Differential Calculus - Lesson 4

Equations of Tangent Lines

LI

- Find equations of tangent lines to curves.

SC

- Differentiation.
- Straight line equation.

To find the equation of a tangent line to a curve $y = f(x)$ at $x = p$:

- Differentiate the function $y = f(x)$.
- Work out this derivative at $x = p$; this *gives* m, the gradient of the tangent line at $x = p$.
- Get the y - coordinate by substituting $x = p$ into $y = f(x)$.
- Use $y - b = m(x - a)$ $(a = p$ and $b = f(p))$ to get the tangent line equation.

Example 1

Find the equation of the tangent to the curve $y = x^2 - 3x + 5$ at the point $(1, 3)$.

$$y(x) = x^2 - 3x + 5$$

$$\therefore \quad y'(x) = 2x - 3$$

$$\therefore \quad y'(1) = 2(1) - 3$$

$$\Rightarrow \quad \underline{y'(1) = -1}$$

$$y(x) = x^2 - 3x + 5$$

$$\therefore \quad y(1) = (1)^2 - 3(1) + 5$$

$$\Rightarrow \quad y(1) = 1 - 3 + 5$$

$$\Rightarrow \quad \underline{y(1) = 3}$$

$$y - b = m(x - a)$$

$$\boxed{\begin{array}{c} m = -1 \\ (\underset{1}{a}, \underset{3}{b}) \end{array}}$$

$$\therefore \quad y - 3 = -1(x - 1)$$

$$\Rightarrow \quad y - 3 = -x + 1$$

$$\Rightarrow \quad \boxed{y = -x + 4}$$

Example 2

At point Q, the tangent to $y = x^2 - 3x + 4$ has gradient 7.

Find the equation of the tangent at Q.

$$y(x) = x^2 - 3x + 4$$

$$\therefore \quad y'(x) = 2x - 3$$

But $y'(x) = 7$. So,

$$2x - 3 = 7$$

$$\Rightarrow \quad \underline{x = 5}$$

$$y(x) = x^2 - 3x + 4$$

$$\therefore \quad y(5) = (5)^2 - 3(5) + 4$$

$$\Rightarrow \quad y(5) = 25 - 15 + 4$$

$$\Rightarrow \quad \underline{y(5) = 14}$$

$$y - b = m(x - a)$$

$m = 7$

(a, b)
$\ \ 5 \ \ 14$

$$\therefore \quad y - 14 = 7(x - 5)$$

$$\Rightarrow \quad y - 14 = 7x - 35$$

$$\Rightarrow \quad \boxed{y = 7x - 21}$$

Example 3

If the gradient of the tangent to the curve $y = kx^2 + x - 5$ at $x = 1$ is 5, find k and find the equation of the tangent to the curve at $x = 1$.

$$y(x) = kx^2 + x - 5$$

$$\therefore \quad y'(x) = 2kx + 1$$

At $x = 1$, $y'(x) = 5$. So,

$$2k(1) + 1 = 5$$

$$\Rightarrow \quad \boxed{k = 2}$$

$$y(x) = 2x^2 + x - 5$$

$$\therefore \quad y(1) = 2(1)^2 + (1) - 5$$

$$\Rightarrow \quad y(1) = 2 + 1 - 5$$

$$\Rightarrow \quad \underline{y(1) = -2}$$

$$y - b = m(x - a)$$

$$\boxed{\begin{array}{c} m = 5 \\ (a, b) \\ 1 \quad -2 \end{array}}$$

$$\therefore \quad y - (-2) = 5(x - 1)$$

$$\Rightarrow \quad y + 2 = 5x - 5$$

$$\Rightarrow \quad \boxed{y = 5x - 7}$$

CfE Higher Maths

pg. 240 - 243 Ex. 10A

Q 1, 3 a, b, 5 a, c, 6, 12

Questions

1 For each of these functions, find the equation of the tangent at the given point.

a $y = x^2 + 3x + 6$; $(1, 10)$
b $y = x^2 - 6x + 4$; $(3, -5)$
c $y = 2x^2 + 3x - 5$; $(-1, -6)$
d $y = x^3 + 3x^2 - 2x + 4$; $(2, 20)$
e $y = x^3 - 2x$; $(-3, -21)$
f $y = 4 - 3x - x^2$ $(2, -6)$

3 For each of these functions, find the equation of the tangent at the given point

a $y = x^2 + 5x + 1$; $x = 1$
b $y = x^2 - x + 6$; $x = 3$

5 a A curve has equation $y = x^2 + 4x - 6$. At P, the gradient of the curve is -6. Find the equation of the tangent to the curve at P.

c A curve has equation $y = 5x^2 - x + 3$. At R, the tangent to the curve has gradient $= -6$. Find the equation of the curve at R.

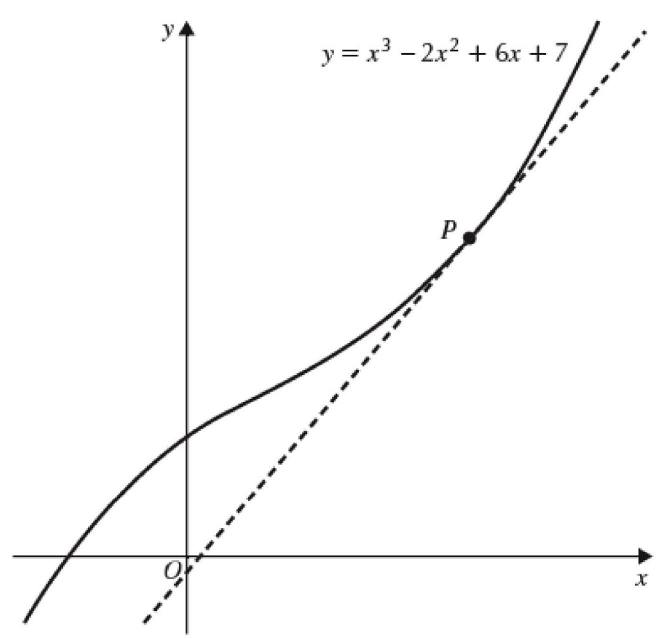
6 For each of these functions, find the equation of the tangent at the given point. **You will need to express each function in differentiable form first.**

a $y = (x - 2)(x + 5)$; $x = 4$
b $y = 2x^2(x - 3)$; $x = -1$
c $y = (x - 2)(x^2 + x - 1)$; $x = 1$
d $y = x(x + 2)^2$; $x = -2$
e $y = \dfrac{4x - 1}{x}$; $x = 2$
f $y = 4\sqrt{x}$; $x = 36$

12 An old road can be represented by the curve with equation $x^3 - 2x^2 + 6x + 7$ as shown in the diagram.

A new road is represented by the dashed line. The line is a tangent to the curve at the point P, where $x = 2$.

Find the equation of the dashed line.



Answers

1 **a** $y = 5x + 5$

b $y = -5$

c $y = -x - 7$

d $y = 22x - 24$

e $y = 25x + 54$

f $y = -7x + 8$

3 **a** $y = 7x$

b $y = 5x - 3$

5 **a** $y = -6x - 31$

c $y = -6x + \frac{7}{4}$

6 **a** $y = 11x - 26$

b $y = 18x + 10$

c $y = -2x + 1$

d $y = 0$

e $y = \frac{1}{4}x + 3$

f $y = \frac{1}{3}x + 12$

12 $y = 10x - 1$