

Differential Calculus - Lesson 2

Derivatives of More Complicated Expressions

LI

- Differentiate expressions involving combinations of powers of x .

SC

- Sum and Difference Rules.
- Indices Rules.
- Expanding brackets.
- Splitting algebraic fractions.

Example 1

Differentiate $y = \frac{4}{x^3}$.

$$y = \frac{4}{x^3}$$

$$y = 4x^{-3}$$

$$\therefore y' = -12x^{-4}$$

$$\left(y' = -\frac{12}{x^4} \right)$$

Example 2

Differentiate $y = -\frac{7}{2x^6}$.

$$y = -\frac{7}{2x^6}$$

$$y = -\frac{7}{2} x^{-6}$$

$$\begin{aligned}\therefore y' &= 21x^{-7} \\ \left(y' = \frac{21}{x^7} \right)\end{aligned}$$

Example 3

Differentiate $y = (x + 2)(x - 7)$.

$$y = (x + 2)(x - 7)$$

$$y = x^2 + 2x - 7x - 14$$

$$y = x^2 - 5x - 14$$

$$\therefore \boxed{y' = 2x - 5}$$

Example 4

Differentiate $y = \frac{3 - x^5}{4x^7}$.

$$y = \frac{3 - x^5}{4x^7}$$

$$y = \frac{3}{4x^7} - \frac{x^5}{4x^7}$$

$$y = \frac{3}{4}x^{-7} - \frac{1}{4}x^{-2}$$

$$\therefore y' = -\frac{21}{4}x^{-8} + \frac{1}{2}x^{-3}$$

$$(y' = -\frac{21}{4x^8} + \frac{1}{2x^3})$$

Example 5

Differentiate $y = \frac{(x - 1)(x + 9)}{x^2}$.

$$y = \frac{(x - 1)(x + 9)}{x^2}$$

$$y = \frac{x^2 - x + 9x - 9}{x^2}$$

$$y = \frac{x^2 + 8x - 9}{x^2}$$

$$y = \frac{x^2}{x^2} + \frac{8x}{x^2} - \frac{9}{x^2}$$

$$y = 1 + 8x^{-1} - 9x^{-2}$$

$$\therefore y' = -8x^{-2} + 18x^{-3}$$

$$\left(y' = -\frac{8}{x^2} + \frac{18}{x^3} \right)$$

Example 6

Differentiate $y = (3 - \sqrt{x})(1 + \frac{1}{\sqrt{x}})$.

$$y = (3 - \sqrt{x})(1 + \frac{1}{\sqrt{x}})$$

$$y = 3 + \frac{3}{\sqrt{x}} - \sqrt{x} - 1$$

$$y = 2 + 3x^{-1/2} - x^{1/2}$$

$$\therefore y' = -\frac{3}{2}x^{-3/2} - \frac{1}{2}x^{-1/2}$$

$$(y' = -\frac{3}{2x^{3/2}} - \frac{1}{2x^{1/2}})$$

$$(y' = -\frac{3}{2\sqrt{x^3}} - \frac{1}{2\sqrt{x}})$$

Example 7

Prove that the derivative of a quadratic function is a linear function.

Let $y = f(x)$ be a quadratic function, i.e. let,

$$y = ax^2 + bx + c \quad (a \neq 0)$$

$$\therefore y' = 2ax + b$$

As $a \neq 0$, $2a \neq 0$; hence, the derivative $y' = 2ax + b$ is a linear function.

Example 8

Prove that the derivative of a cubic function is a quadratic function.

Let $y = f(x)$ be a cubic function, i.e. let,

$$y = ax^3 + bx^2 + cx + d \quad (a \neq 0)$$

$$\therefore y' = 3ax^2 + 2bx + c$$

As $a \neq 0$, $3a \neq 0$; hence, the derivative
 $y' = 3ax^2 + 2bx + c$ is a quadratic function.

CfE Higher Maths

pg. 215 - 217 Ex. 9B

Q 3, 4, 5 a, b, c, e, h, j, k, l, m,
6, 7 a, b, c, f, 8 a, b, d

Questions

3 For each of these functions:

- i** express the function in differentiable form
 - ii** differentiate the function with respect to x .
- | | |
|--------------------------------------|--------------------------------------|
| a $y = (x - 3)(x + 5)$ | b $f(x) = (4x + 1)(2x - 3)$ |
| c $y = x(x + 3)(x - 2)$ | d $y = (x + 2)(x^2 + 3x - 4)$ |
| e $g(x) = 2x^2(x - 1)^2$ | f $y = (x + 1)(x - 3)^2$ |
| g $y = (x + 4)(x + 1)(x - 2)$ | |

4 For each of these functions:

- i** express the function in differentiable form
- ii** differentiate the function with respect to x , expressing the answer with positive indices.

a $y = \frac{5}{x^2}$	b $y = \frac{7}{x^4}$	c $y = \frac{1}{2x^3}$
d $f(x) = \frac{1}{6x^2}$	e $g(x) = 4x^3 - \frac{2}{x^5}$	f $y = \frac{4}{3x}$
g $y = 8x + 5 - \frac{1}{x^2}$	h $y = \frac{4}{x^3} - \frac{3}{x}$	i $f(x) = \frac{3}{2x^4} - 5x - 6$

5 For each of these functions:

- i** express the function in differentiable form
- ii** differentiate the function with respect to x , expressing the answer in root form.

a $y = 8\sqrt{x}$	b $y = \sqrt[3]{x^2}$	c $f(x) = 12\left(\sqrt[4]{x^3}\right)$
e $y = \sqrt{x^7}$	h $y = \frac{4}{\sqrt[4]{x}}$	j $y = \frac{9}{\sqrt[6]{x^5}}$
k $y = \frac{3}{2}\left(\sqrt[9]{x^4}\right)$	l $y = \frac{1}{8\left(\sqrt[5]{x^6}\right)}$	m $g(x) = \frac{5}{2\left(\sqrt[3]{x^2}\right)}$

6 For each of these functions:

- i** express the function in differentiable form
- ii** differentiate the function with respect to x , expressing the answer with positive indices

a $y = \frac{x^2 - 4}{x}$

b $y = \frac{3x^2 - 5x}{x^2}$

c $f(x) = \frac{4 - x^3}{x}$

d $g(x) = \frac{x^2 + 5x - 3}{x^2}$

e $y = \frac{1 - x^4}{2x^3}$

f $y = \frac{x^2 - 3x - 2}{6x^3}$

g $y = \frac{(x+1)(x+4)}{x^2}$

h $y = \frac{(3-x)(1+2x)}{x^2}$

i $y = \frac{(x-2)(2x+1)^2}{x^3}$

7 Differentiate these functions.

a $y = \sqrt{x}(x-3)$

b $f(x) = \frac{3}{x^2} \left(x^2 - \frac{1}{x} \right)$

c $g(x) = \left(1 - \sqrt{x}\right) \left(2 - \frac{1}{\sqrt{x}}\right)$

f $y = \left(\frac{3}{x} - \frac{x}{3}\right)^2$

8 Differentiate these functions.

a $f(x) = \frac{2 - x^3}{\sqrt{x}}$

b $y = \frac{(x-2)^2}{4\sqrt{x}}$

d $h(x) = \frac{(x-2)(\sqrt{x} + 5)}{x\sqrt{x}}$

Answers

3 a $x^2 + 2x - 15$

$2x + 2$

b $8x^2 - 10x - 3$

$16x - 10$

c $x^3 + x^2 - 6x$

$3x^2 + 2x - 6$

d $x^3 + 5x^2 + 2x - 8$

$3x^2 + 10x + 2$

e $2x^4 - 4x^3 + 2x^2$

$8x^3 - 12x^2 + 4x$

f $x^3 - 5x^2 + 3x + 9$

$3x^2 - 10x + 3$

g $x^3 + 3x^2 - 6x - 8$

$3x^2 + 6x - 6$

4 a $5x^{-2}$

$-\frac{10}{x^3}$

b $7x^{-4}$

$-\frac{28}{x^5}$

c $\frac{1}{2}x^{-3}$

$-\frac{3}{2x^4}$

d $\frac{1}{6}x^{-2}$

$-\frac{1}{3x^3}$

e $4x^3 - 2x^{-5}$

$\frac{10}{x^6} + 12x^2$

f $\frac{4}{3}x^{-1}$

$-\frac{4}{3x^2}$

g $8x + 5 - x^{-2}$

$\frac{2}{x^3} + 8$

h $4x^{-3} - 3x^{-1}$

$-\frac{12}{x^4} + \frac{3}{x^2}$

i $\frac{3}{2}x^{-4} - 5x - 6$

$-5 - \frac{6}{x^5}$

5 a $8x^{\frac{1}{2}}$

$\frac{4}{\sqrt{x}}$

b $x^{\frac{2}{3}}$

$\frac{2}{3\sqrt[3]{x}}$

c $12x^{\frac{3}{4}}$

$\frac{9}{4\sqrt{x}}$

e $x^{\frac{7}{2}}$

$\frac{7}{2}\sqrt{x^5}$

h $4x^{-\frac{1}{4}}$

$-\frac{1}{4\sqrt[4]{x^5}}$

j $9x^{-\frac{5}{6}}$

$-\frac{15}{2\sqrt[6]{x^{11}}}$

k $\frac{3}{2}x^{\frac{4}{9}}$

$\frac{2}{3\sqrt[9]{x^5}}$

l $\frac{1}{8}x^{-\frac{6}{5}}$

$-\frac{3}{20\sqrt[5]{x^{11}}}$

m $\frac{5}{2}x^{-\frac{2}{3}}$

$-\frac{5}{3\sqrt[3]{x^5}}$

6 a $x - 4x^{-1}$

$$\frac{4}{x^2} + 1$$

b $3 - 5x^{-1}$

$$\frac{5}{x^2}$$

c $4x^{-1} - x^2$

$$-\frac{4}{x^2} - 2x$$

d $1 + 5x^{-1} - 3x^{-2}$

$$\frac{6}{x^3} - \frac{5}{x^2}$$

e $\frac{1}{2}x^{-3} - \frac{1}{2}x$

$$-\frac{1}{2} - \frac{3}{2x^4}$$

f $-\frac{1}{3}x^{-3} - \frac{1}{2}x^{-2} + \frac{1}{6}x^{-1}$

$$\frac{1}{x^4} + \frac{1}{x^3} - \frac{1}{6x^2}$$

g $1 + 4x^{-2} + 5x^{-1}$

$$-\frac{8}{x^3} - \frac{5}{x^2}$$

h $5x^{-1} + 3x^{-2} - 2$

$$-\frac{6}{x^3} - \frac{5}{x^2}$$

i $4 - 4x^{-1} - 7x^{-2} - 2x^{-3}$

$$\frac{6}{x^4} + \frac{14}{x^3} + \frac{4}{x^2}$$

7 a $-\frac{3}{2\sqrt{x}} + \frac{3\sqrt{x}}{2}$

b $\frac{9}{x^4}$

c $\frac{1}{2\sqrt{x^3}} - \frac{1}{\sqrt{x}}$

f $-\frac{18}{x^3} + \frac{2x}{9}$

8 a $-\frac{1}{x^{\frac{3}{2}}} - \frac{5x^{\frac{3}{2}}}{2}$

b $-\frac{1}{2x^{\frac{3}{2}}} - \frac{1}{2\sqrt{x}} + \frac{3\sqrt{x}}{8}$

d $\frac{-5}{2x^{\frac{3}{2}}} + \frac{2}{x^2} + \frac{15}{x^{\frac{5}{2}}}$