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Unit 1 : Partial Fractions - Lesson 2

Long Division and Partial Fractions

LI

- Long divide polynomials.
- Find partial fractions for improper rational functions.

SC

- Algebra.

Example

Resolve $\frac{x^3 + 4x^2 - x + 2}{x^2 + x}$ into a polynomial

plus partial fractions.

$$\begin{array}{r}
 \begin{array}{c} x^3 \\ x^2 \\ \hline 3x^2 \end{array} \quad x^2 + x \quad \left[\begin{array}{r} x^3 + 4x^2 - x + 2 \\ x^3 + x^2 \\ \hline 3x^2 - x + 2 \\ 3x^2 + 3x \\ \hline -4x + 2 \end{array} \right] \\
 \begin{array}{c} x^3 \\ x^2 \\ \hline 3x^2 \end{array} \quad x^2 + x \quad \begin{array}{c} x^3 + 4x^2 - x + 2 \\ x^3 + x^2 \\ \hline 3x^2 - x + 2 \\ 3x^2 + 3x \\ \hline -4x + 2 \end{array}
 \end{array}$$

CONSTANT

Stop long division when this polynomial has degree less than this one

Hence,

$$\frac{x^3 + 4x^2 - x + 2}{x^2 + x} = (x + 3) + \frac{(-4x + 2)}{x^2 + x}$$

Now do partial fraction on this

$$\frac{(-4x + 2)}{x^2 + x} = \frac{(-4x + 2)}{x(x + 1)} = \frac{A}{x} + \frac{B}{x + 1}$$

$$-4x + 2 = A(x + 1) + Bx$$

$$\underline{x = 0 :}$$

$$2 = A(0 + 1)$$

$$\Rightarrow \underline{2 = A}$$

$$\underline{x = -1 :}$$

$$-4(-1) + 2 = B(-1)$$

$$\Rightarrow 4 + 2 = -B$$

$$\Rightarrow \underline{B = -6}$$

$$\therefore \frac{(-4x + 2)}{x^2 + x} = \frac{2}{x} - \frac{6}{x + 1}$$

$$\Rightarrow \boxed{\frac{x^3 + 4x^2 - x + 2}{x^2 + x} = (x + 3) + \frac{2}{x} - \frac{6}{x + 1}}$$

AH Maths - MiA (2nd Edn.)

- pg. 26-7 Ex. 2.5 All Q.

1 Resolve each of the improper rational functions into a polynomial function plus partial fractions.

a $\frac{x^2 - x + 6}{x^2 + x - 2}$

b $\frac{x^3 - x^2 - 5x - 7}{x^2 - 2x - 3}$

c $\frac{x^3 - 5x^2 + 11x - 12}{x^2 - 5x + 6}$

d $\frac{2x^2 - 7}{x^2 - 4}$

e $\frac{x^3 - 3x}{x^2 - x - 2}$

f $\frac{x^2}{(x - 1)^2}$

g $\frac{x^3 + 2}{x(x^2 - 3)}$

h $\frac{x^4 + 1}{x^3 + 2x}$

i $\frac{3x^4 + 4x^3 + 6x^2 + 2x - 1}{x^3 + x^2}$

j $\frac{(x + 2)(x - 2)}{(x + 1)(x - 1)}$

k $\frac{(x + 3)(x - 1)}{(x + 2)(x + 1)}$

l $\frac{(x + 1)(x - 2)(x + 3)}{(x - 1)(x - 3)}$

2 Assuming $\frac{x^2}{(x + a)(x + b)}$ can be expressed in the form $A + \frac{B}{x + a} + \frac{C}{x + b}$ find expressions for A, B and C in terms of a and b.

Answers to AH Maths (MiA), pg. 26-7, Ex. 2.5

1 a $1 - \frac{4}{x+2} + \frac{2}{x-1}$

b $x+1 - \frac{1}{x-3} + \frac{1}{x+1}$

c $x + \frac{3}{x-3} + \frac{2}{x-2}$

d $2 + \frac{1}{4(x-2)} - \frac{1}{4(x+2)}$

e $x+1 - \frac{2}{3(x+1)} + \frac{2}{3(x-2)}$

f $1 + \frac{2}{x-1} + \frac{1}{(x-1)^2}$

g $1 - \frac{2}{3x} + \frac{2-3\sqrt{3}}{6(x+\sqrt{3})} + \frac{2+3\sqrt{3}}{6(x-\sqrt{3})}$

h $x + \frac{1}{2x} - \frac{5x}{2(x^2+2)}$

i $3x+1 - \frac{1}{x^2} + \frac{3}{x} + \frac{2}{x+1}$

j $1 + \frac{3}{2(x+1)} - \frac{3}{2(x-1)}$

k $1 + \frac{3}{x+2} - \frac{4}{x+1}$

l $x+6 + \frac{4}{x-1} + \frac{12}{x-3}$

2 $\frac{x^2}{(x+a)(x+b)} = A + \frac{B}{x+a} + \frac{C}{x+b} =$
 $\frac{Ax^2 + x(Aa + Ab + B + C) + (Aab + Bb + Ca)}{(x+a)(x+b)}$

Equating coefficients in the numerator gives: $A = 1$,

$B = \frac{-a^2}{a-b}$, $C = \frac{b^2}{a-b}$