Differential Calculus - Lesson 3

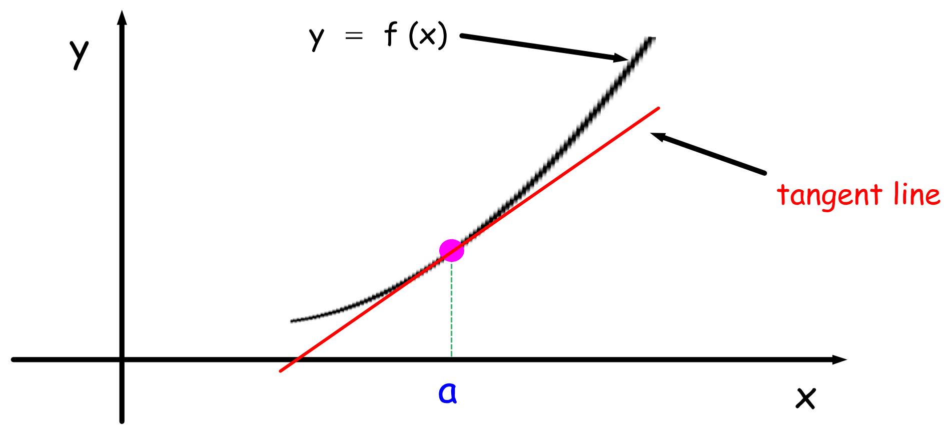
Gradients of Tangent Lines

LI

- Find gradients of tangent lines to curves.
- Find a missing coordinate given the gradient.

SC

- Differentiation.



The rate of change of $y = f(x)$ at $x = a$ (sometimes called the gradient of the curve at $x = a$) is equal to the gradient of the tangent line at $x = a$:

gradient of curve at $x = a$

$$m = \left(\frac{dy}{dx}\right)_{x = a}$$

gradient of tangent line at $x = a$

Common notations for the gradient of the curve $y = f(x)$ at $x = a$ are :

$f'(a)$, $y'(a)$ Lagrange Form

$\left(\frac{dy}{dx}\right)_{x = a}$ Leibniz Form

Useful Things to Remember
(especially for non-calc.)

$$x^{1/2} = \sqrt{x}$$

$$x^{3/2} = x\sqrt{x}$$

$$x^{5/2} = x^2\sqrt{x}$$

Example 1

Find the gradient of the tangent to the curve $y = x^2 - 6x + 8$ at the point $(2, 8)$.

$$y(x) = x^2 - 6x + 8$$

$$\therefore \quad y'(x) = 2x - 6$$

$$\therefore \quad y'(2) = 2(2) - 6$$

$$\Rightarrow \quad \boxed{y'(2) = -2}$$

Example 2

A curve has equation $y = 10\sqrt{x}$.

Find the rate of change of y when $x = 16$.

$$y(x) = 10\sqrt{x}$$

$$y(x) = 10x^{1/2}$$

$$\therefore \quad y'(x) = 5x^{-1/2}$$

$$\Rightarrow \quad y'(x) = \frac{5}{x^{1/2}}$$

$$\Rightarrow \quad y'(x) = \frac{5}{\sqrt{x}}$$

$$\therefore \quad y'(16) = \frac{5}{\sqrt{16}}$$

$$\Rightarrow \quad \boxed{y'(16) = \frac{5}{4}}$$

Example 3

Find the gradient of the curve $y = \dfrac{4}{\sqrt{x}}$

at $x = 4$.

$$y(x) = \frac{4}{\sqrt{x}}$$

$$y(x) = 4x^{-1/2}$$

$$\therefore \quad y'(x) = -2x^{-3/2}$$

$$\Rightarrow \quad y'(x) = -\frac{2}{x^{3/2}}$$

$$\Rightarrow \quad y'(x) = -\frac{2}{x\sqrt{x}}$$

$$\therefore \quad y'(4) = -\frac{2}{4\sqrt{4}}$$

$$\Rightarrow \quad y'(4) = -\frac{2}{8}$$

$$\Rightarrow \quad \boxed{y'(4) = -\frac{1}{4}}$$

Example 4

A curve has equation $y = 3x^2 - 12x + 6$.

Find the x - coordinate of the point at which the tangent to the curve has gradient 12.

$$y(x) = 3x^2 - 12x + 6$$

$$\therefore \quad y'(x) = 6x - 12$$

Gradient $= 12$ means $y'(x) = 12$. So,

$$12 = 6x - 12$$

$$\Rightarrow \quad 6x = 24$$

$$\Rightarrow \quad \boxed{x = 4}$$

CfE Higher Maths

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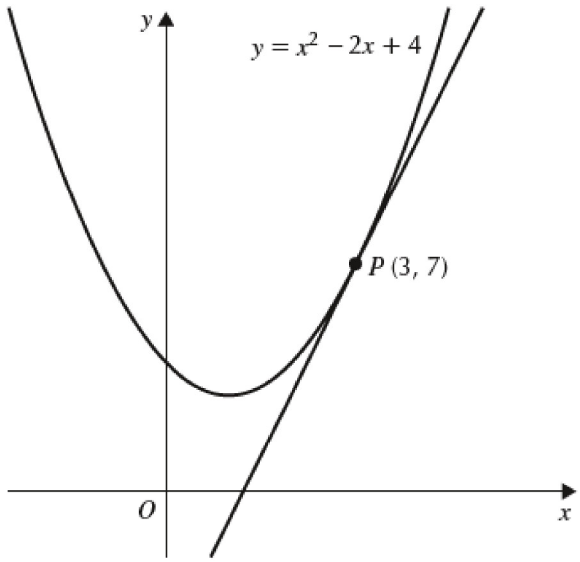
Questions

1 For each of the following, find the gradient of the curve at the given point.

a



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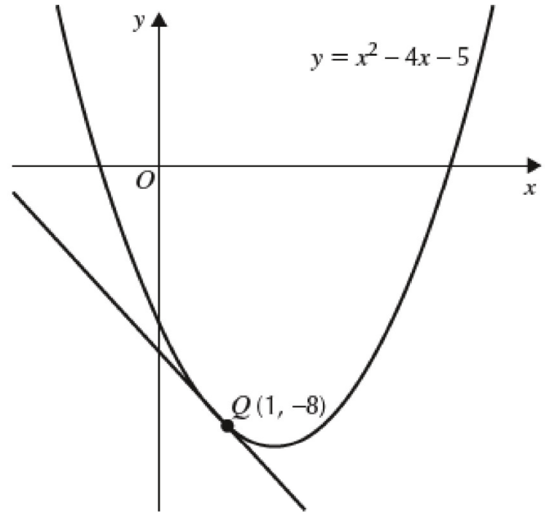
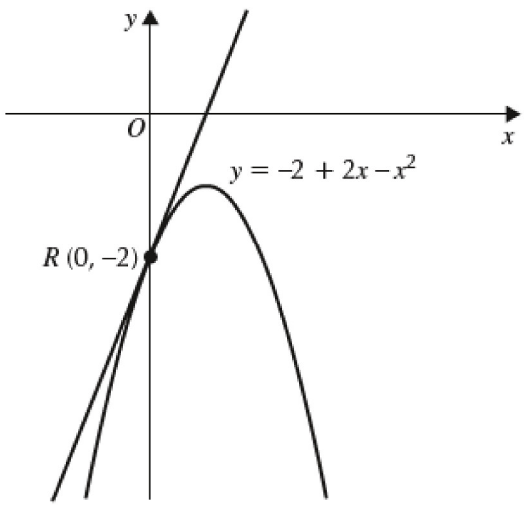


c



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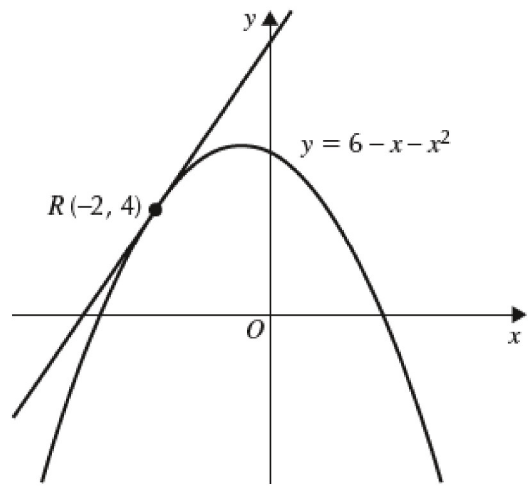


2 **a** Find the gradient of the tangent to the curve $y = x^2 + 4x + 2$ at the point where $x = 3$.

b A curve has equation $y = 5x^2 - 15x$. Find the gradient of the curve at the point where $x = 2$.

c Given $f(x) = x^3 - 4x^2 + 5x + 3$, find the rate of change of f when $x = 1$.

d Find the gradient of the curve $y = (x + 2)(x + 5)$ at the point where $x = -3$

e Given $g(x) = 6x - x^3$, find the value of $g'(-2)$.

f A curve has equation $y = 4x(x^2 - 2)$. Find $\frac{dy}{dx}$ when $x = -1$.

3 A curve has equation $y = \frac{2}{x}$ where $x \neq 0$. Find the gradient of the curve when

a $x = 1$ b $x = -3$ c $x = \frac{1}{2}$

4 On a suitable domain, the function f is defined by $f(x) = 3\sqrt{x}$

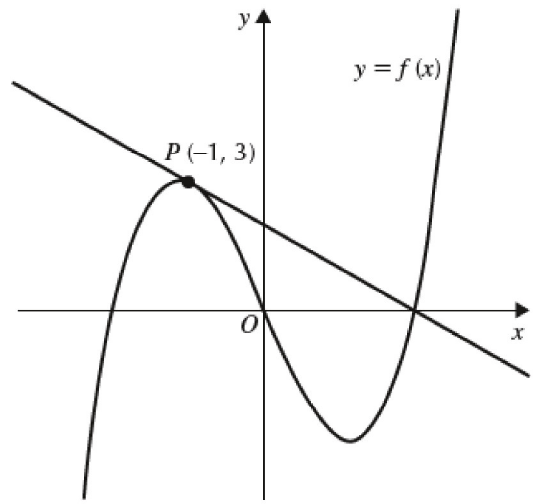
a Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 4$.

b Find the rate of change of f when $x = 9$.

c Evaluate $f'\left(\frac{1}{16}\right)$.

5 The diagram shows part of the graph of the cubic function with equation $f(x) = x(x^2 - 4)$. A tangent to the graph is drawn at P.

Find the gradient of this tangent.



6 A curve has equation $y = \frac{5}{4x^2}$ where $x \neq 0$.

Find the gradient of the curve at the point where $x = -10$.

7 a Find the x–coordinate of the point where the tangent to the curve $y = x^2 + 8x - 3$ has gradient 2.

b The function f is defined by $f(x) = 5 - 4x - x^2$. Determine the value of p, given that $f'(p) = 2$.

8 Find the coordinates of the point where the tangent to the curve $y = 3x^2 - 4x + 1$ has gradient -10.

9 Find the x–coordinate of the point where the tangent to the curve $y = x^4 + 20x$ has gradient -12.

10 a Determine the x–coordinates of the points where the tangent to the curve $y = \frac{1}{3}x^3 - 3x^2 + 12x + 2$ has gradient 4.

b Determine the x–coordinates of the points where the tangent to the curve $y = x^3 + 2x^2 - 7x + 1$ has gradient -3.

18 Find the range of values of x for which the gradient of the curve $y = x^3 + x^2 - 5x + 2$ is greater than 3.

Answers

1 **a** 4
b -2
c 2
d 3

2 **a** 10
b 5
c 0
d 1
e -6
f 4

3 **a** -2
b $-\frac{2}{9}$
c -8

4 **a** $\frac{3}{4}$
b $\frac{1}{2}$
c 6

5 -1

6 $\frac{1}{400}$

7 **a** -3
b -3

8 $(-1,8)$

9 -2

10 **a** $x = 2$
$x = 4$
b $x = -2$
$x = \frac{2}{3}$

18 $x < -2$ and $x > \frac{4}{3}$