Polynomials - Lesson 4

Finding Polynomial Roots

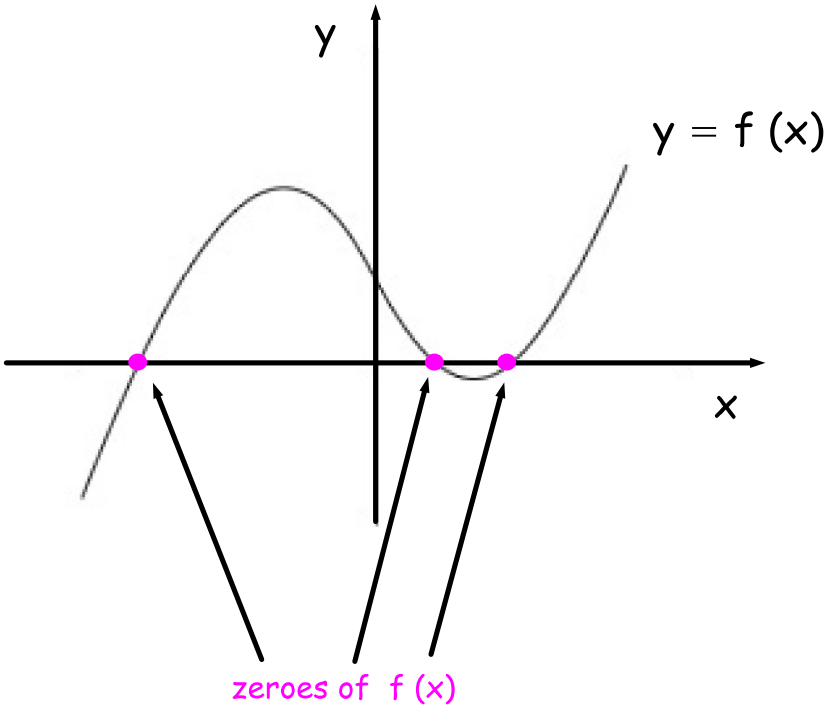
LI

- Find roots of polynomials.

SC

- Fully factorise a polynomial.

A root (aka zero) of $f(x)$ is a solution of the equation $f(x) = 0$



Strategy for Finding Polynomial Roots

- Fully factorise polynomial.
- Solve $f(x) = 0$.

Example 1

Show that $x = 3$ is a root of the polynomial $f(x) = x^3 - 4x^2 + x + 6$ and hence find all the other roots.

	x^3	x^2	x^1	x^0
3	1	− 4	1	6
		3	− 3	− 6
	1	− 1	− 2	0

As $R = 0$, by the Factor Theorem, $(x - 3)$ is a factor of $f(x)$. So,

$$f(x) = (x - 3)(x^2 - x - 2)$$

$$\Rightarrow \quad f(x) = (x - 3)(x - 2)(x + 1)$$

For roots, $f(x) = 0$:

$$(x - 3)(x - 2)(x + 1) = 0$$

$$\Rightarrow \quad x - 3 = 0, \quad x - 2 = 0, \quad x + 1 = 0$$

$$\Rightarrow \quad \boxed{x = 3, \quad x = 2, \quad x = -1}$$

Example 2

Find the value of k given that $x = 4$ is a root of $f(x) = x^3 - 3x^2 + kx - 4$ and hence find all the roots.

	x^3	x^2	x^1	x^0
4	1	− 3	k	− 4
		4	4	4 k + 16
	1	1	k + 4	4 k + 12

As $x = 4$ is a root, the Factor Theorem implies that the remainder equals 0. So,

$$4k + 12 = 0$$

$$\Rightarrow \quad \boxed{k = -3}$$

Hence,

$$f(x) = (x - 4)(x^2 + x + 1)$$

For roots, $f(x) = 0$; hence,

$$(x - 4)(x^2 + x + 1) = 0$$

$$\Rightarrow \quad x - 4 = 0, \quad x^2 + x + 1 = 0$$

The quadratic has discriminant equal to − 3; hence, the quadratic has no real roots. So, the only root of $f(x)$ is,

$$\boxed{x = 4}$$

CfE Higher Maths

pg. 163 - 4 Ex. 7I Q 2 a, 3 e, 4, 9.

Questions - Ex. 7I

2 Solve each polynomial equation.

a $3x^3 + 9x^2 - 3x - 9 = 0$

3 Solve these polynomial equations.

e $x^4 + 2x^3 - 21x^2 + 5x + 8 = x^3 + 6x - 12$

4 Given that the equation $2x^3 + 3x^2 + ax - 2 = 0$ has a root at $x = 1$, determine the value of a and solve for x.

9 Valentine's cookies are packed in gift boxes in the shape of a square based cuboid. The height of the box is 4 cm greater than its width and the volume of the box is $128\,\text{cm}^3$. Determine the dimensions of the box.

Answers

2 **a** $x = -3, -1, 1$

3 **e** $x = -5, -1, 1, 4$

4 $a = -3$

$x = -2, -\frac{1}{2}, 1$

9 $4 \times 4 \times 8$