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Solving Trigonometric Equations - Lesson 2

Solving Linear Trigonometric Equations with Wave Functions

LI

- Solve trigonometric equations of the form $a \sin px + b \cos px + c = 0$ for x in degrees or radians.

SC

- Wave functions.
- Solve linear trig. equations.

Strategy

- Use one of the four addition formulae to write $a \sin px + b \cos px$ in the form of a single sine or single cosine.
- Solve the resulting linear trig. equation.

Example 1

Write $5 \cos x^\circ + 12 \sin x^\circ$ in the form $k \cos(x - \alpha)^\circ$, where $k > 0$ and $0 \leq \alpha < 360$.

Hence solve $5 \cos x^\circ + 12 \sin x^\circ - 13 = 0$
$0 \leq x < 360$.

$$\begin{aligned} k \cos(x - \alpha)^\circ &= k(\cos x^\circ \cos \alpha^\circ + \sin x^\circ \sin \alpha^\circ) \\ &= (k \cos \alpha^\circ) \cos x^\circ + (k \sin \alpha^\circ) \sin x^\circ \\ &= 5 \cos x^\circ + 12 \sin x^\circ \end{aligned}$$

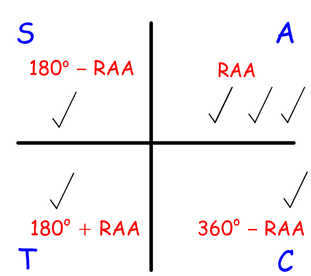
$$\therefore \quad k \sin \alpha^\circ = 12 \quad (1)$$
$$k \cos \alpha^\circ = 5 \quad (2)$$

$$k = \sqrt{12^2 + 5^2}$$
$$\underline{k = 13}$$

$$\tan \alpha^\circ = \frac{12}{5} \quad (3)$$
$$\text{RAA} = \tan^{-1}(12/5)$$
$$\text{RAA} = 67.4^\circ$$

$$k \sin \alpha^\circ = 12 \Rightarrow \sin \alpha^\circ > 0$$
$$k \cos \alpha^\circ = 5 \Rightarrow \cos \alpha^\circ > 0$$
$$\tan \alpha^\circ = \frac{12}{5} \Rightarrow \tan \alpha^\circ > 0$$

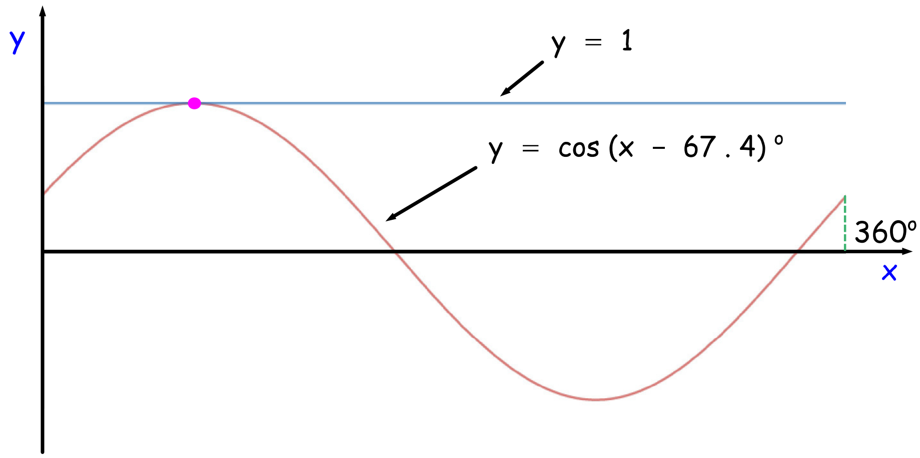
$$\therefore \quad \underline{\alpha^\circ = 67.4^\circ}$$



$$\boxed{5 \cos x^\circ + 12 \sin x^\circ = 13 \cos(x - 67.4)^\circ}$$

To solve $5\cos x^\circ + 12\sin x^\circ - 13 = 0$ $(0 \leq x < 360)$, we use the previous result to write,

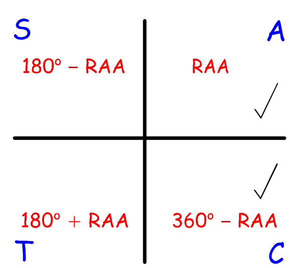
$$13\cos(x - 67.4)^\circ - 13 = 0$$
$$\cos(x - 67.4)^\circ = 1$$



1 solution expected

$$\cos(x - 67.4)^\circ = 1$$
$$\therefore \quad RAA = \cos^{-1} 1$$
$$\Rightarrow \quad \underline{RAA = 0^\circ}$$

cos is + ve



$$\therefore \quad x^\circ - 67.4^\circ = 0^\circ, 360^\circ$$
$$\Rightarrow \quad x^\circ = 67.4^\circ, 427.4^\circ$$

As 427.4° is outside the given range of $0 \leq x < 360$, we reject it.

$$\boxed{x^\circ = 67.4^\circ}$$

Example 2

Solve $8\cos 2x - 6\sin 2x - 5 = 0$,
where $0 \le x \le 2\pi$.

We have a choice of using any one of the four addition formulae; it's best (but not essential) to pick one that has a similar form to 'a cos 2x − b cos 2x', so that we avoid negatives for the 'k sin α' and 'k cos α' equations.

$$\begin{aligned} k\cos(2x + \alpha) &= k(\cos 2x\cos\alpha - \sin 2x\sin\alpha) \\ &= (k\cos\alpha)\cos 2x - (k\sin\alpha)\sin 2x \\ &= 8\cos 2x - 6\sin 2x \end{aligned}$$

$$\therefore \quad \begin{aligned} k\sin\alpha &= 6 \qquad (1) \\ k\cos\alpha &= 8 \qquad (2) \end{aligned}$$

$$k = \sqrt{6^2 + 8^2} \qquad\qquad \tan\alpha = \frac{6}{8} \qquad (3)$$

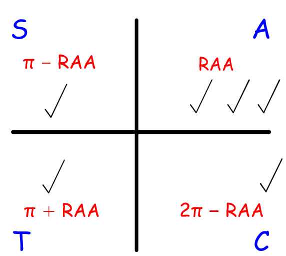
$$\underline{k = 10} \qquad\qquad RAA = \tan^{-1}(3/4)$$

$$RAA = 0.643\ldots$$

$$\begin{aligned} k\sin\alpha &= 6 &&\Rightarrow &\sin\alpha &> 0 \\ k\cos\alpha &= 8 &&\Rightarrow &\cos\alpha &> 0 \\ \tan\alpha &= \frac{3}{4} &&\Rightarrow &\tan\alpha &> 0 \end{aligned}$$

everything here is positive, thanks to our initial choice of addition formula

$$\therefore \quad \underline{\alpha = 0.643\ldots}$$

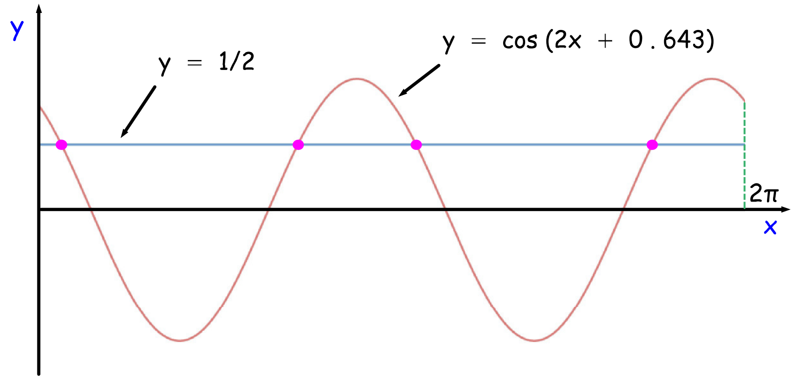


$$\boxed{8\cos 2x - 6\sin x = 10\cos(2x + 0.643\ldots)}$$

To solve $8\cos 2x - 6\sin 2x - 5 = 0$ $(0 \le x \le 2\pi)$, we use the previous result to write,

$$10\cos(2x + 0.643\ldots) - 5 = 0$$

$$\cos(2x + 0.643\ldots) = 1/2$$



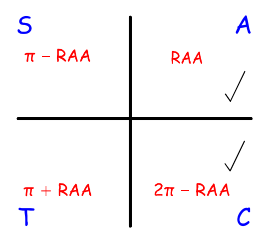
4 solutions expected

$$\cos(2x + 0.643\ldots) = 1/2$$

$\therefore$ $\text{RAA} = \cos^{-1}(1/2)$

$\Rightarrow$ $\underline{\text{RAA} = \pi/3 = 1.047\ldots}$

cos is + ve



$\therefore$ $2x + 0.643\ldots = 1.047\ldots, 2\pi - 1.047\ldots$

$\Rightarrow$ $2x = 0.403\ldots, 4.592\ldots$

There are 2 more solutions, so keep adding 2π until $2x$ is between 0 and $4\pi = 12.566\ldots$ (as $0 \le x \le 2\pi, 0 \le 2x \le 4\pi$). So,

$$2x = 0.403\ldots, 4.592\ldots$$
$$0.403\ldots + 2\pi,$$
$$5.235\ldots + 2\pi$$

$$\boxed{x = 0.202, 2.296, 3.343, 5.438 \text{ (to 3 d.p.)}}$$

Mar 3-16:58

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