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Exponentials and Logarithms - Lesson 2

Solving Logarithmic Equations

LI

- Solve equations involving logarithms with the same base.

SC

- Logarithmic Rules.

If $\log_a b = \log_a c$, what is the connection between b and c ?

$$b = c$$

('Cancel' the logs)

Example 1

Solve $\log_2 x = 5$ for x .

Method 1

$$\log_2 x = 5$$

$$\therefore 2^5 = x$$

$$\Rightarrow \boxed{x = 32}$$

Method 2

$$\log_2 x = 5$$

$$\therefore \log_2 x = 5 \log_2 2$$

$$\Rightarrow \log_2 x = \log_2 2^5$$

$$\therefore x = 2^5$$

$$\Rightarrow \boxed{x = 32}$$

Example 2

Solve $\log_e (x / 5) = 3$, writing the answer as an exact value.

$$\log_e (x / 5) = 3$$

$$\therefore \log_e (x / 5) = 3 \log_e e$$

$$\Rightarrow \log_e (x / 5) = \log_e e^3$$

$$\therefore x / 5 = e^3$$

$$\Rightarrow \boxed{x = 5 e^3}$$

Example 3

Solve $9^x = 4$ for x (to 3 d.p.).

$$9^x = 4$$

$$\therefore \log_e 9^x = \log_e 4$$

$$\Rightarrow x (\log_e 9) = \log_e 4$$

$$\Rightarrow x = (\log_e 4) / (\log_e 9)$$

$$\Rightarrow x = 0.631$$

(Try it with base 10 - should get same answer)

Example 4

Solve $2 \log_a x + \log_a 4 = \log_a 8$ for $x > 0$.

$$2 \log_a x + \log_a 4 = \log_a 8$$

$$\Rightarrow \log_a x^2 + \log_a 4 = \log_a 8$$

$$\Rightarrow \log_a 4x^2 = \log_a 8$$

$$\therefore 4x^2 = 8$$

$$\Rightarrow x^2 = 2$$

$$\Rightarrow \underline{x = \pm\sqrt{2}}$$

As $x > 0$, $x = \sqrt{2}$.

Example 5

Solve $\log_2 (3x - 5) - \log_2 (x + 2) = 1$ for x .

$$\log_2 (3x - 5) - \log_2 (x + 2) = 1$$

$$\Rightarrow \log_2 [(3x - 5) / (x + 2)] = \log_2 2$$

$$\therefore (3x - 5) / (x + 2) = 2$$

$$\Rightarrow 3x - 5 = 2(x + 2)$$

$$\Rightarrow 3x - 5 = 2x + 4$$

$$\Rightarrow x = 9$$

Example 6

Solve $\log_a (2x + 1) + \log_a (3x - 10) = \log_a (11x)$
for $x > 0$.

$$\log_a (2x + 1) + \log_a (3x - 10) = \log_a (11x)$$

$$\Rightarrow \log_a [(2x + 1)(3x - 10)] = \log_a (11x)$$

$$\therefore (2x + 1)(3x - 10) = 11x$$

$$\Rightarrow 6x^2 - 17x - 10 = 11x$$

$$\Rightarrow 6x^2 - 28x - 10 = 0$$

$$\Rightarrow 3x^2 - 14x - 5 = 0$$

$$\Rightarrow (3x + 1)(x - 5) = 0$$

$$\therefore \underline{x = -1/3, x = 5}$$

As $x > 0$, $x = 5$.

CfE Higher Maths

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pg. 13 - 14 Ex. 1F All Q (except 2 g)

pg. 15 Ex. 1G All Q (except 2 d)

