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Differentiation and Properties of Functions - Lesson 2

Increasing and Decreasing Functions

LI

- Decide whether or not a function is increasing or decreasing at a specific x - value.
- Decide for which x - values a function is increasing or decreasing.

SC

- Differentiate functions.
- Graphs of linear, quadratic and cubic functions.
- Solving quadratic inequations.

A function $y = f(x)$ is increasing at $x = a$ if : $\left(\frac{dy}{dx}\right)_{x=a} > 0$	A function $y = f(x)$ is decreasing at $x = a$ if : $\left(\frac{dy}{dx}\right)_{x=a} < 0$

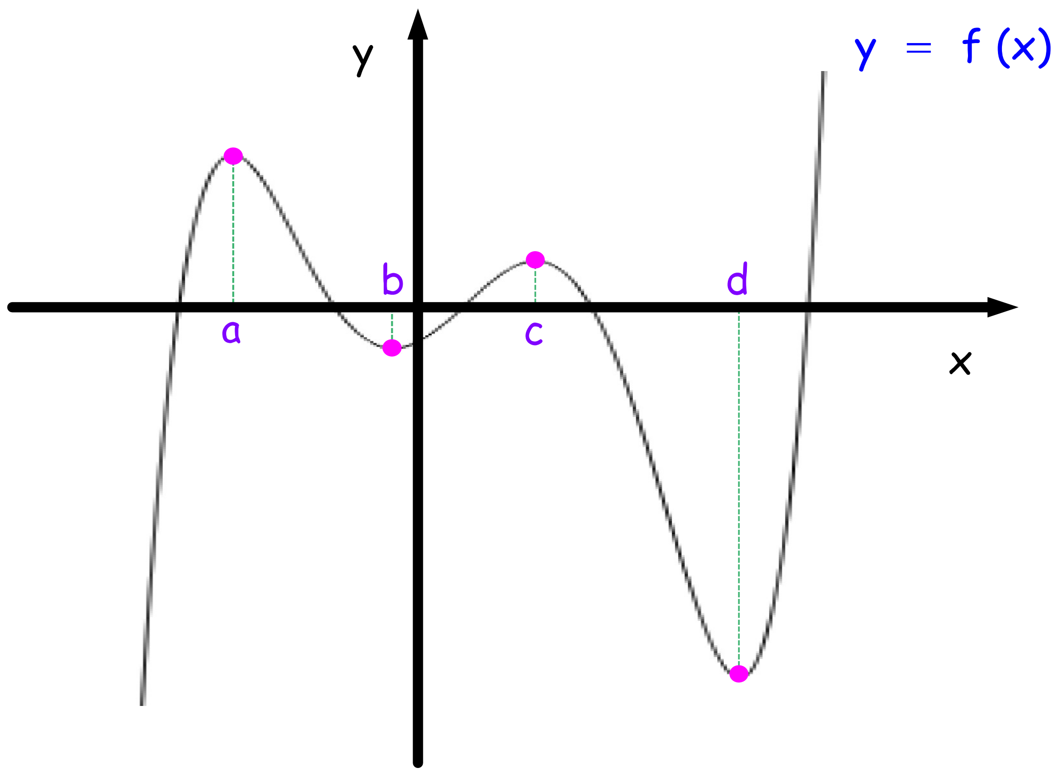
An interval is a set of values (usually x)

A function is increasing on an interval if the derivative is positive for every x - value in that interval	A function is decreasing on an interval if the derivative is negative for every x - value in that interval

A function is stationary at $x = a$ if :

$$\left(\frac{dy}{dx}\right)_{x=a} = 0$$

Non-increasing	Non-decreasing
$\left(\frac{dy}{dx}\right) \leq 0$	$\left(\frac{dy}{dx}\right) \geq 0$



f is stationary when $x = a, b, c, d$.

f is increasing when $x < a$, $b < x < c$ and $x > d$.

f is decreasing when $a < x < b$ and $c < x < d$.

f is non-increasing when $a \leq x \leq b$ and $c \leq x \leq d$.

f is non-decreasing when $x \leq a$, $b \leq x \leq c$ and $x \geq d$.

Example 1

Show that the function $f(x) = x^3 - 6x + 15$ is increasing when $x = 2$.

$$f(x) = x^3 - 6x + 15$$

$$\therefore \quad f'(x) = 3x^2 - 6$$

$$\therefore \quad f'(2) = 3(2)^2 - 6$$

$$\Rightarrow \quad \underline{f'(2) = 6}$$

$$\boxed{\text{As } f'(2) = 6 > 0,\ f \text{ is increasing at } x = 2}$$

Example 2

Show that the function $g(x) = x^2 + \frac{64}{x}$

is decreasing when $x = -4$.

$$g(x) = x^2 + \frac{64}{x}$$

$$g(x) = x^2 + 64x^{-1}$$

$$\therefore \quad g'(x) = 2x - 64x^{-2}$$

$$\Rightarrow \quad g'(x) = 2x - \frac{64}{x^2}$$

$$\therefore \quad g'(-4) = 2(-4) - \frac{64}{(-4)^2}$$

$$\Rightarrow \quad g'(-4) = -8 - 4$$

$$\Rightarrow \quad \underline{g'(-4) = -12}$$

As $g'(-4) = -12 < 0$, g is decreasing at $x = -4$.

Example 3

Show that $m(x) = \dfrac{11}{x^7}$ is decreasing for all $x \neq 0$.

$$m(x) = \frac{11}{x^7}$$

$$m(x) = 11x^{-7}$$

$$\therefore \quad m'(x) = -77x^{-8}$$

$$\Rightarrow \quad m'(x) = -\frac{77}{x^8}$$

When x is positive or negative, $x^8 > 0$; hence,

$\dfrac{77}{x^8} > 0$; hence, $-\dfrac{77}{x^8} = m'(x) < 0$.

Thus, m is decreasing for all $x \neq 0$.

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Example 4

Show that $Q(x) = x^3 + 3x^2 + 4x - 19$ is always increasing.

$$Q(x) = x^3 + 3x^2 + 4x - 19$$

$$\therefore \quad Q'(x) = 3x^2 + 6x + 4$$

To show that Q is always increasing, we must show that the quadratic $3x^2 + 6x + 4$ is always positive; this means showing that this quadratic is always above the x- axis; this means showing that this quadratic has no real roots; this means showing that the discriminant of this quadratic is always negative.

Discriminant (D) of $3x^2 + 6x + 4$ is,

$$D = 6^2 - 4(3)(4)$$

$$\Rightarrow D = 36 - 48$$

$$\Rightarrow D = -12 < 0$$

As $D < 0$ for all x values, $3x^2 + 6x + 4 = 0$ has no real roots; hence, $3x^2 + 6x + 4$ does not cross the x - axis; hence, $m'(x) > 0$ for all x; hence, m is always increasing.

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Example 5

Find the range of values of x for which $r(x) = x(x^3 - 32)$ is increasing.

$$r(x) = x(x^3 - 32)$$

$$r(x) = x^4 - 32x$$

$$\therefore \quad r'(x) = 4x^3 - 32$$

r increasing means $r'(x) > 0$. So,

$$4x^3 - 32 > 0$$

$$\Rightarrow \quad 4x^3 > 32$$

$$\Rightarrow \quad x^3 > 8$$

$$\Rightarrow \quad \boxed{x > 2}$$

Example 6

Find the range of values of x for which

$T(x) = x^3 + \frac{5}{2}x^2 - 2x + 7$ is :

(a) decreasing.

(b) increasing.

(c) non-decreasing.

(d) non-increasing.

$$T(x) = x^3 + \frac{5}{2}x^2 - 2x + 7$$

$$\therefore \quad T'(x) = 3x^2 + 5x - 2$$

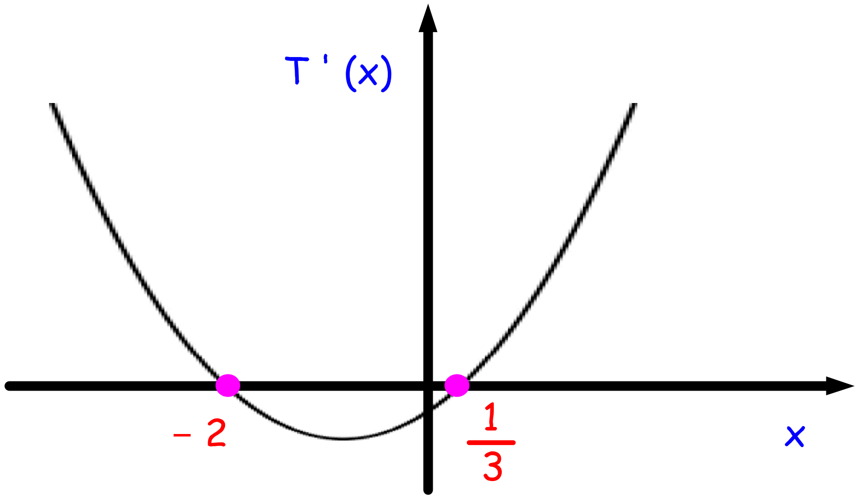
To find where T' is $< 0, > 0, \geq 0$ and ≤ 0, we need to sketch the graph of the quadratic $T'(x) = 3x^2 + 5x - 2$.

To do this, find the roots (if any) of $T'(x) = 0$. The discriminant of $3x^2 + 5x - 2$ is $49 > 0$, so there are 2 distinct real roots.

$$3x^2 + 5x - 2 = 0$$

$$\therefore \quad (3x - 1)(x + 2) = 0$$

$$\Rightarrow \quad \underline{x = -2, x = \frac{1}{3}}$$



(a) T is decreasing when $T' < 0$, so,

$$\boxed{\text{T is decreasing when } -2 < x < \frac{1}{3}}$$

(b) T is increasing when $T' > 0$, so,

$$\boxed{\text{T is increasing when } x < -2 \text{ and } x > \frac{1}{3}}$$

(c) T is non-decreasing when $T' \geq 0$, so,

$$\boxed{\text{T is non-decreasing when } x \leq -2 \text{ and } x \geq \frac{1}{3}}$$

(d) T is non-increasing when $T' \leq 0$, so,

$$\boxed{\text{T is non-increasing when } -2 \leq x \leq \frac{1}{3}}$$

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