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Unit 2 : Sequences and Series - Lesson 3

Geometric Sequences

LI

- Know what a Geometric Sequence is.
- Find the n^{th} term formula for a geometric sequence.
- Solve problems involving geometric sequences.

SC

- Arithmetic of real numbers.

A geometric sequence is a sequence where the ratio of any two successive terms is constant :

$$\frac{u_{n+1}}{u_n} = r \quad (\text{for all } n \in \mathbb{N})$$

(r is called the common ratio)

The n^{th} term of a geometric sequence is :

$$u_n = a\, r^{n-1}$$

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Example 1

Show that 2, 8, 16, . . . cannot be the first three terms of a geometric sequence.

$$8 \div 2 = 4$$

$$16 \div 8 = 2$$

As successive ratios are not constant, these 3 numbers cannot form the start of a geometric sequence.

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Example 2

Find the n^{th} term formula for the geometric sequence that begins 6, 30, 150, ...

The first term is 6; the common ratio is $30 \div 6 = 5$. So,

$$u_n = a r^{n-1}$$

$$\therefore \quad \boxed{u_n = 6 . 5^{n-1}}$$

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Example 3

A geometric sequence has third term 12 and eighth term 3/8.

Find the n^{th} term formula and also the 10^{th} term.

$$u_n = a r^{n-1}$$

$$u_3 = 12 \qquad\qquad u_8 = 3/8$$

$$12 = a r^{3-1} \qquad\qquad 3/8 = a r^{8-1}$$

$$\Rightarrow \underline{a r^2 = 12} \qquad\qquad \Rightarrow \underline{a r^7 = 3/8}$$

$$a r^2 = 12$$

$$\underline{a r^7 = 3/8}$$

$$\therefore \quad r^5 = 3/8 \div 12$$

$$\Rightarrow \quad r^5 = 1/32$$

$$\Rightarrow \quad \underline{r = 1/2}$$

$$a r^2 = 12$$

$$\Rightarrow \quad a\,(1/4) = 12$$

$$\Rightarrow \quad \underline{a = 48}$$

$$u_n = a r^{n-1}$$

$$\therefore \quad \boxed{u_n = 48 \,.\, (1/2)^{n-1}}$$

$$u_{10} = 48 \,.\, (1/2)^{10-1}$$

$$\Rightarrow \quad \boxed{u_{10} = 3/32}$$

Example 4

Given a geometric sequence 6, 12, 24, 48, . . . , find the value of n for which $u_n = 49\,152$.

$$u_n = a\,r^{n-1}$$

$$\therefore \quad \underline{u_n = 6 \, . \, 2^{n-1}}$$

As $u_n = 49\,152$ we have,

$$6 \, . \, 2^{n-1} = 49\,152$$

$$\Rightarrow \quad 2^{n-1} = 8\,192$$

$$\therefore \quad (n - 1) \ln 2 = \ln 8\,192$$

$$\Rightarrow \quad n - 1 = (\ln 8\,192)/(\ln 2)$$

$$\Rightarrow \quad n - 1 = 13$$

$$\Rightarrow \quad \boxed{n = 14}$$

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Example 5

Show that $e^{2x}, e^{5x}, e^{8x}, \ldots$ could be the first three terms of a geometric sequence.

Hence show that $u_n = e^{f(n)x}$, stating explicitly the function $f(n)$.

$$e^{5x} \div e^{2x} = e^{3x}$$

$$e^{8x} \div e^{5x} = e^{3x}$$

As successive ratios are constant ($r = e^{3x}$), these 3 numbers could be the first three terms of a geometric sequence.

$$u_n = a r^{n-1}$$

$$\therefore \quad u_n = e^{2x} . (e^{3x})^{n-1}$$

$$\Rightarrow \quad u_n = e^{2x} . e^{3x(n-1)}$$

$$\Rightarrow \quad u_n = e^{2x} . e^{3xn} . e^{-3x}$$

$$\Rightarrow \quad u_n = e^{3xn - x}$$

$$\Rightarrow \quad \boxed{u_n = e^{(3n-1)x} \quad (f(n) = 3n - 1)}$$

AH Maths - MiA (2nd Edn.)

- pg. 156-8 Ex. 9.3 Q 1, 2, 5, 6 b, c, 8, 11.

Ex. 9.3

1 For each of these geometric sequences **i** identify a and r **ii** find an expression for the nth term.

a $1, 4, 16, 64, \ldots$

b $3, -12, 48, -192$

c $1536, 768, 384, 192, \ldots$

d $3645, -1215, 405, -135, \ldots$

e $1, 0.1, 0.01, 0.001, \ldots$

f $\frac{1}{2}, \frac{3}{8}, \frac{9}{32}, \frac{27}{128}, \ldots$

g $0.12, 0.048, 0.0192, 0.00768, \ldots$

h $18.4, 20.24, 22.264, 24.4904, \ldots$

2 a The first term of a geometric sequence is 3. The common ratio is 6. Calculate the sixth term.

b In a geometric sequence, $u_1 = 0.5, u_2 = 0.3$. What is term 5?

c In a geometric sequence, $u_2 = 12, u_3 = 24$. Calculate u_{10}.

5 a The first term of a geometric sequence is 3. The 10th term is 1536.
Calculate the common ratio.

b The common ratio of a geometric sequence is 0.7 and the 23rd term is 0.4
Calculate the first term correct to the nearest whole number.

6 b The terms $a, a + d, a + xd$ form the start of a geometric sequence.

i Express x in terms of a and d.

ii Express the common ratio in terms of x.

c $a, ar, a + 2d$ are the first three terms of a geometric sequence.

Given that $r > 0$, express r in terms of a and d.

8 A gearing system works best when the number of teeth on the gear train form a geometric sequence. The terms, of course, must be rounded to the nearest whole number.

a Calculate the unknown number of teeth in each of these three-wheel gear systems.

i 8 teeth, x teeth, 18 teeth

ii 14 teeth, 21 teeth, y teeth

iii z teeth, 14 teeth, 49 teeth

b A certain type of gear has four wheels in the train. Again, working to the nearest whole number, calculate the unknown terms in each of the trains.

i $16, x, y, 54$

ii $20, 30, p, q$

iii $25, 30, a, b$

11 In an experiment, a ball of radius 1 cm, made of 'super rubber', is dropped from a height.
On its first bounce it reached the height of 8 m. On its second it reached the height of 6.4 m.

a Successive bounces form a geometric sequence.
Calculate the height reached on the seventh bounce (to 1 dp).

b The ball effectively stops when the bounce is less than the radius of the ball (1 cm). After how many bounces will this happen?

Answers to AH Maths (MiA), pg. 156-8, Ex. 9.3

1 **a** **i** $1, 4$ **ii** $u_n = ar^{n-1} = 1 \times 4^{n-1}$

b **i** $3, -4$ **ii** $u_n = 3 \times (-4)^{n-1}$

c **i** $1536, \frac{1}{2}$ **ii** $u_n = 1536 \times \left(\frac{1}{2}\right)^{n-1}$

d **i** $3645, -\frac{1}{3}$ **ii** $u_n = 3645 \times \left(-\frac{1}{3}\right)^{n-1}$

e **i** $1, \frac{1}{10}$ **ii** $u_n = 1 \times \left(\frac{1}{10}\right)^{n-1}$

f **i** $\frac{1}{2}, \frac{3}{4}$ **ii** $u_n = \frac{1}{2} \times \left(\frac{3}{4}\right)^{n-1}$

g **i** $0{\cdot}12, 0{\cdot}4$ **ii** $u_n = 0{\cdot}12 \times (0{\cdot}4)^{n-1}$

h **i** $18{\cdot}4, 1{\cdot}1$ **ii** $u_n = 18{\cdot}4 \times 1{\cdot}1^{n-1}$

2 **a** 23 328 **b** $\frac{81}{1\,250}$

c 3072

5 **a** 2 **b** 1023

6 **b** **i** $x = \frac{2a + d}{a}$ **ii** $r = x - 1$ **c** $r = \sqrt{\frac{2d + a}{a}}$

8 **a** **i** 12 **ii** 31 or 32 **iii** 4

b **i** 24, 36 **ii** 45, 67 or 68 **iii** 36, 43

11 **a** $2{\cdot}1$ m **b** 31st bounce