20 / 9 / 16

Vectors - Lesson 3

Collinearity and Dividing a Line in a Ratio

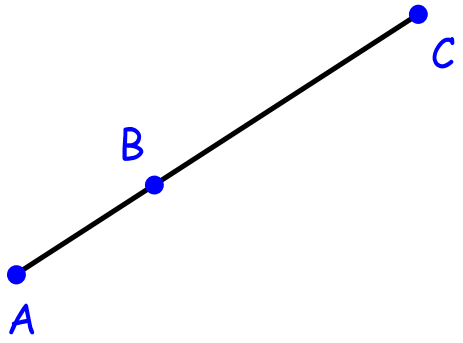
LI

- Know what it means for points to be Collinear.
- Show that 3 or more points are collinear.
- Divide a line in a given ratio.

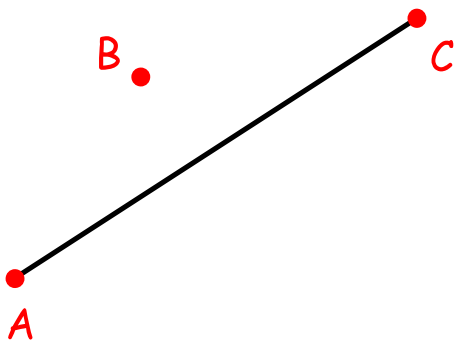
SC

- Arithmetic.

Points are collinear if they lie on the same straight line



A, B and C are collinear



A, B and C are not collinear

To show that points A, B and C are collinear, show that two out of three vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ (or negatives of any of these) are parallel

To show that points A, B and C are not collinear, show that no two of the three vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ (or negatives of any of these) are parallel to each other

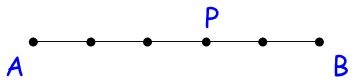
Feb 10-14:11

Example 1

A is the point $(4, -6, 12)$ and B is the point $(4, 4, -3)$. P divides AB in the ratio $3 : 2$.

Find the coordinates of P.

If P divides AB in the ratio $3 : 2$, then P lies $3/5$ of the way from A to B.



In other words, the vector $\overrightarrow{AP}$ is $3/5$ of the vector $\overrightarrow{AB}$.

$$\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB}$$

$$\Rightarrow \quad \mathbf{p} - \mathbf{a} = \frac{3}{5}(\mathbf{b} - \mathbf{a})$$

$$\Rightarrow \quad 5\mathbf{p} - 5\mathbf{a} = 3\mathbf{b} - 3\mathbf{a}$$

$$\Rightarrow \quad \mathbf{p} = \frac{1}{5}(3\mathbf{b} + 2\mathbf{a})$$

$$\therefore \quad \mathbf{p} = \frac{1}{5}\left[3\begin{pmatrix} 4 \\ 4 \\ -3 \end{pmatrix} + 2\begin{pmatrix} 4 \\ -6 \\ 12 \end{pmatrix}\right]$$

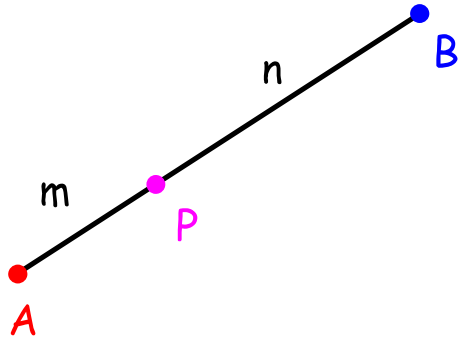
$$\Rightarrow \quad \mathbf{p} = \frac{1}{5}\begin{pmatrix} 20 \\ 0 \\ 15 \end{pmatrix}$$

$$\Rightarrow \quad \mathbf{p} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$$

P (4, 0, 3)

Sep 14-08:47

Alternative way (Section Formula)



If P divides AB in the ratio m : n, then :

$$p = \left(\frac{n}{m+n}\right)a + \left(\frac{m}{m+n}\right)b$$

Feb 10-15:14

Example 2

Show that $A(-1, 4, 2)$, $B(5, 1, 11)$ and $C(7, 0, 14)$ are collinear; find the ratio in which B divides AC.

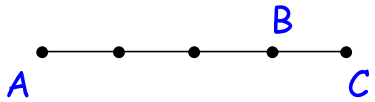
We have,

$$\overrightarrow{AB} = \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

As $\overrightarrow{AB} = 3\overrightarrow{BC}$, and B is common to AB and BC, A, B and C are collinear.

As $\overrightarrow{AB} = 3\overrightarrow{BC}$, the following diagram shows where B is in relation to A and C.



AB is 3 times the length of BC.

$$AB : BC = 3 : 1$$
(B divides AC in the ratio $3 : 1$)

CfE Higher Maths

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pg. 112-3 Ex. 5E All Q

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