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Solving Trigonometric Equations - Lesson 3

Solving Simple Quadratic Trigonometric Equations

LI

- Solve trigonometric equations of the form :

$$a \sin^2 (b x + c) + d = 0$$
$$a \cos^2 (b x + c) + d = 0$$
$$a \tan^2 (b x + c) + d = 0$$

for various ranges of x (in degrees or radians).

SC

- Square roots.
- Solve linear trig. equations.

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Strategy

- If f (x) is one of sin (b x + c), cos (b x + c) or tan (b x + c), get equation into the form :

$$(f(x))^2 = k$$

- Take square roots of the above equation and solve the resulting 2 linear trig. equations :

$$f(x) = \sqrt{k}$$

$$f(x) = -\sqrt{k}$$

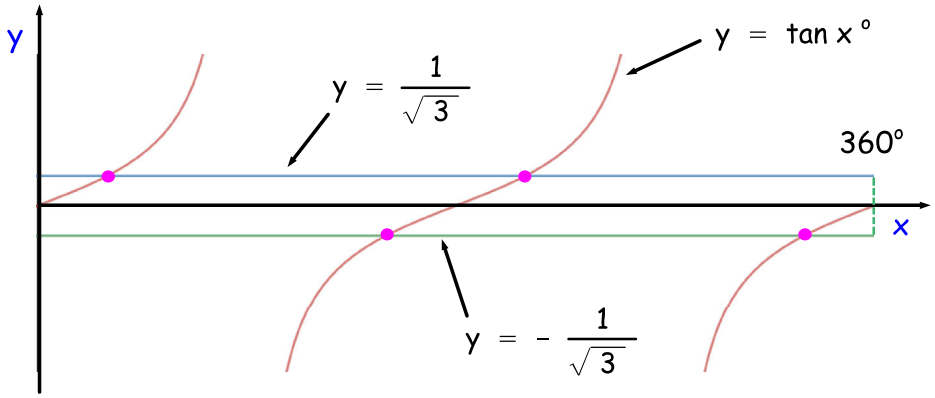
Example 1 (non-calculator)

Solve $3\tan^2 x° = 1 \quad (0 \leq x \leq 360)$.

$$3\tan^2 x° = 1$$

$$\tan^2 x° = \frac{1}{3}$$

$$\tan x° = \pm\frac{1}{\sqrt{3}}$$



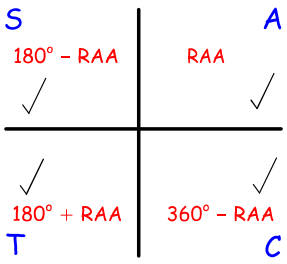
4 solutions expected

$$\tan x° = \frac{1}{\sqrt{3}} \qquad \tan x° = -\frac{1}{\sqrt{3}}$$

$$RAA = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad RAA = 30°$$

tan is + ve (1st equation)
and
tan is − ve (2nd equation)



$$\therefore \quad x° = 30°, 150°, 210°, 330°$$

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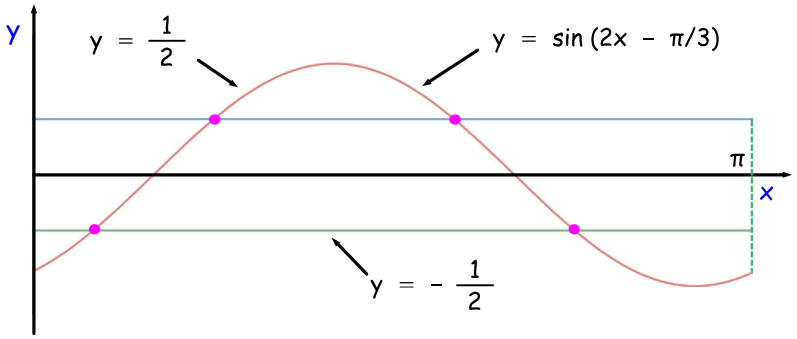
Example 2 (non-calculator)

Solve $\sin^2(2x - \pi/3) = 1/4$ $(0 \leq x \leq \pi)$.

$$\sin^2(2x - \pi/3) = 1/4$$

$$\sin(2x - \pi/3) = \pm 1/2$$



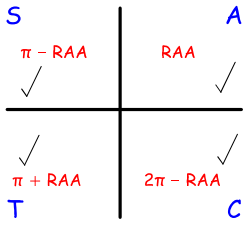
4 solutions expected

$\sin(2x - \pi/3) = 1/2$ $\quad$ $\sin(2x - \pi/3) = -1/2$

$$RAA = \sin^{-1}(1/2)$$

$$\Rightarrow \quad RAA = \pi/6$$

sin is + ve (1st equation)
and
sin is − ve (2nd equation)



As, $0 \leq x \leq \pi$, $0 \leq 2x \leq 2\pi$ and hence $-\pi/3 \leq 2x - \pi/3 \leq 5\pi/3$; alternatively, $-2\pi/6 \leq 2x - \pi/3 \leq 10\pi/6$.

$\therefore \quad 2x - \pi/3 = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

$11\pi/6$ is too big, so subtract 2π from it to get $-\pi/6$.

$\therefore \quad 2x - \pi/3 = -\pi/6, \pi/6, 5\pi/6, 7\pi/6$

$\Rightarrow \quad 2x = \pi/6, \pi/2, 7\pi/6, 3\pi/2$

$\Rightarrow \quad \boxed{x = \pi/12, \pi/4, 7\pi/12, 3\pi/4}$

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CfE Higher Maths

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