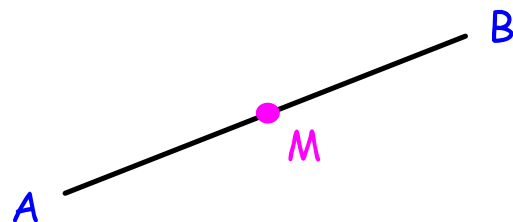## 24 / 3 / 16

Straight Lines - Lesson 4

Medians, Perpendicular Bisectors and Altitudes

<u>LI</u>

- Calculate equations of medians, perpendicular bisectors and altitudes.

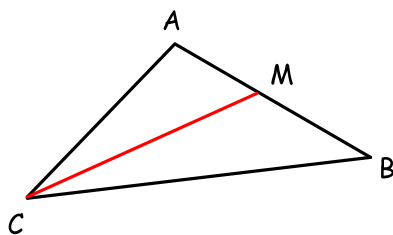- Know the concurrency properties of the above lines.

<u>SC</u>

- Midpoint formula.
- Straight line equation.

Jan 10-12:11

1

B

M

A

To work out the coordinates of the halfway point (aka midpoint)
on a line joining 2 points A (x_1, y_1) and B (x_2, y_2),
use the midpoint formula :

$$\text{Midpoint M} = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

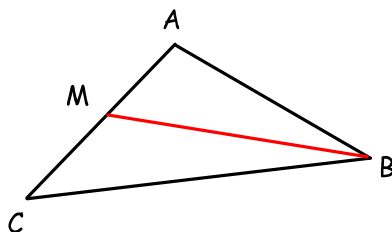Jun 21-11:21

Medians

A

M

B

C

From a vertex of a triangle
to midpoint of opposite side

- Use A and B to get midpoint M.
- Then use C and M to get gradient and equation of median.

<u>Example 1</u>

Find the equation of the median from B in the
triangle with vertices A (2, 1), B (5, − 3)
and C (0, 3).

A

M

B

C

$$\text{Midpoint M} = \left(\frac{2 + 0}{2} , \frac{1 + 3}{2} \right)$$

$$\therefore \qquad M (1, 2)$$

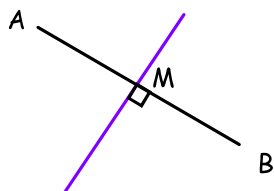$$m_{BM} = \frac{-3 - 2}{5 - 1} = \frac{-5}{4} \Rightarrow m_{BM} = -\frac{5}{4}$$

$$y - b = m (x - a)$$

$$y - (-3) = -\frac{5}{4} (x - 5)$$

$$4y + 12 = -5 (x - 5)$$

$$4y + 12 = -5x + 25$$

$$5x + 4y = 13$$

Jun 5-12:32
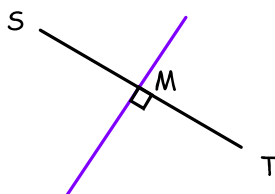
3

Perpendicular Bisectors

Bisects a line at right angles

- Use A and B to get midpoint M and gradient of AB.
- Then use perp. gradients to get gradient of perp. bisector; then equation of perp. bisector.

Example 2

Find the equation of the perpendicular bisector of the line joining S (3, 8) and T (4, – 4).

$$\text{Midpoint M} = \left(\frac{3 + 4}{2} , \frac{8 + (-4)}{2} \right)$$

$$\therefore \quad M \left(\tfrac{7}{2} , 2 \right)$$

$$m_{ST} = \frac{-4 - 8}{4 - 3} = \frac{-12}{1} \Rightarrow m_{ST} = -12$$
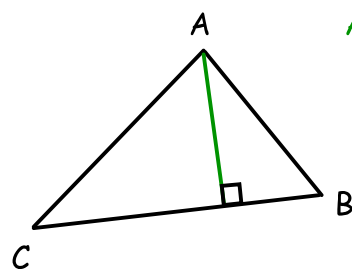
$$\therefore \quad m_{PERP.} = \frac{1}{12}$$

$$y - b = m(x - a)$$

$$y - 2 = \frac{1}{12}(x - \tfrac{7}{2})$$

$$12y - 24 = x - \tfrac{7}{2}$$

$$24y - 48 = 2x - 7$$

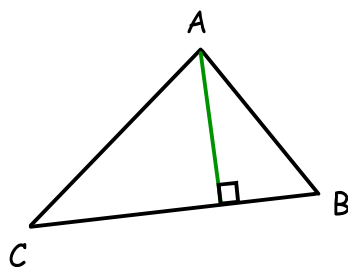$$\boxed{2x - 24y = -41}$$

Jun 5-12:32

4

A   Altitudes

B

C

From a vertex to opposite
side, cutting at right angles

- Use C and B to get gradient of line BC.
- Then use perp. gradients to get gradient of altitude;
  then equation of altitude.

Example 3

Find the equation of the altitude from A in the
triangle with vertices A $(2, -4)$, B $(-2, -8)$
and C $(5, 1)$.

A

B

C

$$m_{BC} = \frac{1 - (-8)}{5 - (-2)} = \frac{9}{7} \Rightarrow m_{BC} = \frac{9}{7}$$

$$\therefore \quad m_{PERP.} = -\frac{7}{9}$$

$$y - b = m(x - a)$$

$$y - (-4) = -\frac{7}{9}(x - 2)$$

$$9y + 36 = -7(x - 2)$$

$$9y + 36 = -7x + 14$$

$$\boxed{7x + 9y = -22}$$

Jun 5-12:32

Type of Line	What u need 2 work out the equation
Median	Midpoint of opposite side and opposite vertex
Perp. Bis.	Midpoint of given line and gradient perp. to given line
Altitude	Gradient perp. to opposite side and opposite vertex

Jun 24-09:51

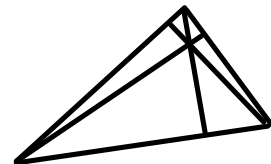Concurrent means meeting at a common point

- The 3 medians of a triangle are
  concurrent at the centroid.
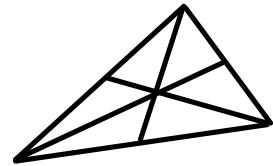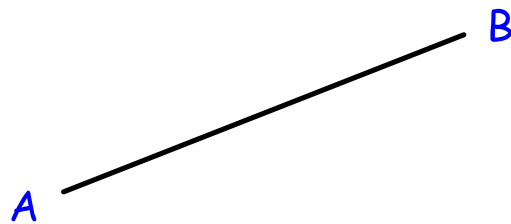
- The 3 perpendicular bisectors
  of a triangle are concurrent
  at the circumcentre.
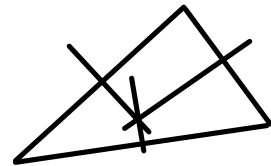
- The 3 altitudes of a triangle are
  concurrent at the orthocentre.

Jun 20-17:09

B

A

To work out the distance between 2 points A (x_1, y_1) and B (x_2, y_2), use the distance formula :

$$d_{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Jun 26-11:42

CfE Higher Maths

pg. 304 - 5 Ex. 13F All Q

Oct 29-09:14