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Geometry of Complex Numbers - Lesson 2

Complex Loci

LI

- Describe Complex Loci algebraically and geometrically.

SC

- Modulus and Argument.

Jan 10-12:11

The word 'locus' (pl. 'loci') means 'place' or 'location'

Complex Loci are points in the complex plane satisfying an equation or inequation involving modulus or argument

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Example 1

Describe (and sketch) the complex loci given by,

$$|z - 3| = 2$$

Let $z = x + iy$. Then,

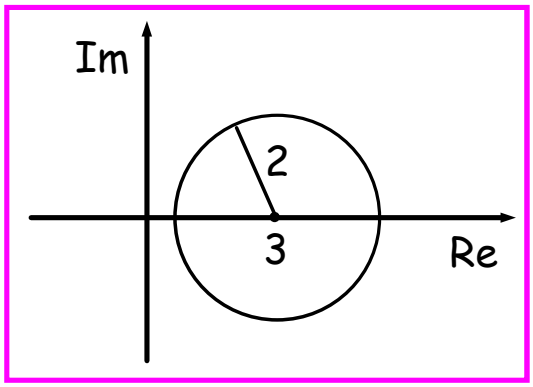
$$|x + iy - 3| = 2$$

$$\Rightarrow \quad |(x - 3) + iy| = 2$$

$$\therefore \quad \sqrt{(x - 3)^2 + y^2} = 2$$

$$\Rightarrow \quad \boxed{(x - 3)^2 + y^2 = 4}$$

All points on the circle, centre (3, 0), radius 2



Example 2

Describe (and sketch) the complex loci given by,

$$|z - 1 + 2i| < 3$$

Let $z = x + iy$. Then,

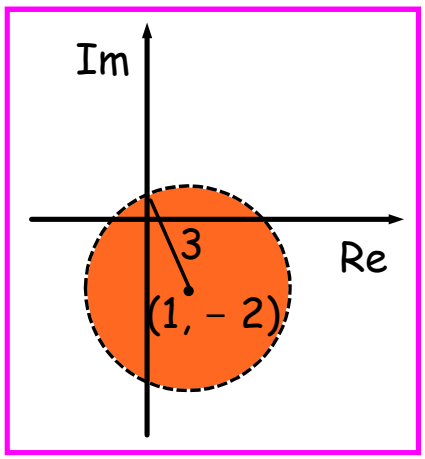
$$|x + iy - 1 + 2i| < 3$$

$$\Rightarrow \quad |(x - 1) + i(y + 2)| < 3$$

$$\therefore \quad \sqrt{(x - 1)^2 + (y + 2)^2} < 3$$

$$\Rightarrow \quad \boxed{(x - 1)^2 + (y + 2)^2 < 9}$$

All points inside the circle, centre $(1, -2)$, radius 3



Example 3

Identify and sketch the complex loci,

$$\arg z = -\pi/4$$

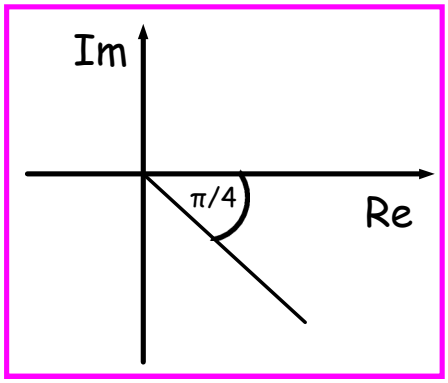
Let $z = x + iy$. Then, $\tan\theta = y/x$, with $\theta = -\pi/4$. So,

$$y/x = \tan(-\pi/4)$$

$$\Rightarrow \quad y/x = -1$$

$$\Rightarrow \quad \underline{y = -x}$$

$$\boxed{\text{All points on the line } y = -x \quad (x > 0)}$$



Example 4

Identify and sketch the complex loci,

$$|z - 4| = |z + i|$$

Let $z = x + iy$. Then,

$$|x + iy - 4| = |x + iy + i|$$

$$\Rightarrow \quad |(x - 4) + iy| = |x + i(y + 1)|$$

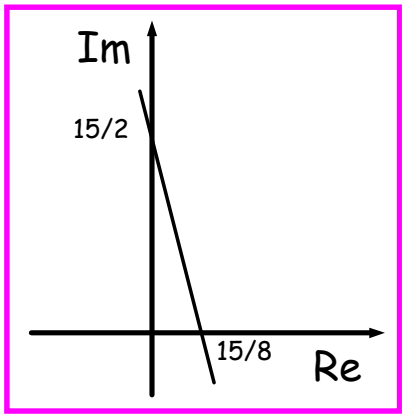
$$\therefore \quad \sqrt{(x - 4)^2 + y^2} = \sqrt{(x^2 + (y + 1)^2}$$

$$\Rightarrow \quad (x - 4)^2 + y^2 = x^2 + (y + 1)^2$$

$$\Rightarrow \quad x^2 - 8x + 16 + y^2 = x^2 + y^2 + 2y + 1$$

$$\Rightarrow \quad \boxed{8x + 2y - 15 = 0}$$

All points on the line $8x + 2y - 15 = 0$



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AH Maths - MiA (2nd Edn.)

- pg. 213 Ex. 12.4 All Q.

Feb 10-15:47

Ex. 12.4

1 Given that $z = x + iy$, for each of these equations

i find the equation of the locus **ii** draw the locus on an Argand diagram.

a $|z| = 5$ **b** $|z - 3| = 2$ **c** $|z + 1| = 4$

d $|z + i| = 3$ **e** $|z - 2i| = 3$ **f** $|z + 1 + 2i| = 3$

g $|2z + 3i| = 5$ **h** $|3z - i| = 5$ **i** $|3z + 3 - 2i| = 4$

j $\arg z = \frac{\pi}{6}$ **k** $\arg z = \frac{\pi}{4}$ **l** $\arg z = \frac{2\pi}{3}$

m $\arg z = 1$ **n** $2 \arg z = \frac{\pi}{4}$ **o** $\arg z = -\frac{\pi}{3}$

2 Explore the loci of the form

a $|z - a| = b$ **b** $|z - ai| = b$ **c** $|z - ai - b| = c$

3 Given that $z = x + iy$ find the equation of the locus in each case.

a $|z - 1| = |z - i|$ **b** $|z - 2| = |z - i|$

c $|z - 3| = |z - 2i|$ **d** $|z - a| = |z - bi|$

4 Sketch these loci, given that $z = x + iy$.

a $|z| \leqslant 3$ **b** $|z - 3| \leqslant 2$ **c** $|z + 2| \geqslant 5$

Answers to AH Maths (MiA), pg. 213, Ex. 12.4

1 a i $x^2 + y^2 = 25$ ii Circle centre $(0, 0)$; radius 5

b i $(x - 3)^2 + y^2 = 4$ ii Circle centre $(3, 0)$; radius 2

c i $(x + 1)^2 + y^2 = 16$

ii Circle centre $(-1, 0)$; radius 4

d i $x^2 + (y + 1)^2 = 9$

ii Circle centre $(0, -1)$; radius 3

e i $x^2 + (y - 2)^2 = 9$ ii Circle centre $(0, 2)$; radius 3

f i $(x + 1)^2 + (y + 2)^2 = 9$

ii Circle centre $(-1, -2)$; radius 3

g i $x^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{4}$

ii Circle centre $\left(0, -\frac{3}{2}\right)$; radius $\frac{5}{2}$

h i $x^2 + \left(y - \frac{1}{3}\right)^2 = \frac{25}{9}$

ii Circle centre $\left(0, \frac{1}{3}\right)$; radius $\frac{5}{3}$

i i $(x + 1)^2 + \left(y - \frac{2}{3}\right)^2 = \frac{16}{9}$

ii Circle centre $\left(-1, \frac{2}{3}\right)$; radius $\frac{4}{3}$

j i $y = \frac{1}{\sqrt{3}}x \quad (x > 0)$

ii Straight line through $(0, 0)$ gradient $\frac{1}{\sqrt{3}} \quad (x > 0)$

k i $y = x \quad (x > 0)$

ii Straight line through $(0, 0)$ gradient 1 $\quad (x > 0)$

l i $y = -\sqrt{3}x \quad (x > 0)$

ii Straight line through $(0, 0)$ gradient $-\sqrt{3} \quad (x > 0)$

m i $y = 1{\cdot}56x$ (to 3 sf) $\quad (x > 0)$

ii Straight line through $(0, 0)$ gradient tan 1 $\quad (x > 0)$

n i $y = 0{\cdot}414x$ (to 3 sf) $\quad (x > 0)$

ii Straight line through $(0, 0)$ gradient $\tan\left(\frac{\pi}{8}\right) \quad (x > 0)$

o i $y = -\sqrt{3}x \quad (x > 0)$

ii Straight line through $(0, 0)$ gradient $-\sqrt{3} \quad (x > 0)$

2 a Circle centre $(a, 0)$; radius b

b Circle centre $(0, a)$; radius b

c Circle centre (b, a); radius c

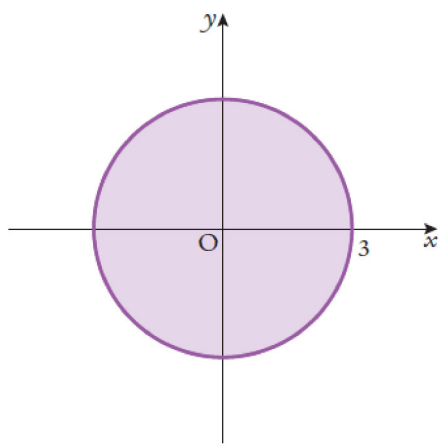
3 a $x = y$

b $y = 2x - \frac{3}{2}$

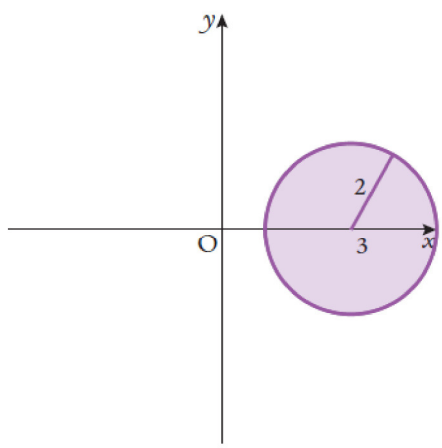
c $y = \frac{3}{2}x - \frac{5}{4}$

d $y = \frac{a}{b}x - \frac{(a^2 - b^2)}{2b}$

4 a



b



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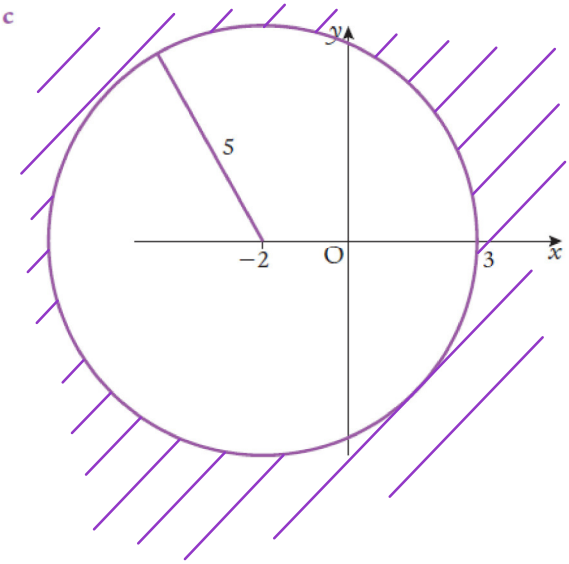

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