8 / 11 / 17

Unit 2 : Sequences and Series - Lesson 4

Finite Geometric Series

LI

- Know what a Finite Geometric Series is.
- Find the n^{th} term formula for a finite geometric series.
- Solve problems involving finite geometric series.

SC

- Arithmetic of real numbers.

A finite series is obtained by adding finitely many terms of a sequence

A finite geometric series (up to n terms) is obtained by adding the first n terms of a geometric sequence

The sum to n terms of a geometric sequence is :

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad (r \neq 1)$$

Oct 9-15:09

Example 1

Find, to 4 s.f., the sum to eight terms of the geometric sequence that starts 5, 6, 36/5, ...

We have $a = 5$ and $r = 6/5$.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\therefore \quad S_8 = \frac{5(1 - (6/5)^8)}{1 - (6/5)}$$

$$\Rightarrow \quad S_8 = 82.495\ldots$$

$$\therefore \quad \boxed{S_8 = 82.50 \text{ (4 s.f.)}}$$

Oct 2-23:10

Example 2

A geometric sequence begins 4, 6, 9, ...

Find the smallest value of n for which $S_n > 200$.

We have $a = 4$ and $r = 6/4 \Rightarrow r = 3/2$.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\therefore \quad S_n = \frac{4(1 - (3/2)^n)}{1 - (3/2)}$$

$$\Rightarrow \quad \underline{S_n = 8((3/2)^n - 1)}$$

$$S_n > 200$$

$$\therefore \quad 8((3/2)^n - 1) > 200$$

$$\Rightarrow \quad (3/2)^n - 1 > 25$$

$$\Rightarrow \quad (3/2)^n > 26$$

$$\therefore \quad n \ln(3/2) > \ln 26$$

$$\Rightarrow \quad n > (\ln 26)/(\ln(3/2))$$

$$\Rightarrow \quad n > 8.035\ldots$$

$$\therefore \quad \boxed{n = 9}$$

Oct 2-23:10

Example 3

Show that the geometric sequence that starts 12, 3, 3/4, . . . has sum to n terms given by $S_n = 16\,p(n)$, stating explicitly the function $p(n)$.

If $S_n = 63/4$, find n.

We have $a = 12$ and $r = 3/12 \Rightarrow r = 1/4$.

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\therefore \quad S_n = \frac{12(1 - (1/4)^n)}{1 - (1/4)}$$

$$\Rightarrow \quad S_n = \frac{12(1 - (1/4^n))}{3/4}$$

$$\Rightarrow \quad \boxed{\begin{array}{l} S_n = 16(1 - (1/4^n)) \\ (p(n) = 1 - (1/4^n)) \end{array}}$$

$$S_n = 63/4$$

$$\therefore \quad 16(1 - (1/4^n)) = 63/4$$

$$\Rightarrow \quad 1 - (1/4^n) = 63/64$$

$$\Rightarrow \quad 1/4^n = 1/64$$

$$\Rightarrow \quad 4^n = 64$$

$$\Rightarrow \quad \boxed{n = 3}$$

Oct 2-23:10

AH Maths - MiA (2nd Edn.)

- pg. 159-161 Ex. 9.4 Q 1, 2 a, b, 3, 4, 6, 7, 9.

Feb 10-15:47

Ex. 9.4

1 Find the sum of each geometric sequence to the required number of terms.

a 3, 6, 12, ... to eight terms

b 5, 20, 80, ... to seven terms

c 4, −12, 36, ... to 10 terms

d 3, 12, 48, ... to six terms

e 2, −4, 8, ... to 12 terms

f −3, 6, −12, ... to 10 terms

2 Evaluate each geometric series to the number of specified terms.

a $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ to eight terms

b $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$ to nine terms

3 a How many terms of the series 8 + 24 + 72 + ... must be added to get a sum of 26 240?

b At which term does the sum 1.5 + 6 + 24 + ... exceed one million?

4 A geometric series has a common ratio of 3. Its sum to eight terms is 39 360.

a Calculate the first term.

b Calculate the sum to six terms.

6 a The sum of the first three terms of a GP is 744. The sum of the next three is 93 000. Find the series.

b Find the sum of the seventh, eighth and ninth terms.

7 a, $a - 12$, $a + 12$ are the first three terms of a geometric sequence.

a What is the value of a?

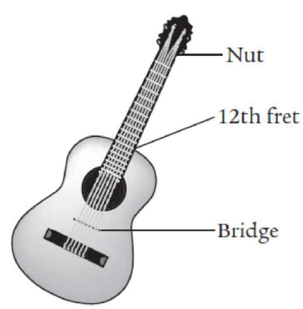
b Calculate the sum of the first 10 terms.

9 When making a guitar, the spacings between the frets on the neck are mathematically fixed.

Each spacing is $\frac{17}{18}$ of the previous spacing.

a If the first spacing (between the nut and the first fret) is 4 cm, calculate the distance between the sixth and ninth frets.

b If the 12th fret is placed half way between the nut and the bridge, what is the distance between the nut and the bridge?



Answers to AH Maths (MiA), pg. 159-161, Ex. 9.4

1	a	765	b	27 305	c	−59 048
	d	4095	e	−2730	f	1023
2	a	$\frac{255}{256}$	b	$\frac{171}{256}$		
3	a	8	b	11th		
4	a	12	b	4368		
6	a	$a = 24, r = 5$	b	11 625 000		
7	a	4	b	−1364		
9	a	8·05 cm	b	71·48 cm (2 dp)		