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Functions - Lesson 3

Functions - Inverses

LI

- Know what the inverse of a functions is.
- Know that a function does not always have an inverse.
- Know the graphical interpretation for a function to have an inverse.
- Find the inverse of a function.

SC

- Solving linear, quadratic and cubic equations.

Suppose that all the inputs and all the outputs of a function $f : A \longrightarrow B$ have been worked out.

If every element of dom f is matched to exactly one element of ran f and every element of ran f is matched to exactly one element of dom f, then we say that there is 1 - 1 ('one to one') correspondence between dom f and ran f.

Another way of expressing this is as follows. If $f : A \longrightarrow B$ and $g : B \longrightarrow A$ are two functions satisfying the property that

$$f(g(x)) = x \qquad \text{or} \qquad g(f(x)) = x$$

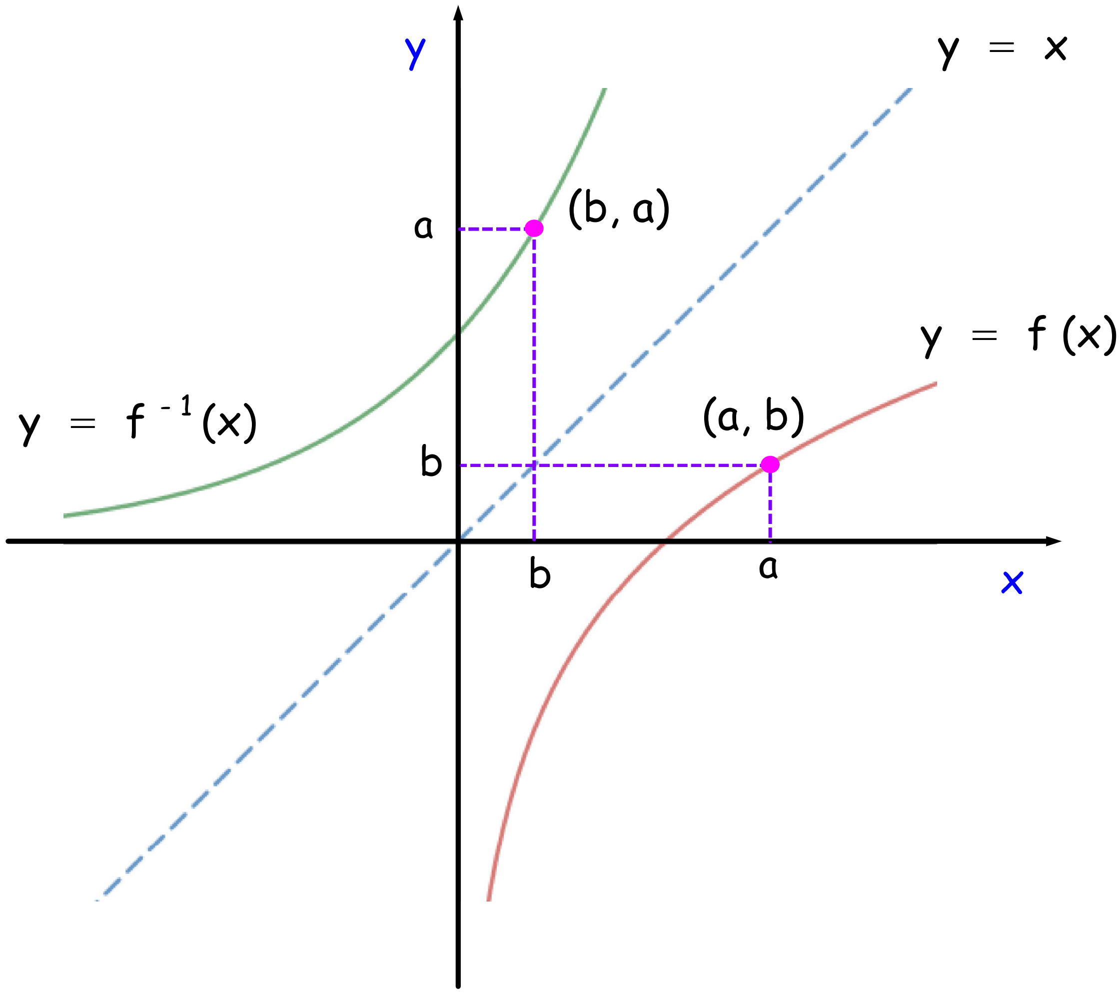
then we say that g is the inverse of f and write $g = f^{-1}$.

A function only has one inverse.

We also say that f is the inverse of g and write $f = g^{-1}$.

WARNING : The '-1' is not an index (power), just a special way of indicating the inverse

Graphical Interpretation of the Inverse



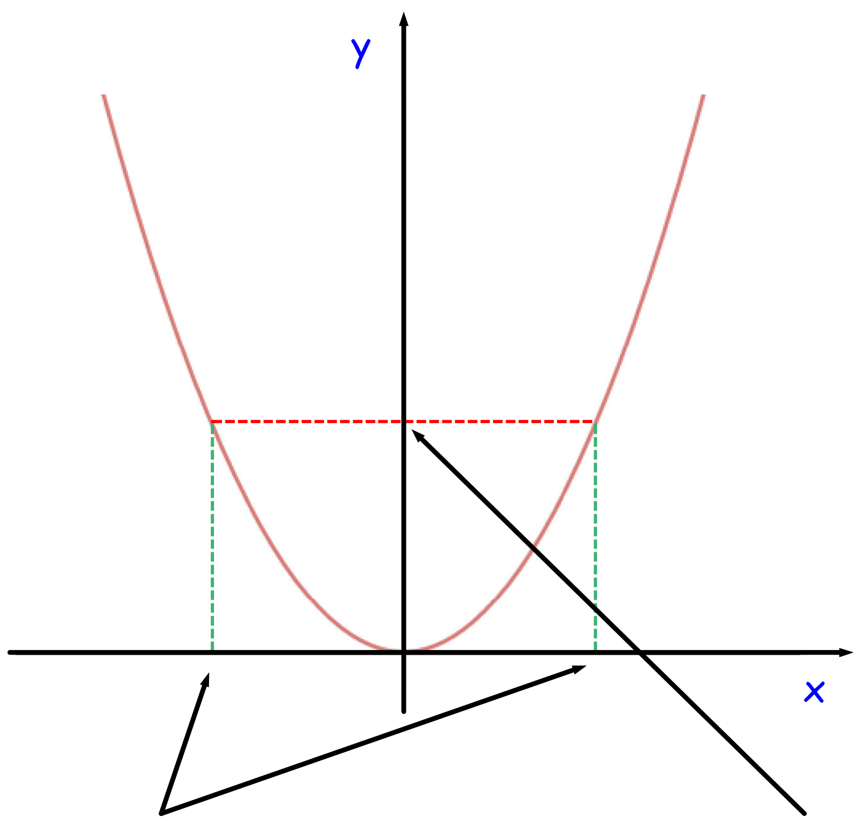
To sketch the graph of the inverse of a function, reflect the graph in the line $y = x$

If $f(a) = b$, then $f^{-1}(b) = a$ and vice versa

Knowing When a Function Does Not Have an Inverse

From the graphical interpretation, it is clear that a function will not have an inverse when at least 2 x - values give the same y - value.

For example, for the function $f(x) = x^2$ - with domain $\mathbb{R}$, every y - value (except for 0) has 2 x - values matching to it. This means that there is no inverse for the function $f(x) = x^2$.



These 2 x - values give the same y - value

However, restricting the function $y = x^2$ to non-negative values of x makes each x - value match up to exactly one y - value and vice versa. So, by restricting the domain to non-negative x - values, the function $y = x^2$ will have an inverse.

Procedure for Working Out the Inverse of a Function

- Write the function in the form $y = f(x)$.
- Swap the letters x and y.
- Solve for y.
- Write $f^{-1}(x) =$ the result obtained in the last bullet point.

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Example 1

If $f(x) = 3x + 7$, find f^{-1}.

$$y = 3x + 7$$

Interchange x and y:

$$x = 3y + 7$$

$$x - 7 = 3y$$

$$y = \frac{x-7}{3}$$

$$\therefore \quad \boxed{f^{-1}(x) = \frac{x-7}{3}}$$

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Example 2

If $p(x) = 2x^3 - 9$, find p^{-1}.

$$y = 2x^3 - 9$$

Interchange x and y:

$$x = 2y^3 - 9$$

$$x + 9 = 2y^3$$

$$y^3 = \frac{x+9}{2}$$

$$y = \sqrt[3]{\frac{x+9}{2}}$$

$$\therefore \boxed{p^{-1}(x) = \sqrt[3]{\frac{x+9}{2}}}$$

Example 3

Find the inverse of $r(x) = x^2 + 6 \ (x \geq 0)$.
Sketch the graphs of $y = r(x)$ and $y = r^{-1}(x)$.

$$y = x^2 + 6$$

Interchange x and y:

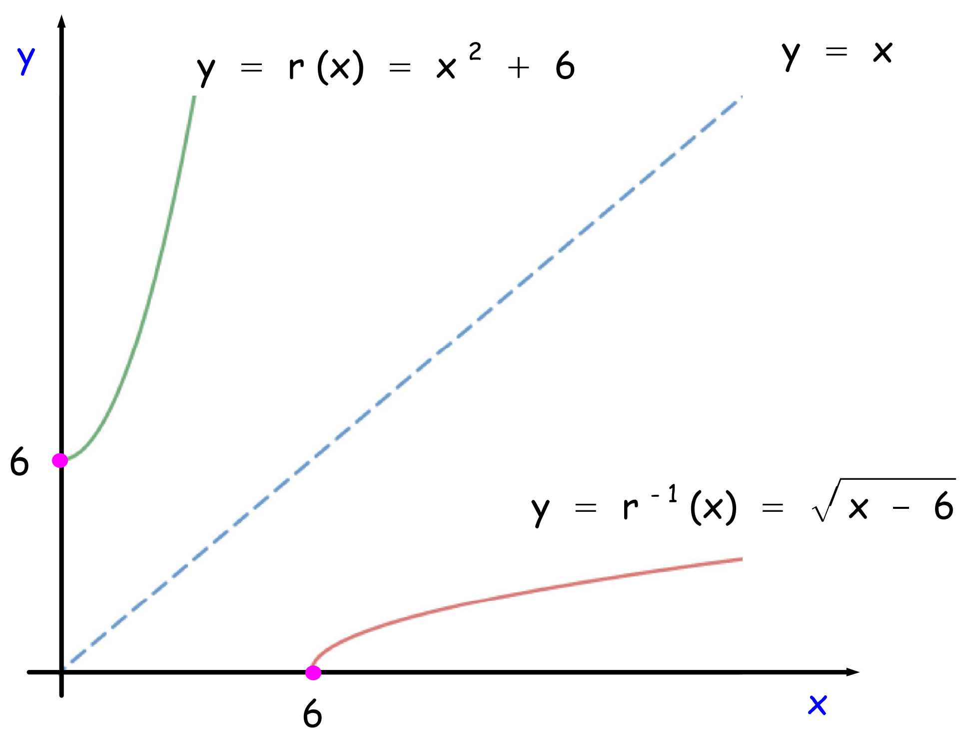
$$x = y^2 + 6$$

$$x - 6 = y^2$$

$$y = \sqrt{x - 6}$$

positive root, as $y > 0$

$$\therefore \quad \boxed{r^{-1}(x) = \sqrt{x - 6}}$$



Find the inverses of these functions :

1) $f(x) = 4x^3 + 11.$

2) $g(x) = (2x + 1)^3.$

3) $h(x) = (4 - 7x)^{1/3}.$

$$f^{-1}(x) = \sqrt[3]{\frac{x - 11}{4}}$$

$$g^{-1}(x) = \frac{x^{1/3} - 1}{2}$$

$$h^{-1}(x) = \frac{4 - x^{3}}{7}$$

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