

Unit 2 : Sequences and Series - Lesson 3

Geometric Sequences

LI

- Know what a Geometric Sequence is.
- Find the nth term formula for a geometric sequence.
- Solve problems involving geometric sequences.

<u>SC</u>

• Arithmetic of real numbers.

A geometric sequence is a sequence where the ratio of any two successive terms is constant :

$$\frac{\mathsf{u}_{n+1}}{\mathsf{u}_n} = r \quad \text{(for all } n \in \mathbb{N}\text{)}$$

(r is called the common ratio)

The nth term of a geometric sequence is :

$$u_n = a r^{n-1}$$

Example 1

Show that 2, 8, 16, ... cannot be the first three terms of a geometric sequence.

 $8 \div 2 = 4$ $16 \div 8 = 2$

As successive ratios are not constant, these 3 numbers cannot form the start of a geometric sequence.

Example 2

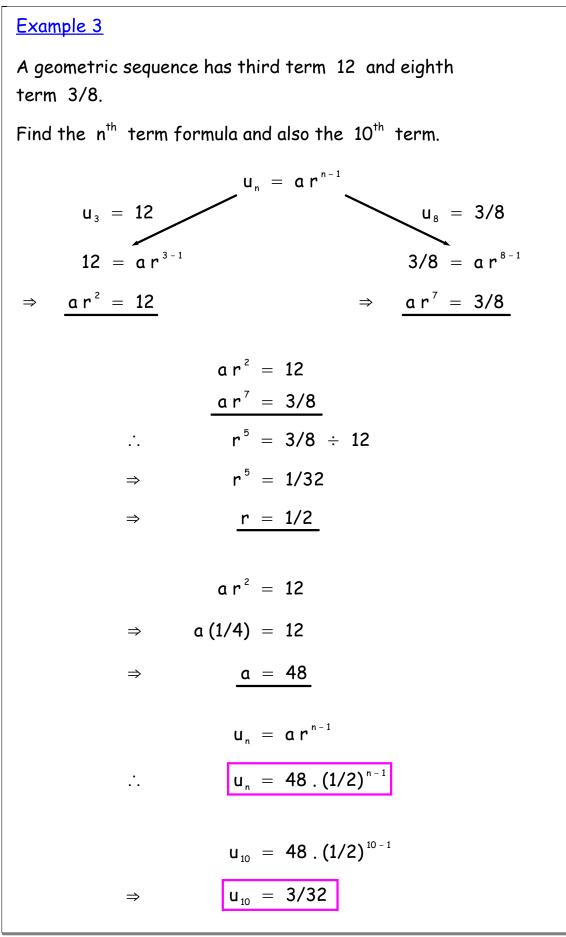
Find the n^{th} term formula for the geometric sequence that begins 6, 30, 150,

The first term is 6; the common ratio is $30 \div 6 = 5$. So,

$$u_n = a r^{n-1}$$

 $u_n = 6.5^{n-1}$

. .



Example 4 Given a geometric sequence 6, 12, 24, 48, ..., find the value of n for which $u_n = 49152$. $u_n = a r^{n-1}$ $u_n = 6.2^{n-1}$ As $u_n = 49152$ we have, $6 \cdot 2^{n-1} = 49152$ $2^{n-1} = 8.192$ \Rightarrow \therefore (n - 1) ln 2 = ln 8 192 n - 1 = (ln 8 192)/(ln 2) \Rightarrow n - 1 = 13⇒ n = 14 \Rightarrow

Example 5

Show that e^{2x} , e^{5x} , e^{8x} ,... could be the first three terms of a geometric sequence.

Hence show that $u_n = e^{f(n)x}$, stating explicitly the function f(n).

$$e^{5x} \div e^{2x} = e^{3x}$$

 $e^{8x} \div e^{5x} = e^{3x}$

As successive ratios are constant ($r = e^{3x}$), these 3 numbers could be the first three terms of a geometric sequence.

$$u_{n} = a r^{n-1}$$

$$\therefore \qquad u_{n} = e^{2x} \cdot (e^{3x})^{n-1}$$

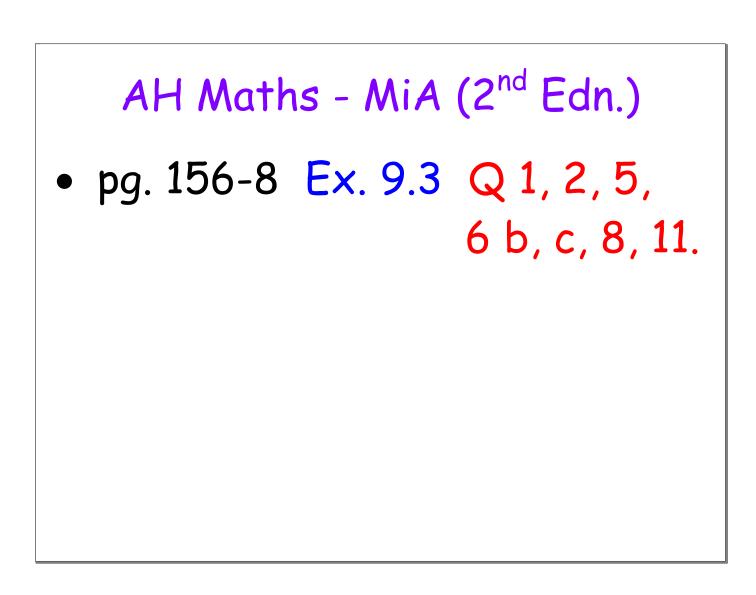
$$\Rightarrow \qquad u_{n} = e^{2x} \cdot e^{3x(n-1)}$$

$$\Rightarrow \qquad u_{n} = e^{2x} \cdot e^{3xn} \cdot e^{-3x}$$

$$\Rightarrow \qquad u_{n} = e^{3xn-x}$$

$$\Rightarrow \qquad u_{n} = e^{(3n-1)x}$$

$$(f(n) = 3n - 1)$$



	Ex. 9.3
	For each of these geometric sequences i identify a and r ii find an expression for the nth term.a 1, 4, 16, 64,b 3, -12, 48, -192c 1536, 768, 384, 192,d 3645, -1215, 405, -135,e 1, 0.1, 0.01, 0.001,f $\frac{1}{2}, \frac{3}{8}, \frac{9}{32}, \frac{27}{128},$
2	 g 0.12, 0.048, 0.0192, 0.00768, h 18.4, 20.24, 22.264, 24.4904, a The first term of a geometric sequence is 3. The common ratio is 6. Calculate the sixth term. b In a geometric sequence, u₁ = 0.5, u₂ = 0.3. What is term 5? c In a geometric sequence, u₂ = 12, u₃ = 24. Calculate u₁₀.
5 4	 a The first term of a geometric sequence is 3. The 10th term is 1536. Calculate the common ratio. b The common ratio of a geometric sequence is 0.7 and the 23rd term is 0.4
6 1	Calculate the first term correct to the nearest whole number. b The terms a , $a + d$, $a + xd$ form the start of a geometric sequence. i Express x in terms of a and d .
	ii Express the common ratio in terms of <i>x</i> . a, ar, a + 2d are the first three terms of a geometric sequence. Given that $r > 0$, express <i>r</i> in terms of <i>a</i> and <i>d</i> .
t	 A gearing system works best when the number of teeth on the gear train form a geometric sequence. The terms, of course, must be rounded to the nearest whole number. a Calculate the unknown number of teeth in each of these three-wheel gear systems. i 8 teeth, x teeth, 18 teeth ii 14 teeth, 21 teeth, y teeth
1	 iii z teeth, 14 teeth, 49 teeth b A certain type of gear has four wheels in the train. Again, working to the nearest whole number, calculate the unknown terms in each of the trains. i 16, x, y, 54 ii 20, 30, p, q iii 25, 30, a, b
	 In an experiment, a ball of radius 1 cm, made of 'super rubber', is dropped from a height. On its first bounce it reached the height of 8 m. On its second it reached the height of 6.4 m. a Successive bounces form a geometric sequence. Calculate the height reached on the seventh bounce (to 1 dp). b The ball offectively stops when the bounce is less than the radius of the ball (1 cm). After how many
	b The ball effectively stops when the bounce is less than the radius of the ball (1 cm). After how many bounces will this happen?

