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Vectors - Lesson 2

Position Vectors, Coordinates and Equilibrium

LI

- Know what a Position Vector is.
- Know what is means for vectors to be in Equilibrium.

SC

- Arithmetic.

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A position vector is a vector relative to a reference position

$\overrightarrow{OA}$ is the position vector of A relative to O.

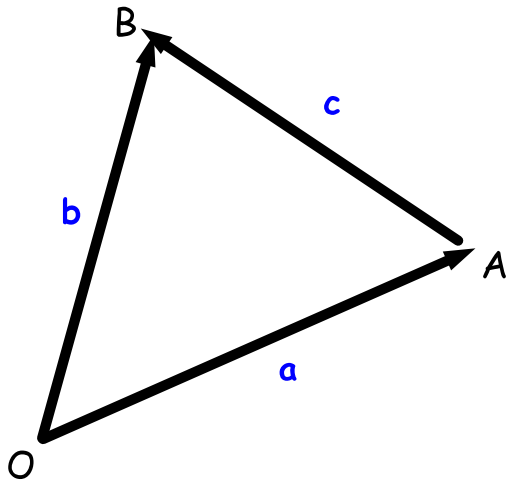
$\overrightarrow{OB}$ is the position vector of B relative to O.

$\overrightarrow{AB}$ is the position vector of B relative to A.

$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$ Head-to-Tail Rule

$\therefore \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\Rightarrow \quad \boxed{\overrightarrow{AB} = \mathbf{b} - \mathbf{a}}$



Vectors are in Equilibrium if their resultant equals the zero vector

Example 1

If A, B and C are the points $(-3, 2)$, $(1, 0)$ and $(6, 5)$, find $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$.

The position vectors of points A, B and C are :

$$\mathbf{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{c} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$

So,

$$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \boxed{\begin{pmatrix} 4 \\ -2 \end{pmatrix}}$$

$$\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \boxed{\begin{pmatrix} 5 \\ 5 \end{pmatrix}}$$

$$\overrightarrow{CA} = \mathbf{a} - \mathbf{c} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 6 \\ 5 \end{pmatrix} = \boxed{\begin{pmatrix} -9 \\ -3 \end{pmatrix}}$$

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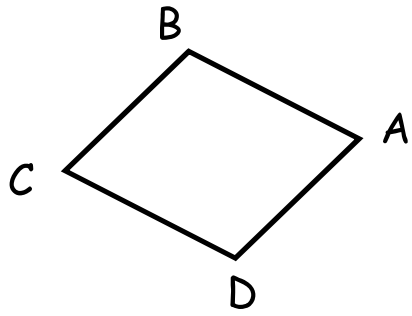
Example 2

ABCD is a parallelogram with A (3, 8), B (– 4, 11), and C (– 1, 6). Find the coordinates of D.

Making a sketch shows that the vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are equal.



$$\overrightarrow{AB} = \overrightarrow{DC}$$

$$\therefore \quad \mathbf{b} - \mathbf{a} = \mathbf{c} - \mathbf{d}$$

$$\Rightarrow \quad \mathbf{d} = \mathbf{c} + \mathbf{a} - \mathbf{b}$$

$$\Rightarrow \quad \mathbf{d} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -4 \\ 11 \end{pmatrix}$$

$$\Rightarrow \quad \mathbf{d} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

D (6, 3)

Example 3

The three forces $\begin{pmatrix} -6 \\ -4 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

are in equilibrium.

Find the values of a, b and c.

As the three forces are in equilibrium, their resultant equals the zero vector. So,

$$\begin{pmatrix} -6 \\ -4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} -3 \\ -2 \\ 7 \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -7 \end{pmatrix}$$

$$a = 3, b = 2, c = -7$$

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