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Unit 2 : Arithmetic and Algebra of Complex Numbers - Lesson 2

Solving Complex Quadratics and Other Simple Equations

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- Solve complex quadratics of the form $a z^2 + b z + c = 0$.
- Solve complex equations of the form $z^2 = a + b i$.
- Solve other simple equations.

<u>SC</u>

- Quadratic formula.
- Equating real and imaginary parts.

Example 1
Solve
$$z^2 - 2z + 5 = 0$$
 for z.

$$z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$\Rightarrow z = \frac{2 \pm \sqrt{-16}}{2}$$

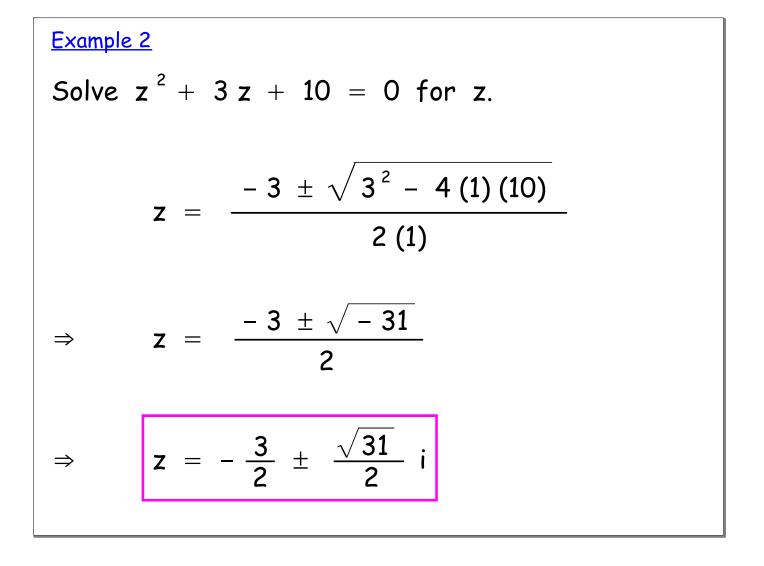
$$\Rightarrow z = \frac{2 \pm \sqrt{-16}}{2}$$

$$\Rightarrow z = \frac{2 \pm \sqrt{(16)(-1)}}{2}$$

$$\Rightarrow z = \frac{2 \pm 4\sqrt{-1}}{2}$$

$$\Rightarrow z = \frac{2 \pm 4i}{2}$$

$$\Rightarrow z = 1 \pm 2i$$



Example 3

Find two numbers that have a sum of 12 and a product of 40.

Let the two numbers be a and b. We require,

a + b = 12a b = 40

Solving the second equation for b and substituting into the first equation gives,

 $a + \frac{40}{a} = 12$ $\Rightarrow a^{2} + 40 = 12 a$ $\Rightarrow a^{2} - 12 a + 40 = 0$

This is a quadratic in a. Solving it gives (check !),

a = 6 + 2i, 6 - 2i

The first equation (a + b = 12) then gives the corresponding solutions for b:

- $b = 12 (6 + 2i) \Rightarrow b = 6 2i$
- $b = 12 (6 2i) \Rightarrow b = 6 + 2i$

So, $a = 6 + 2i \Rightarrow b = 6 - 2i$ (and vice versa); the numbers are thus,

6 + 2 i, 6 - 2 i

Example 4 Find the square roots of 5 + 12 i. Let a + bi be a square root of 5 + 12 i, i.e., $a + bi = \sqrt{5 + 12 i}$ $\therefore \qquad (a + bi)^2 = 5 + 12 i$ $\Rightarrow \qquad (a + bi)(a + bi) = 5 + 12 i$ $\Rightarrow \qquad (a^2 - b^2) + (2 a b)i = 5 + 12 i$

Equating real and imaginary parts gives,

$$a^{2} - b^{2} = 5$$

2 a b = 12

Solving the second equation for b (b = 6/a) and substituting into the first equation gives,

$$a^{2} - \frac{36}{a^{2}} = 5$$

$$\Rightarrow \qquad a^{4} - 36 = 5a^{2}$$

$$\Rightarrow \qquad a^{4} - 5a^{2} - 36 = 0$$

This is a quadratic in the variable a². Factorising gives,

$$(a^{2} - 9)(a^{2} + 4) = 0$$

 $\Rightarrow a^{2} - 9 = 0, a^{2} + 4 = 0$

The second equation $(a^2 = -4)$ has no solutions for a (as a is a real number). So,

$$\underline{a = 3} \Rightarrow b = 6/3 \Rightarrow \underline{b = 2}$$

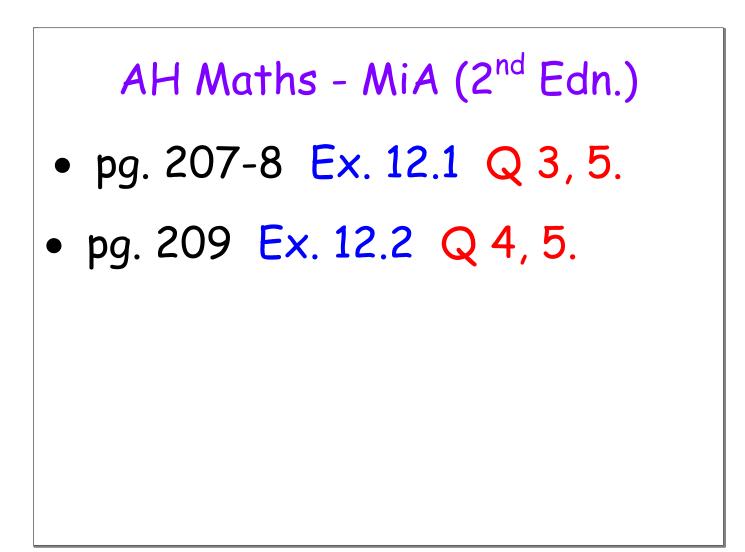
$$\underline{a = -3} \Rightarrow b = 6/(-3) \Rightarrow \underline{b = -2}$$

$$\therefore \sqrt{5 + 12i} = 3 + 2i, -3 - 2i$$

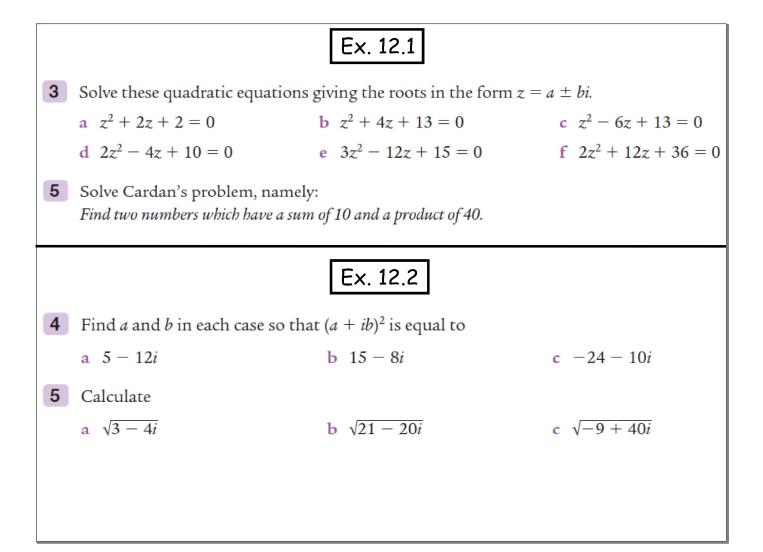
Example 5
Solve for z,

$$z + 2\overline{z} = 6 + 8i$$

Let $z = x + yi$. Then,
 $x + yi + 2(x - yi) = 6 + 8i$
 $\Rightarrow x + 2x + yi - 2yi = 6 + 8i$
 $\Rightarrow 3x + (-y)i = 6 + 8i$
Equating real and imaginary parts gives,
 $3x = 6 \Rightarrow x = 2$
 $-y = 8 \Rightarrow y = -8$
 $\therefore z = 2 - 8i$



Solve for z: 1) $z - 3\overline{z} = 9 + 12i$. 2) $3\overline{z} + iz = 10 - 18i$.



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Answers to AH Maths (MiA), pg. 207-8, Ex. 12.1
3 a
$$-1 \pm i$$
 b $-2 \pm 3i$ c $3 \pm 2i$
d $1 \pm 2i$ e $2 \pm i$ f $-3 \pm 3i$
5 $a + b = 10$ and $ab = 40 \Rightarrow a^2 - 10a + 40 = 0 \Rightarrow$
 $a = 5 \pm \sqrt{15}i$
Answers to AH Maths (MiA), pg. 209, Ex. 12.2
4 a $-3 + 2i$ and $3 - 2i$ b $-4 + i$ and $4 - i$
c $-1 + 5i$ and $1 - 5i$
5 a $-2 + i$ and $2 - i$ b $-5 + 2i$ and $5 - 2i$
c $4 + 5i$ and $-4 - 5i$

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Solve for z: 1) $z - 3\overline{z} = 9 + 12i$. z = -(9/2) + 3i2) $3\overline{z} + iz = 10 - 18i$. z = 6 + 8i