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Graphs of Related Functions - Lesson 4

Quadratic Graphs

LI

- State the maximum or minimum turning point coordinates of a quadratic using completed the square form.
- Sketch quadratic graphs, especially using completed the square form.
- Find the equation of a quadratic given its graph and 2 coordinates.

SC

- Complete the Square.

Reminder on Completed the Square Form

Any quadratic can be written in completed the square form :

$$P(x + Q)^2 + R$$

The quadratic then has turning point coordinates $(-Q, R)$ and symmetry axis equation $x = -Q$.

- Minimum TP if $P > 0$.
- Maximum TP if $P < 0$.

To sketch the graph of $y = P(x + Q)^2 + R$:

- $y = x^2$ is x - translated to $y = (x + Q)^2$.
- $y = (x + Q)^2$ is y - scaled to $y = P(x + Q)^2$.
- $y = P(x + Q)^2$ is y - translated to $y = P(x + Q)^2 + R$.

Example 1

Write down the coordinates of the turning point of $y = 3(7 - x)^2 - 4$ and state if it is a maximum or a minimum.

Comparing with the general form, we have,

$$3(7 - x)^2 - 4 = 3(x - 7)^2 - 4$$
$$= P(x + Q)^2 + R$$

$$\therefore \quad \underline{P = 3,\ Q = -7,\ R = -4}$$

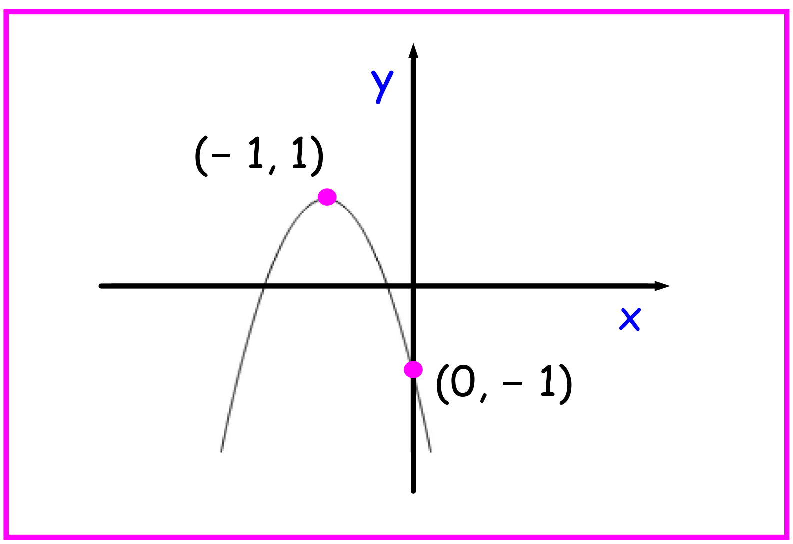
Turning point is $(7, -4)$ and it is a minimum, as $P = 3 > 0$.

Example 2

Sketch the graph of $y = 1 - 2(x + 1)^2$, showing the turning point coordinates and the y - intercept.

The TP is $(-1, 1)$ and it is a maximum, as $P = -2 < 0$.

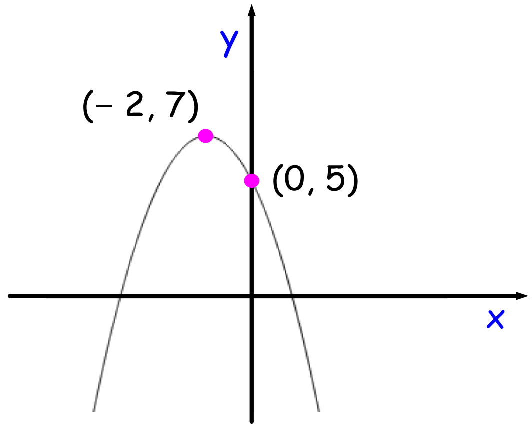
To get the y - intercept, put $x = 0$: $y = -1$.



Or, start with $y = x^2$, shift it 1 unit to the left, then y - scale by -2, then y - translate 1 unit up the y - axis.

Example 3

Find the equation of the following quadratic graph :



Since every quadratic can be written in completed the square form, the general equation is,

$$y = P(x + Q)^2 + R$$

The TP is $(-2, 7)$, so, $\underline{Q = 2 \text{ and } R = 7.}$

So,

$$y = P(x + 2)^2 + 7$$

Using the other coordinate $(0, 5)$ in this last equation gives,

$$5 = P(0 + 2)^2 + 7$$

$$\Rightarrow \quad 4P = -2$$

$$\Rightarrow \quad \underline{P = -1/2}$$

$$\therefore \quad \boxed{y = -\frac{1}{2}(x + 2)^2 + 7}$$

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Example 4

Write $y = 5 + 8x - x^2$ in the form $p - q(x + r)^2$; hence state the minimum value of

$$\frac{1}{5 + 8x - x^2}$$

Completing the square gives,

$$\begin{aligned} & 5 + 8x - x^2 \\ &= -x^2 + 8x + 5 \\ &= -(x^2 - 8x) + 5 \\ &= -[(x - 4)^2 - 16] + 5 \\ &= -(x - 4)^2 + 16 + 5 \\ &= \boxed{21 - (x - 4)^2} \end{aligned}$$

The expression $21 - (x - 4)^2$ has a maximum value of 21; hence,

$$\boxed{\frac{1}{5 + 8x - x^2} \text{ has a minimum value of } \frac{1}{21}}$$

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