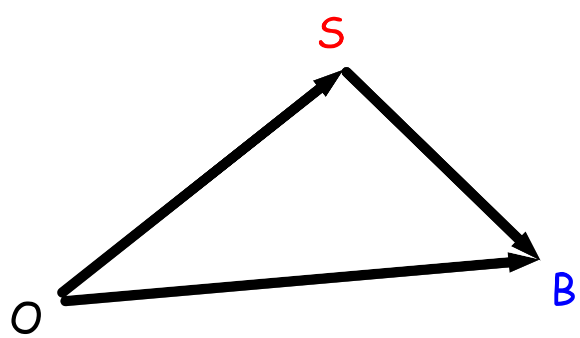
LI

- Calculate relative displacement, velocity and acceleration.

SC

- Diff. and Int. vector functions.
- Sine and Cosine Rules.

Relative Motion



$$\boxed{{}_{B}\underline{\mathbf{v}}_{S} + {}_{S}\underline{\mathbf{v}}_{O} = {}_{B}\underline{\mathbf{v}}_{O}}$$

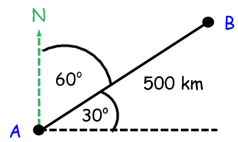
$\boxed{{}_{B}\underline{\mathbf{v}}_{S} \text{ is the relative velocity of } B \text{ wrt } S}$

${}_{B}\underline{\mathbf{v}}_{O}$ is the true velocity of B.

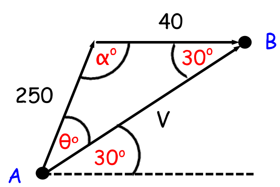
Example 1

Airfields A and B are 500 km apart with B on a bearing of 060° from A. An aircraft which can travel at 250 km/h in still air is to be flown from A to B.

If there is a wind of 40 km/h blowing from the West, find the course the pilot should set to reach B and find, to the nearest minute, the time taken.

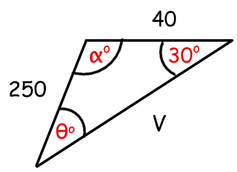


Velocity Triangle :



If the pilot aims to fly straight from A to B, the wind will tend to push the plane to the right of line AB; so, the pilot must aim to fly slightly to the left of AB to go straight from A to B.

Speed Triangle :



$$\frac{\sin \theta^\circ}{40} = \frac{\sin 30^\circ}{250}$$

$$\Rightarrow \quad \sin \theta^\circ = \frac{40 \sin 30^\circ}{250}$$

$$\Rightarrow \quad \sin \theta^\circ = 0.08$$

$$\Rightarrow \quad \theta^\circ = 4.58\ldots^\circ$$

$$\therefore \quad \alpha^\circ = 180^\circ - 30^\circ - 4.58\ldots^\circ$$

$$\Rightarrow \quad \alpha^\circ = 145.41\ldots^\circ$$

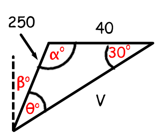
$$\frac{V}{\sin \alpha^\circ} = \frac{250}{\sin 30^\circ}$$

$$\Rightarrow \quad V = \frac{250 \sin \alpha^\circ}{\sin 30^\circ}$$

$$\Rightarrow \quad V = \frac{250 \sin 145.41\ldots^\circ}{\sin 30^\circ}$$

$$\Rightarrow \quad V = 283.83\ldots$$

$$\therefore \quad V = 284 \text{ (nearest km/h)}$$



$$\beta^\circ = 90^\circ - 30^\circ - 4.58\ldots^\circ$$

$$\Rightarrow \quad \beta^\circ = 55.4^\circ$$

Course : 284 km/h on a bearing of 055°

$$t_{AB} = \frac{500}{283.83\ldots} = 1.76\ldots \text{ h}$$

$$\therefore \quad \boxed{t_{AB} = 1 \text{ h } 46 \text{ min}}$$

Example 2

A river flows at speed u m/s. How long will a motor-boat whose speed in still water is v m/s $(v > u)$ take to complete a return journey to a point a distance d metres upstream
(i.e. there and back).

Upstream means against the river direction flow;
downstream means in the same direction as the river flow.

RIVER FLOW DIRECTION

$$\longrightarrow \quad +$$

Taking the river flow direction to be positive to the right, we have, where B stands for boat, R for river and G for ground,

Upstream

Net direction of boat travel is to the left.

$$_B\underline{\mathbf{v}}_G = {}_B\underline{\mathbf{v}}_R + {}_R\underline{\mathbf{v}}_G$$

$$_B\underline{\mathbf{v}}_G = -v\,\underline{\mathbf{i}} + u\,\underline{\mathbf{i}}$$

$$_B\underline{\mathbf{v}}_G = (u - v)\,\underline{\mathbf{i}}$$

$$_B v_G = v - u$$

$$t_{UP} = \frac{d}{v - u}$$

Downstream

Net direction of boat travel is to the right.

$$_B\underline{\mathbf{v}}_G = {}_B\underline{\mathbf{v}}_R + {}_R\underline{\mathbf{v}}_G$$

$$_B\underline{\mathbf{v}}_G = v\,\underline{\mathbf{i}} + u\,\underline{\mathbf{i}}$$

$$_B\underline{\mathbf{v}}_G = (v + u)\,\underline{\mathbf{i}}$$

$$_B v_G = v + u$$

$$t_{DOWN} = \frac{d}{v + u}$$

If T is the total time to go upstream and downstream, we have,

$$T = t_{UP} + t_{DOWN}$$

$$\Rightarrow \quad T = \frac{d}{v - u} + \frac{d}{v + u}$$

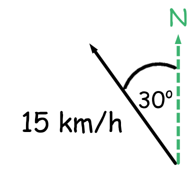
$$\Rightarrow \quad T = \frac{dv + du + dv - du}{(v - u)(v + u)}$$

$$\Rightarrow \quad \boxed{T = \frac{2dv}{(v^2 - u^2)}}$$

Example 3

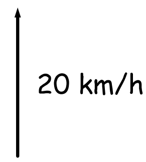
What is the velocity of a cruiser moving at 20 km/h due North as seen by an observer on a liner moving at 15 km/h in a direction 30° W of N ?

$_{L}\underline{\mathbf{V}}_{S}$

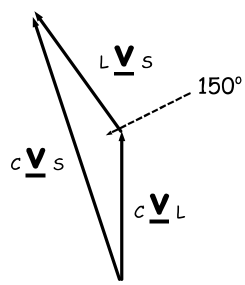


C : Cruiser
L : Liner
S : Sea

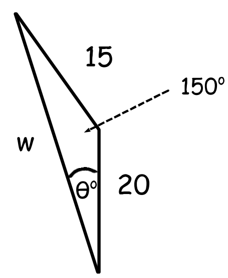
$_{C}\underline{\mathbf{V}}_{L}$



Velocity Triangle :



Speed Triangle :



$$_{C}\underline{\mathbf{V}}_{S} = {}_{C}\underline{\mathbf{V}}_{L} + {}_{L}\underline{\mathbf{V}}_{S}$$

$$w^2 = 15^2 + 20^2 - (2 \times 15 \times 20 \times \cos 150°)$$

$$\Rightarrow \quad w^2 = 1\,144\,.\,61\ldots$$

$$\Rightarrow \quad \underline{w = 33\,.\,83\ldots}$$

$$\frac{\sin \theta°}{15} = \frac{\sin 150°}{33\,.\,83\ldots}$$

$$\Rightarrow \quad \sin \theta° = \frac{15 \sin 150°}{33\,.\,83\ldots}$$

$$\Rightarrow \quad \sin \theta° = 0\,.\,22\ldots$$

$$\Rightarrow \quad \underline{\theta° = 12\,.\,8\ldots°}$$

The velocity of the liner (relative to the sea) is 33 . 8 km/h in a direction 12 . 8° W of N

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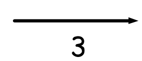
Example 4

To a cyclist riding at 3 m/s due East, the wind appears to come from the South with speed $3\sqrt{3}$ m/s.

Find the true speed and direction of the wind.

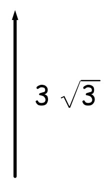
C : Cyclist
W : Wind
G : Ground

${}_{C}\underline{\mathbf{V}}_{G}$



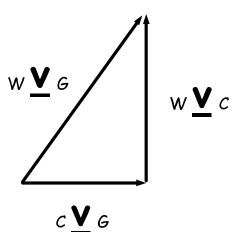
$${}_{C}\underline{\mathbf{V}}_{G} = 3\,\underline{\mathbf{i}}$$

${}_{W}\underline{\mathbf{V}}_{C}$

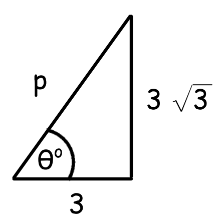


$${}_{W}\underline{\mathbf{V}}_{C} = 3\sqrt{3}\,\underline{\mathbf{j}}$$

Velocity Triangle :



Speed Triangle :

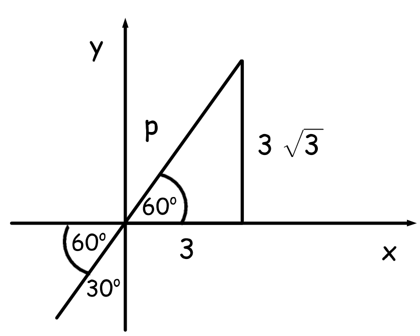


$$p^2 = 9 + 27$$

$$\Rightarrow \quad \underline{p = 6}$$

$$\theta° = \tan^{-1}\sqrt{3}$$

$$\Rightarrow \quad \underline{\theta° = 60°}$$



The true speed and direction of the wind is 6 m/s coming from a direction 30° W of S

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Blue Book

- pg. 223-224 Ex. 10 A Q 6, 9, 11, 13, 15.
- pg. 229-231 Ex. 10 B Q 3, 4, 6, 10, 12, 14, 15, 18, 19, 20, 22.