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Linear and Parabolic Motion - Lesson 2

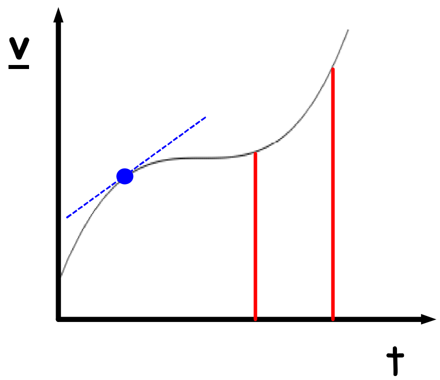
Graphs and Equations of Motion

LI

- Draw distance-time, velocity-time and acceleration-time graphs.
- Use these graphs to solve problems.
- Know, derive and use the 4 equations of motion for constant acceleration.

SC

- Area under a graph.
- Calculus.

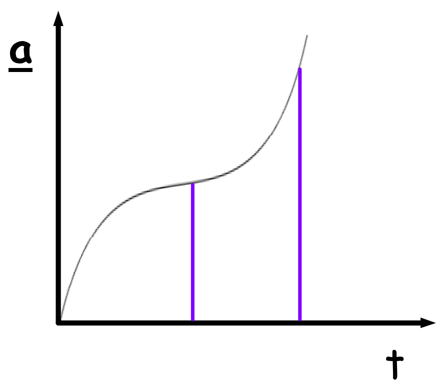


Gradient at a point on a velocity-time graph gives acceleration :

$$\underline{a} = \frac{d\underline{v}}{dt}$$

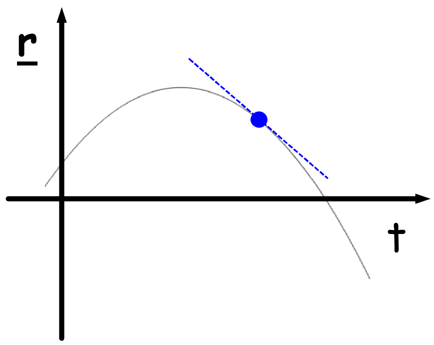
Area under a velocity (speed)-time graph gives displacement (distance) :

$$\underline{r} = \int \underline{v}\ dt$$



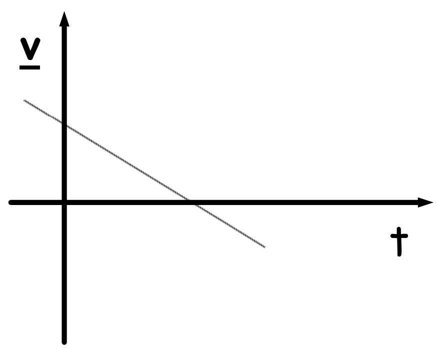
Area under an $\underline{a}$-t graph gives velocity :

$$\underline{v} = \int \underline{a}\ dt$$



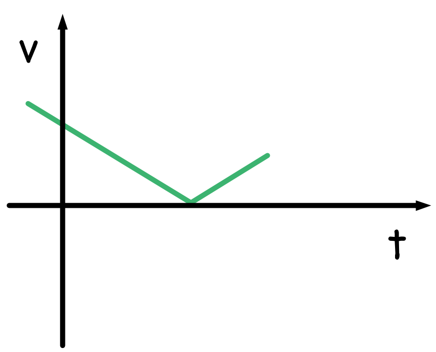
Gradient at a point on a displacement-time graph gives velocity :

$$\underline{v} = \frac{d\underline{r}}{dt}$$



Change any negatives to positive

Modulus of velocity-time graph gives speed-time graph

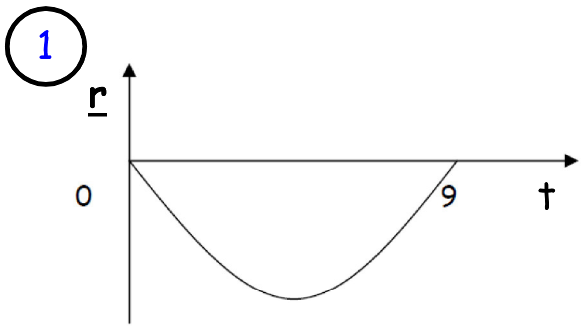


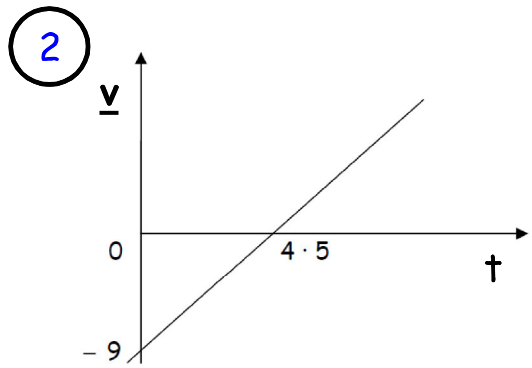
The gradient at a point on a distance-time graph ***does not always give speed***

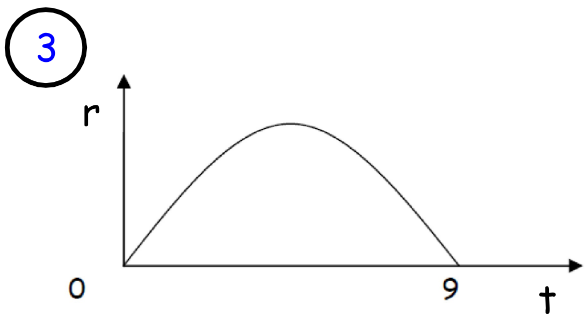
Example 1

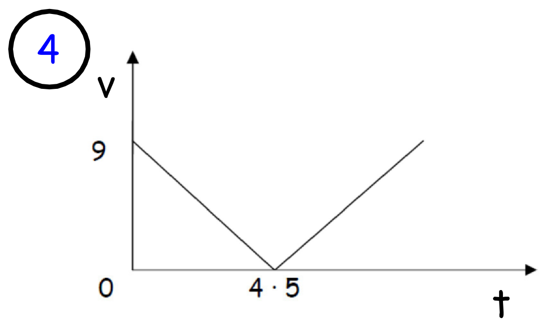
Sketch displacement-time, distance-time, velocity-time and speed-time graphs for a particle that has displacement vector $\underline{\mathbf{r}}(t) = (t^2 - 9t)\,\mathbf{i}$.

$$\underline{\mathbf{v}}(t) = (2t - 9)\,\mathbf{i}$$









To get from graph 1 to graph 4, the correct way is :

1 ⟶ 2 ⟶ 4

Differentiate displacement, then modulus
(not modulus first, then differentiate)

Standard Notation for Kinematical Quantities

For the above rectilinear motion :

- t : time (taken to go from A to B)
- u : initial speed (at A)
- v : final speed (at B)
- a : acceleration (between A and B)
- s : displacement (between A and B)

Derivation of Equations of Motion
for constant acceleration
(using calculus for the first 2 equations)

1st E of M :

$$\frac{dv}{dt} = a$$

$$\therefore \quad v = \int a \, dt$$

$$\Rightarrow \quad v = a t + C$$

Assuming that $v = u$ when $t = 0$, we have $C = u$. So,

$$\boxed{v = u + a t} \qquad \text{(no } s\text{)}$$

2nd E of M :

$$\frac{ds}{dt} = v$$

$$\therefore \quad s = \int v \, dt$$

$$\Rightarrow \quad s = \int (u + a t) \, dt$$

$$\Rightarrow \quad s = u t + (1/2) a t^2 + D$$

Assuming that $s = 0$ when $t = 0$, we have $D = 0$. So,

$$\boxed{s = u t + (1/2) a t^2} \qquad \text{(no } v\text{)}$$

3rd E of M :

Squaring the 1st E of M,

$$\Rightarrow \quad v^2 = (u + a t)^2$$

$$\Rightarrow \quad v^2 = u^2 + 2 u a t + a^2 t^2$$

$$\Rightarrow \quad v^2 = u^2 + 2 a (u t + (1/2) a t^2)$$

The bracketed term can be rewritten using the 2nd E of M. So,

$$\boxed{v^2 = u^2 + 2 a s} \qquad \text{(no } t\text{)}$$

4th E of M :

Rewriting the 3rd E of M and using a difference of 2 squares gives,

$$2 a s = v^2 - u^2$$

$$2 a s = (v - u)(v + u)$$

$(v - u)$ can be rewritten using the 1st E of M. So,

$$2 a s = a t (v + u)$$

$$\Rightarrow \quad \boxed{s = (1/2)(v + u) t} \qquad \text{(no } a\text{)}$$

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Example 2

A vehicle travelling at 54 km h^{-1} is brought to rest with uniform deceleration in 5 seconds.

Find the deceleration and the distance travelled during this deceleration.

$$+ \longrightarrow$$

$u = 54 \text{ km h}^{-1} = (54 \times 1\,000 \div 3\,600) \text{ m s}^{-1} = 15 \text{ m s}^{-1}$

$v = 0 \text{ m s}^{-1}$

$t = 5 \text{ s}$

$a =$

$s =$

To work out the acceleration first, use the 1st E of M :

$$v = u + at$$

$$\therefore \quad 0 = 15 + 5a$$

$$\Rightarrow \quad \underline{a = -3 \text{ m s}^{-2}}$$

$$\boxed{\text{Deceleration} = 3 \text{ m s}^{-2}}$$

To get the distance covered, it's best to use the 4th E of M (as it doesn't contain the acceleration) :

$$s = (1/2)(v + u)t$$

$$\therefore \quad s = (1/2)(0 + 15)(5)$$

$$\Rightarrow \quad \boxed{s = 37.5 \text{ m}}$$

Example 3

A ball is thrown vertically upwards from ground level with an initial velocity of 10 m/s, reaching a maximum height of 5 . 1 m and then returning to ground level.

Calculate the time of flight and the velocity upon reaching the ground.

Also, sketch displacement-time, velocity-time and acceleration-time graphs for the above motion.

$+\uparrow$

For the journey from the ground to the maximum height,

$$\begin{aligned} u &= 10 \text{ m s}^{-1} \\ v &= 0 \text{ m s}^{-1} \\ s &= 5.1 \text{ m} \\ a &= -9.8 \text{ m s}^{-2} \\ t &= \end{aligned}$$

The 1^{st} E of M gives,

$$v = u + at$$

$$\therefore \quad 0 = 10 - 9.8t$$

$$\Rightarrow \quad \underline{t = 50/49}$$

$$\therefore \quad \boxed{\text{Total flight time} = 100/49 \ (\approx 2.04) \text{ s}}$$

This can also be obtained directly from the 2^{nd} E of M :

$$\begin{aligned} u &= 10 \text{ m s}^{-1} \\ v &= \\ s &= 0 \text{ m} \\ a &= -9.8 \text{ m s}^{-2} \\ t &= \end{aligned}$$

$$s = ut + (1/2)at^2$$

$$\therefore \quad 0 = 10t - (1/2)(9.8)t^2$$

$$\Rightarrow \quad 0 = 10t - 4.9t^2$$

$$\Rightarrow \quad 0 = t(10 - 4.9t)$$

$$\Rightarrow \quad \underline{t = 0, \ t = (10/4.9)}$$

The first time refers to the start of motion; the second time is,

$$\boxed{t = 100/49 \ (\approx 2.04) \text{ s}}$$

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For the velocity upon reaching the ground, symmetry of motion gives the answer directly, but can be calculated too :

$u = 10 \text{ m s}^{-1}$
$s = 0 \text{ m}$
$a = -9.8 \text{ m s}^{-2}$
$t = 100/49 \text{ s}$
$v =$

The 1st E of M gives,

$$v = u + at$$

$$\therefore \quad v = 10 - 9.8\,(100/49)$$

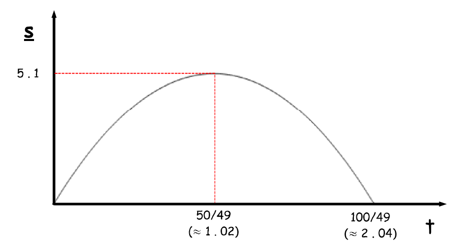
$$\Rightarrow \quad \boxed{v = -10 \text{ m s}^{-1}}$$

The displacement vector is,

$$\underline{s}(t) = (ut + (1/2)\,a t^2)\,\underline{j}$$

$$\Rightarrow \quad \underline{s}(t) = (10t - 4.9t^2)\,\underline{j}$$

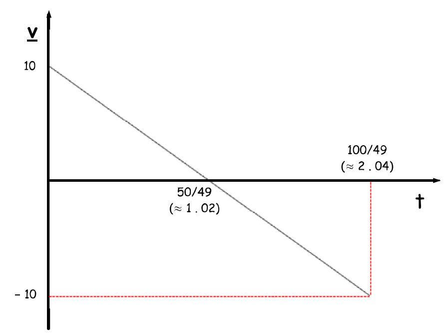
This is a quadratic in t :



The velocity is obtained by differentiating the displacement :

$$\underline{v}(t) = (10 - 9.8t)\,\underline{j}$$

This is a straight line with negative gradient :



The acceleration is obtained by differentiating the velocity :

$$\underline{a}(t) = -9.8\,\underline{j}$$

This is a constant function (horizontal line) :

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Example 4

A sprinter accelerates from rest at a constant rate of $2\ \mathrm{m\ s^{-2}}$ to $8\ \mathrm{m\ s^{-1}}$.

Find the time taken to reach $8\ \mathrm{m\ s^{-1}}$ and the distance covered during the 4^{th} second.

$u = 0\ \mathrm{m\ s^{-1}}$
$v = 8\ \mathrm{m\ s^{-1}}$
$a = 2\ \mathrm{m\ s^{-2}}$
$t =$
$s =$

To work out the time, use the 1^{st} E of M :

$$v = u + at$$

$$\therefore \quad 8 = 0 + 2t$$

$$\Rightarrow \quad \boxed{t = 4\ \mathrm{s}}$$

To get the distance covered during the 4^{th} second, calculate the distance travelled up to 4 s and subtract off the distance travelled up to 3 s.

$$s(t) = ut + (1/2)\,a\,t^2$$

$$\therefore \quad s(4) = (0)(4) + (1/2)(2)(4)^2$$

$$\Rightarrow \quad \underline{s(4) = 16}$$

$$s(3) = (0)(3) + (1/2)(2)(3)^2$$

$$\Rightarrow \quad \underline{s(3) = 9}$$

$$\therefore \quad s(4) - s(3) = 16 - 9$$

$$\Rightarrow \quad \underline{s(4) - s(3) = 7}$$

$$\therefore \quad \boxed{\text{Distance covered during } 4^{th} \text{ second} = 7\ \mathrm{m}}$$

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Blue Book

- pg. 27-29 Ex. 2 C Q 1, 23, 24, 26, 29, 32.
- pg. 31-32 Ex. 2 D Q 2, 4, 13, 15.
- pg. 36-38 Ex. 2 E Q 1 c, 2 a, 3, 6.