

22 / 8 / 16

*Graphs of Related Functions - Lesson 3*

## Functional Transformations - Combinations

### LI

- Know that the order in which functions are transformed sometimes matters.
- Sketch graphs of functions that involve a combination of shifts and scalings.

### SC

- Know the standard transformations (shifts and scalings).

## How Coordinates Change Under Translations and Scalings

### x - translation of a

$$(x, y) \longrightarrow (x + a, y)$$

### y - translation of a

$$(x, y) \longrightarrow (x, y + a)$$

### y - scaling of a

$$(x, y) \longrightarrow (x, ay)$$

### x - scaling of a

$$(x, y) \longrightarrow (x/a, y)$$

The **order** in which transformations are done does not matter **apart from 2 cases :**

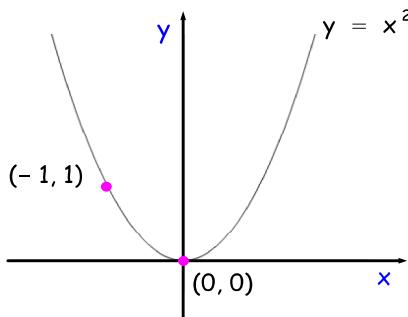
- x-translation with x-scaling.
- y-translation with y-scaling.

Given the function  $y = f(x)$ , to sketch  
 $y = P f(Q(x + R)) + S$ ,  
do the steps in the following order :

- $y = f(x)$  is x - scaled to  $y = f(Qx)$ .
- $y = f(Qx)$  is x - translated to  $y = f(Q(x + R))$ .
- $y = f(Q(x + R))$  is y - scaled to  $y = Pf(Q(x + R))$ .
- $y = Pf(Q(x + R))$  is y - translated to  $y = Pf(Q(x + R)) + S$ .

Example 1

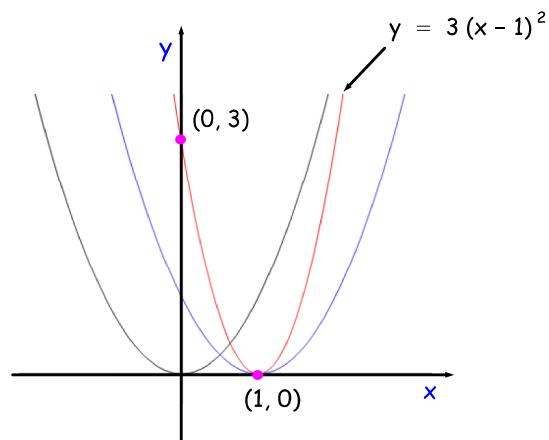
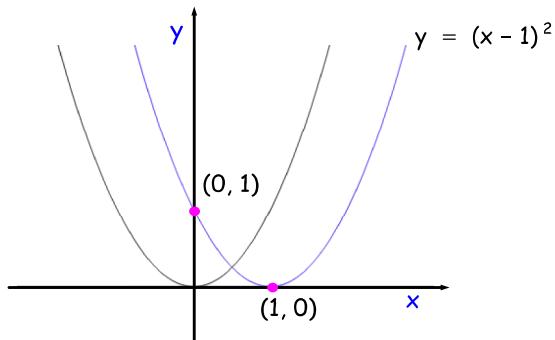
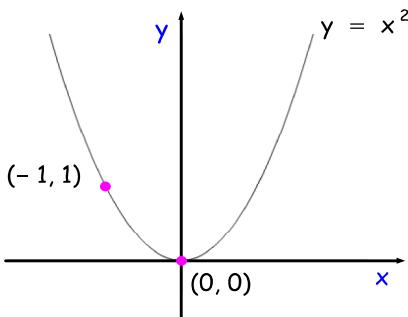
Shown below is the graph of  $y = x^2$ .



Sketch the graph of  $y = 3(x - 1)^2$ .

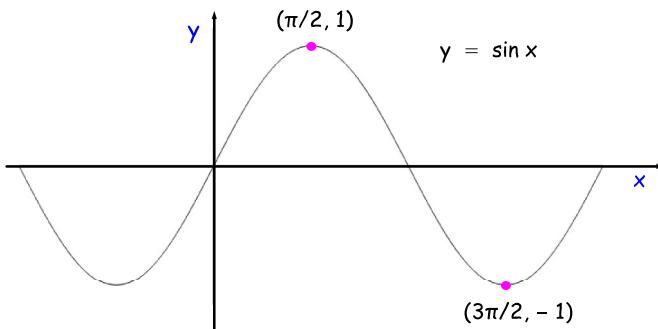
Strategy:

- $y = x^2$ .
- $y = (x - 1)^2$ .
- $y = 3(x - 1)^2$ .



Example 2

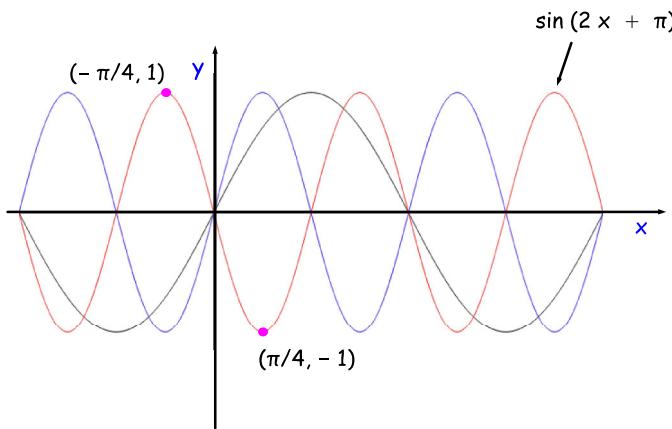
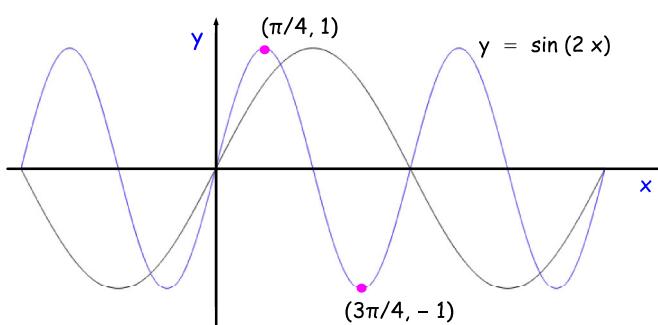
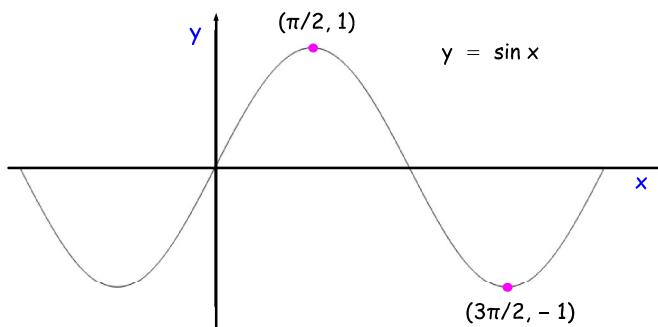
Shown below is the graph of  $y = \sin x$  ( $-\pi \leq x \leq 2\pi$ ).



Sketch the graph of  $y = \sin(2x + \pi)$ .

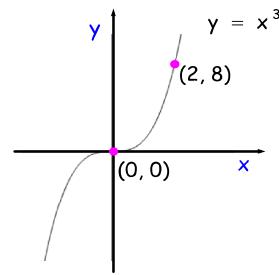
Strategy:

- $y = \sin x$ .
- $y = \sin(2x)$ .
- $y = \sin(2x + \pi) = \sin(2(x + \pi/2))$ .



Example 3

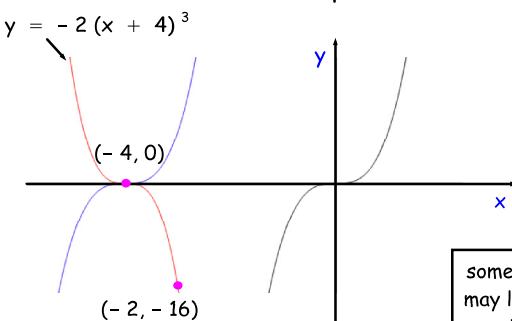
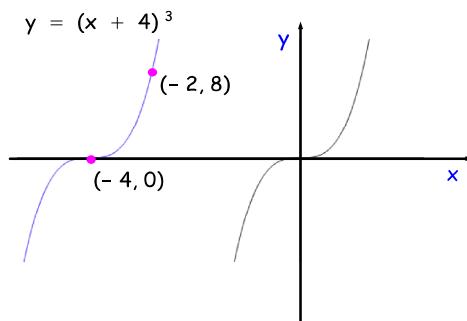
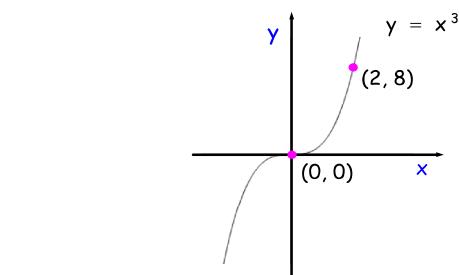
Shown below is the graph of  $y = x^3$ .



Sketch the graph of  $y = 1 - 2(x + 4)^3$ .

Strategy:

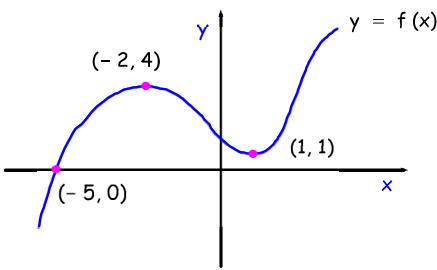
- $y = x^3$ .
- $y = (x + 4)^3$ .
- $y = -2(x + 4)^3$ .
- $y = 1 - 2(x + 4)^3$ .



some coordinates  
may look like they  
are in the wrong  
place, but we are  
only sketching

Example 4

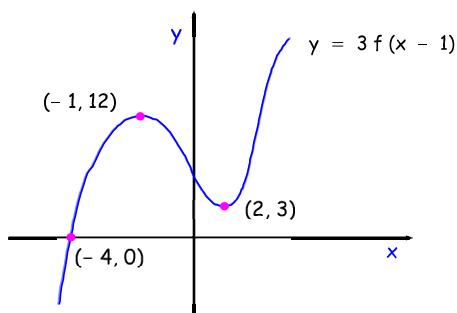
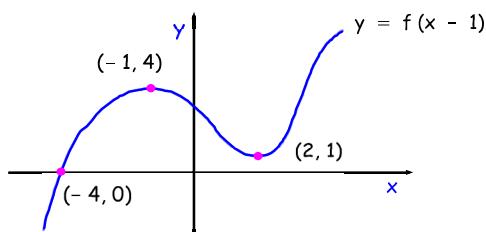
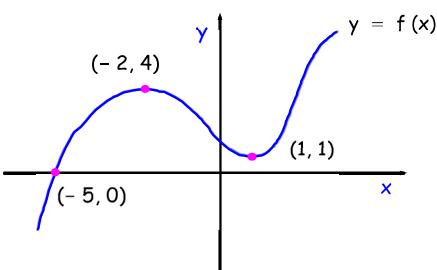
Shown below is the graph of  $y = f(x)$ .



Sketch the graph of  $y = 3f(x - 1) + 2$ .

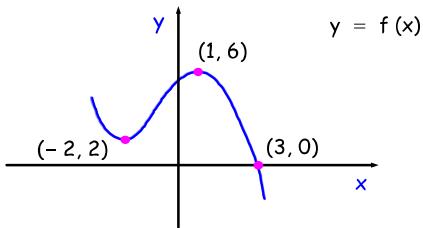
Strategy:

- $y = f(x)$ .
- $y = f(x - 1)$ .
- $y = 3f(x - 1)$ .
- $y = 3f(x - 1) + 2$ .



Example 5

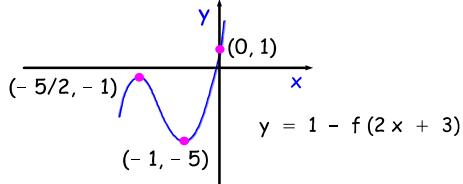
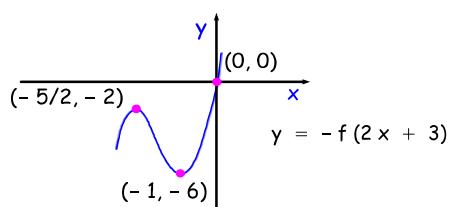
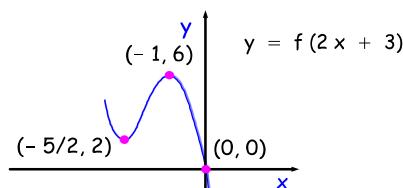
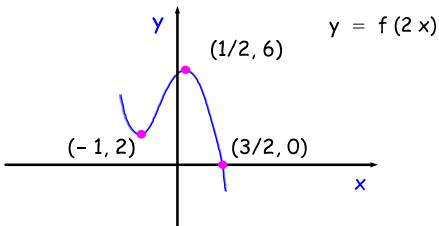
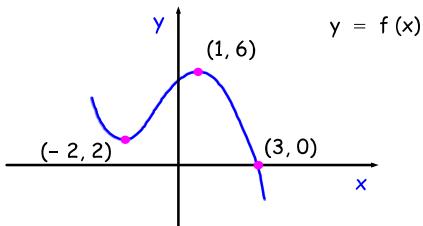
Shown below is the graph of  $y = f(x)$ .



Sketch the graph of  $y = 1 - f(2x + 3)$ .

Strategy:

- $y = f(x)$ .
- $y = f(2x)$ .
- $y = f(2x + 3)$ .
- $y = -f(2x + 3)$ .
- $y = 1 - f(2x + 3) = 1 - f(2(x + 3/2))$ .



## CfE Higher Maths

pg. 58 - 59 Ex. 3A All Q