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Applications of Calculus - Lesson 4

Area Under a Curve

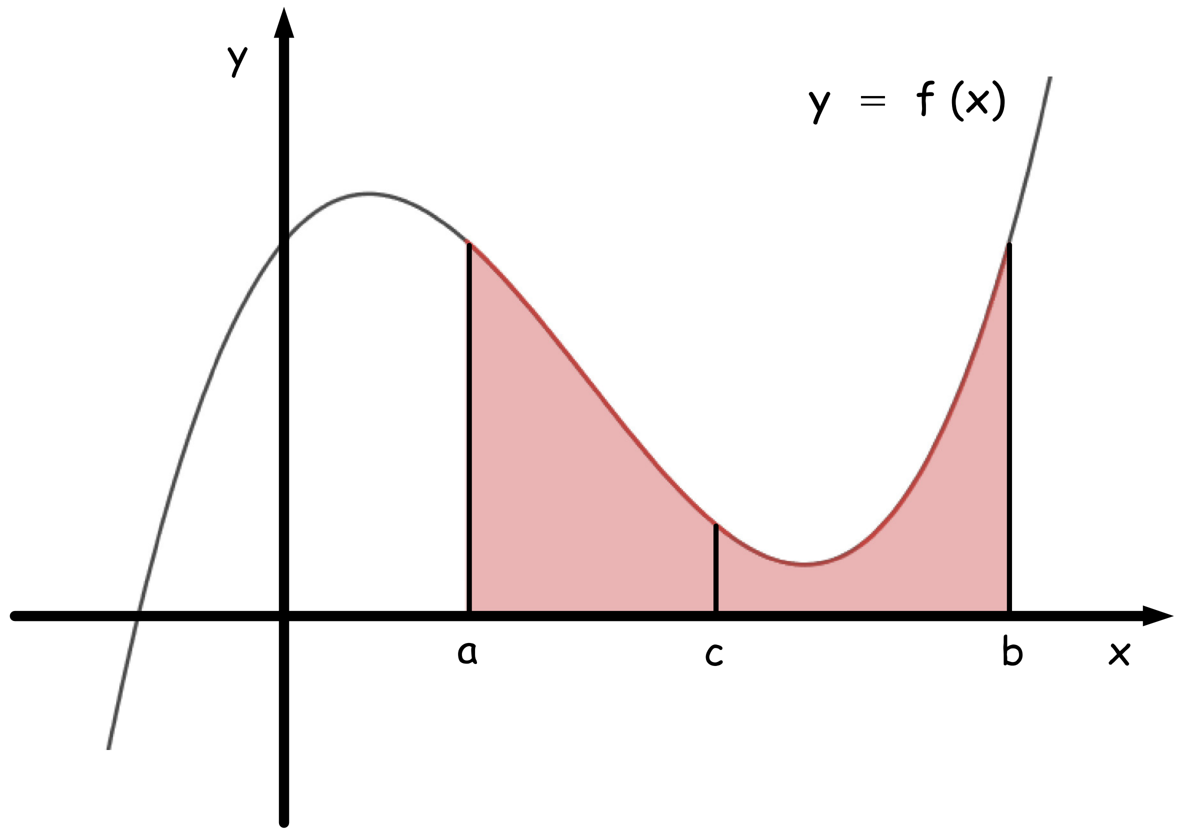
LI

- Calculate the area under a curve.

SC

- Definite Integration.

Consider the area A bounded by a curve $y = f(x)$, the x - axis and the points $x = a$ and $x = b$ (shown below by the shaded area), where the curve is above the x - axis between $x = a$ and $x = b$, but with a point c between a and b :

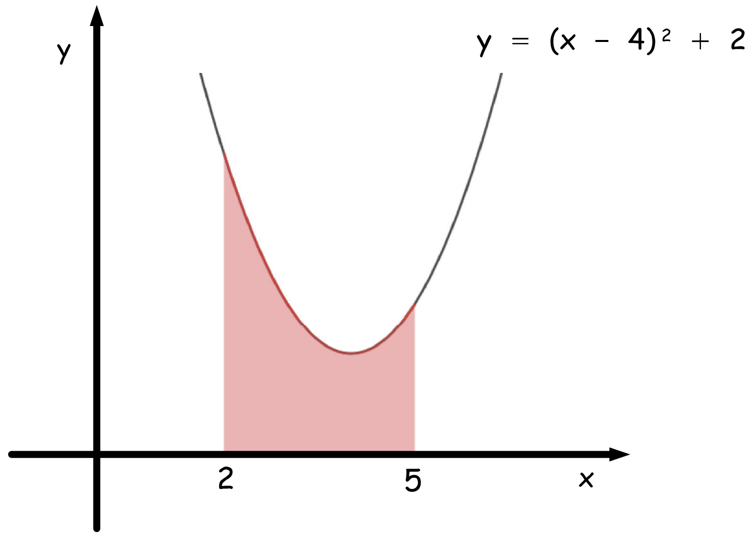


The total area between a and b is clearly given by the area between a and c plus the area between c and b :

$$A = \int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$$

Example 1 (Non-Calc)

Find the following shaded area :



$$A = \int_2^5 (x - 4)^2 + 2 \; dx$$

$$= \int_2^5 (x^2 - 8x + 18) \; dx$$

$$= \left[\frac{x^3}{3} - 4x^2 + 18x \right]_2^5$$

$$= \left(\frac{5^3}{3} - 4(5)^2 + 18(5) \right)$$

$$- \left(\frac{2^3}{3} - 4(2)^2 + 18(2) \right)$$

$$= \frac{125 - 8}{3} - 100 + 90 + 16 - 36$$

$$= 39 - 30$$

$$= \boxed{9 \text{ square units}}$$

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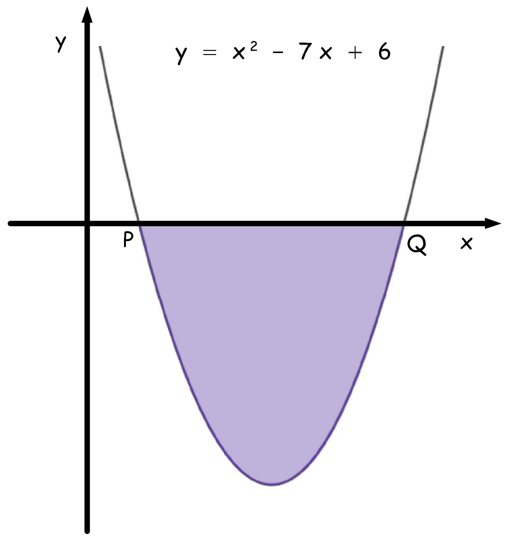
Example 2 (Non-Calc)

Find the following shaded area :



The values of P and Q must first be found :

$$x^2 - 7x + 6 = 0$$
$$(x - 1)(x - 6) = 0$$
$$\underline{x = 1, 6}$$

So, P (1, 0) and Q (6, 0). Hence,

$$A = -\int_1^6 (x^2 - 7x + 6)\ dx$$

$$= \left[\frac{x^3}{3} - \frac{7x^2}{2} + 6x\right]_1^6$$

$$= -\left(\frac{6^3}{3} - \frac{7(6)^2}{2} + 6(6)\right)$$
$$+ \left(\frac{1^3}{3} - \frac{7(1)^2}{2} + 6(1)\right)$$

$$= \frac{1 - 216}{3} + \frac{252 - 7}{2} - 36 + 6$$

$$= -\frac{215}{3} + \frac{245}{2} - \frac{30}{1}$$

$$= \frac{-430 + 735 - 180}{6}$$

$$= \boxed{\frac{125}{6}\text{ units}^2}$$

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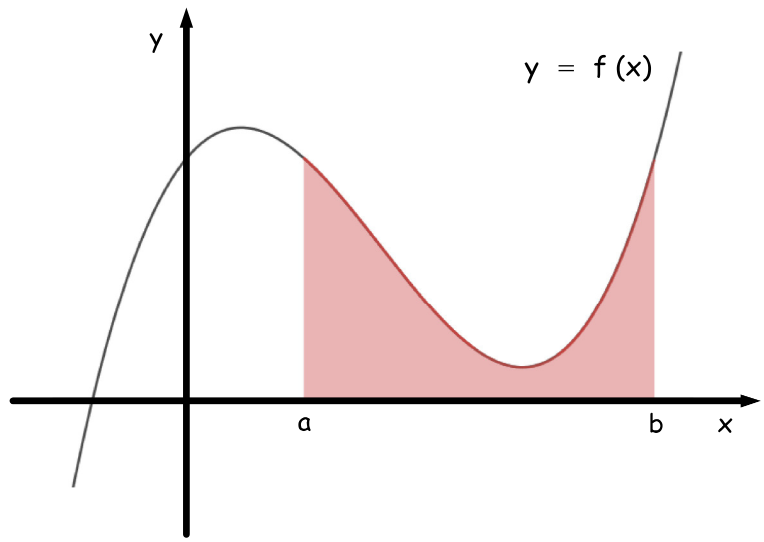
The area A bounded by a curve $y = f(x)$, the x - axis and the points $x = a$ and $x = b$ (shown below by the shaded area), where the curve is above the x - axis between $x = a$ and $x = b$,



is given by the definite integral,

$$A = \int_a^b f(x)\, dx$$

If the area between the curve $y = g(x) = -f(x)$, the x - axis and the points $x = a$ and $x = b$ is considered, the area is obviously the same as before (reflect the above graph in the x - axis) and is given by the integral,

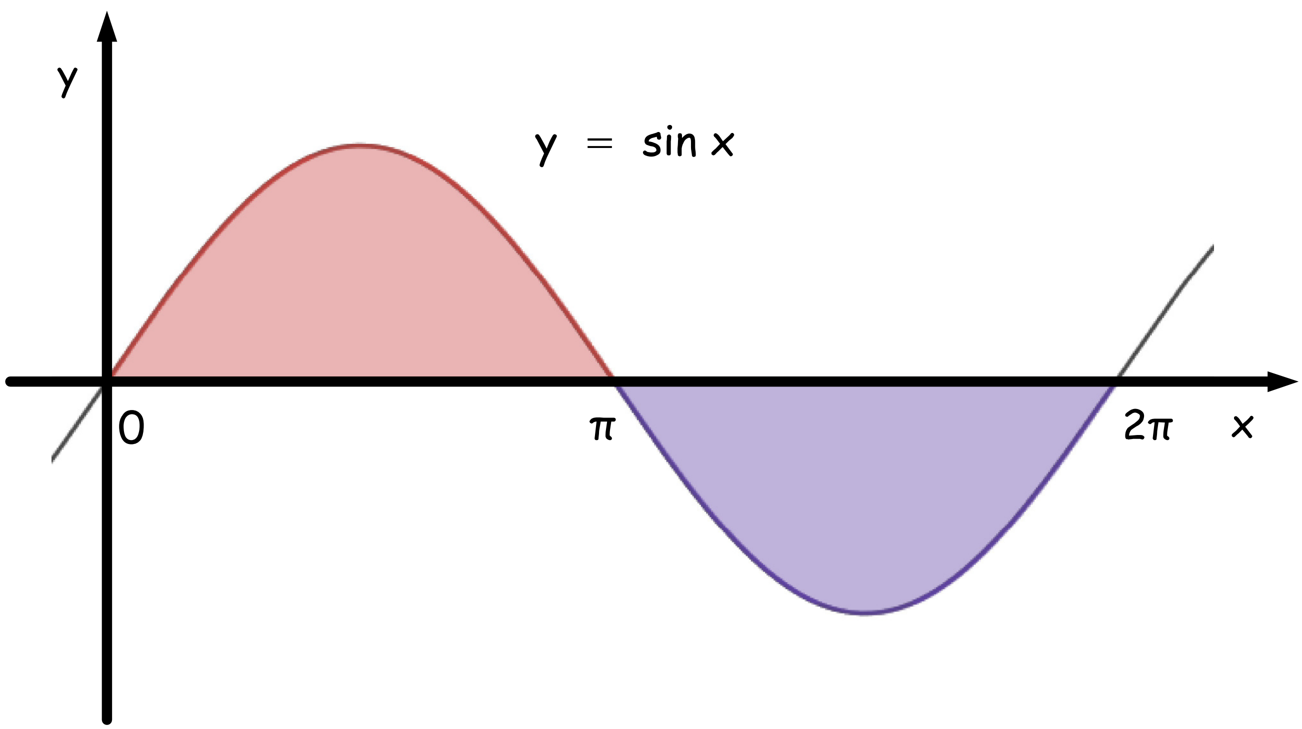
$$A = -\int_a^b g(x)\, dx$$

Thus, when computing an area completely below the x - axis, just negative the answer for the definite integral.

Also note that, by looking at the order in which the limits a and b are evaluated,

$$\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$$

Consider the graph of $y = \sin x$ between the x - axis and the points $x = 0$ and $x = 2\pi$. To calculate the area between this curve, the x - axis and the given points, we must consider the two shaded areas separately (otherwise, the area would be zero - they would cancel out).

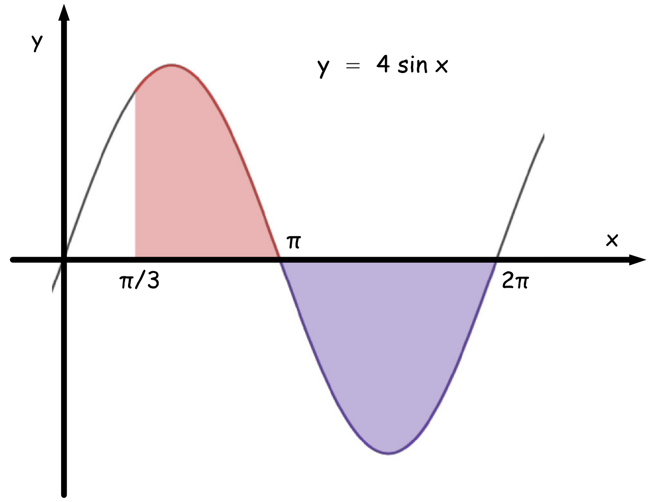


The total area is thus given by,

$$A = \int_0^{\pi} \sin x \; dx \; - \int_{\pi}^{2\pi} \sin x \; dx$$

Example 3 (Non-Calc)

Find the area bounded by the curve $y = 4 \sin x$ and the x - axis between $x = \pi/3$ and $x = 2\pi$.



Denote the area above the x - axis by A_1 and that below the x - axis by A_2.

$$A_1 = \int_{\pi/3}^{\pi} 4 \sin x \, dx$$

$$= -4 \Big[\cos x \Big]_{\pi/3}^{\pi}$$

$$= -4 (\cos \pi - \cos (\pi/3))$$

$$= -4 (-1 - 1/2)$$

$$= \underline{6}$$

$$A_2 = -\int_{\pi}^{2\pi} 4 \sin x \, dx$$

$$= 4 \Big[\cos x \Big]_{\pi}^{2\pi}$$

$$= 4 (\cos 2\pi - \cos \pi)$$

$$= 4 (1 - (-1))$$

$$= \underline{8}$$

Total Area $= A_1 + A_2$

Total Area $= 6 + 8$

$\boxed{\text{Total Area} = 14 \text{ u}^2}$

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