26 / 1 / 18

Matrices and Systems of Equations - Lesson 2

Ill-Conditioned Systems

LI

- Determine when a system of equations ill-conditioned.

SC

- Gradients of straight lines.

A system of equations is ill-conditioned if changing some of the entries of the Augmented Matrix by a small amount results in a large change in the solution of the system

Example 1

The system,

$$\begin{aligned} 2x + y &= 4 \\ 2x + 1.01y &= 4.02 \end{aligned}$$

has solutions $x = 1, y = 2$. Changing the coefficients slightly to give the system,

$$\begin{aligned} 2x + y &= 3.8 \\ 2.02x + y &= 4.02 \end{aligned}$$

gives solutions $x = 11, y = -18.2$. As a small change in the coefficients of the original system gives rise to a large change to the solutions of the new system, the system is ill-conditioned.

Graphical Interpretation

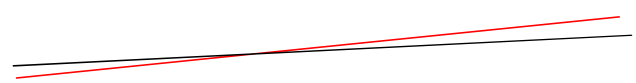
A system of 2 equations in 2 variables represents two straight lines; ill-conditioned systems can be related to the gradients of the lines.

$$a x + b y = e \Rightarrow y = -(a/b)\,x + e/b$$

$$c x + d y = f \Rightarrow y = -(c/d)\,x + f/d$$

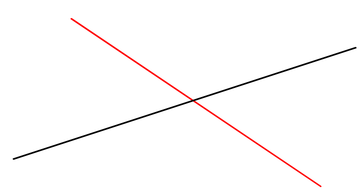
Changing a, b, c or d, means changing the gradient of at least one line.

If the original system has lines that are almost parallel (almost same gradients), then changing the gradient(s) results in a large change in the intersection point - the system is ill-conditioned :



$$-(a/b) \approx -(c/d) \text{ or } a d \approx b c$$

If the original system has lines that are not almost parallel (very different gradients), then changing the gradient(s) results in a small change in the intersection point - the system is not ill-conditioned :



$$-(a/b) \not\approx -(c/d) \text{ or } a d \not\approx b c$$

Example 2

Determine whether or not the following system is ill-conditioned :

$$2x + 9y = 17$$
$$3x - 5y = 4$$

We have,

$$ad = -10, \; bc = 27$$

$\boxed{\text{As } -10 \text{ is not close to } 27\text{, the system is not ill-conditioned}}$

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Example 3

Determine whether or not the following system is ill-conditioned :

$$2x + 9y = 17$$
$$3x + 14y = 4$$

We have,

$$ad = 28, bc = 27$$

As 28 is close to 27, the system is ill-conditioned

AH Maths - MiA (2^{nd} Edn.)

- pg. 273-4 Ex. 14.9 Q 2.

Ex. 14.9

2 Identify which of these systems you feel are ill-conditioned.

a $2x + 9y = 17$
$3x - 5y = 7$

b $9x + 8y = 1$
$8x + 7y = 1$

c $8x + 13y = 18$
$3x + 5y = 7$

d $9x - 10y = 91$
$8x - 9y = 82$

Answers to AH Maths (MiA), pg. 273-4, Ex. 14.9

2 a No
b Yes
c Yes
d Yes

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