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Graphs of Related Functions - Lesson 4

Quadratic Graphs

**LT**
- State the maximum or minimum turning point coordinates of a quadratic using completed the square form.
- Sketch quadratic graphs, especially using completed the square form.
- Find the equation of a quadratic given its graph and 2 coordinates.

**SC**
- Complete the Square.
Reminder on Completed the Square Form

Any quadratic can be written in completed the square form:

\[ P(x + Q)^2 + R \]

The quadratic then has turning point coordinates \((- Q, R)\) and symmetry axis equation \(x = -Q\).

- Minimum TP if \(P > 0\).
- Maximum TP if \(P < 0\).
To sketch the graph of \( y = P(x + Q)^2 + R \):

- \( y = x^2 \) is \( x \)-translated to \( y = (x + Q)^2 \).
- \( y = (x + Q)^2 \) is \( y \)-scaled to \( y = P(x + Q)^2 \).
- \( y = P(x + Q)^2 \) is \( y \)-translated to \( y = P(x + Q)^2 + R \).
Example 1

Write down the coordinates of the turning point of \( y = 3 (7 - x)^2 - 4 \) and state if it is a maximum or a minimum.

Comparing with the general form, we have,

\[
3 (7 - x)^2 - 4 = 3 (x - 7)^2 - 4 = P (x + Q)^2 + R
\]

\[
\therefore \quad P = 3, \quad Q = -7, \quad R = -4
\]

Turning point is \((7, -4)\) and it is a minimum, as \(P = 3 > 0\).
Example 2

Sketch the graph of $y = 1 - 2(x + 1)^2$, showing the turning point coordinates and the $y$-intercept.

The TP is $(-1, 1)$ and it is a maximum, as $P = -2 < 0$.

To get the $y$-intercept, put $x = 0$: $y = -1$.

Or, start with $y = x^2$, shift it 1 unit to the left, then $y$-scale by $-2$, then $y$-translate 1 unit up the $y$-axis.
Example 3

Find the equation of the following quadratic graph:

Since every quadratic can be written in completed the square form, the general equation is,

\[ y = P(x + Q)^2 + R \]

The TP is \((-2, 7)\), so, \(Q = 2\) and \(R = 7\).

So,

\[ y = P(x + 2)^2 + 7 \]

Using the other coordinate \((0, 5)\) in this last equation gives,

\[ 5 = P(0 + 2)^2 + 7 \]

\[ \Rightarrow 4P = -2 \]

\[ \Rightarrow P = -\frac{1}{2} \]

\[ \therefore y = -\frac{1}{2}(x + 2)^2 + 7 \]
Example 4

Write \( y = 5 + 8x - x^2 \) in the form \( p - q(x + r)^2 \); hence state the minimum value of

\[
\frac{1}{5 + 8x - x^2}
\]

Completing the square gives,

\[
5 + 8x - x^2
\]

\[
= -x^2 + 8x + 5
\]

\[
= -(x^2 - 8x) + 5
\]

\[
= -[(x - 4)^2 - 16] + 5
\]

\[
= -(x - 4)^2 + 16 + 5
\]

\[
= 21 - (x - 4)^2
\]

The expression \( 21 - (x - 4)^2 \) has a maximum value of 21; hence,

\[
\frac{1}{5 + 8x - x^2} \text{ has a minimum value of } \frac{1}{21}
\]
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