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Circles - Lesson 2

Circles and Lines

LI

- Find the equation of a tangent line to a circle.
- Determine whether or not a straight line intersects a circle.

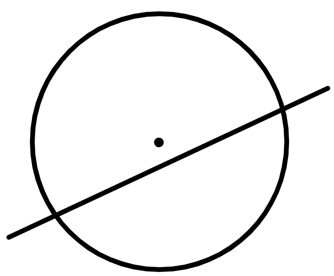
SC

- Equation of a circle.
- Equation of a straight line.
- Discriminants.
- Solving quadratic equations.

To find whether or not a line meets a circle, substitute the straight line equation into the circle equation.

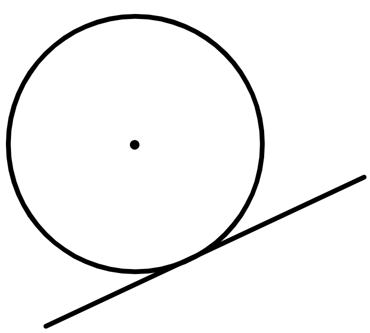
This gives a quadratic (in x or y).

Analyse discriminant to determine possibilities.



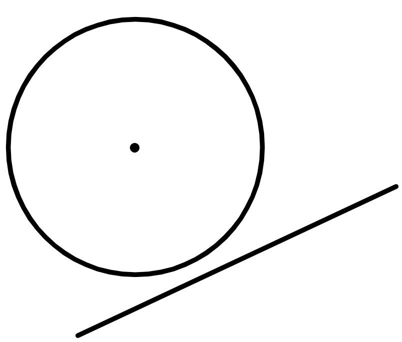
$D > 0$

Line meets circle twice



$D = 0$

Line meets circle once (tangency)

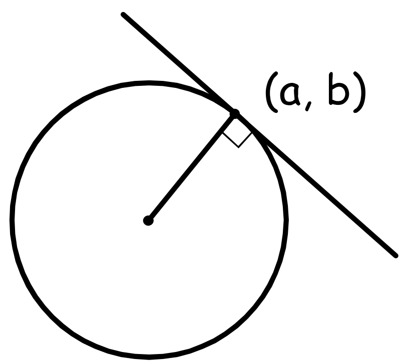


$D < 0$

Line does not meet circle

To find the equation of the tangent
to a circle, we need :

- Gradient of line.
- Intersection point of line and circle.



Strategy

- Calculate gradient of radius (need centre of circle coords.).
- Perpendicularise this to get gradient of line : $m_1 \times m_2 = -1$.
- $y - b = m(x - a)$.

Example 1

Find the equation of the tangent to the circle $x^2 + y^2 - 4x - 2y - 3 = 0$ at $(4, 3)$.

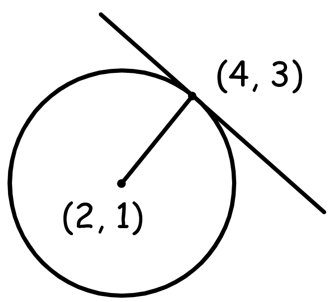
$$x^2 + y^2 - 4x - 2y - 3 = 0$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -4 \Rightarrow g = -2$$

$$2f = -2 \Rightarrow f = -1$$

$$\text{Centre}: (-g, -f) = \underline{(2, 1)}$$



$$m_{\text{RAD.}} = \frac{3 - 1}{4 - 2} = 1$$

$$\therefore \underline{m_{\text{TAN.}} = -1}$$

$$y - b = m(x - a)$$

$$y - 3 = -1(x - 4)$$

$$y - 3 = -x + 4$$

$$\boxed{y = -x + 7}$$

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Example 2

Show that the line $x + 3y - 11 = 0$ is a tangent to the circle $x^2 + y^2 + 2x + 12y - 53 = 0$ and find the point of contact.

$$x^2 + y^2 + 2x + 12y - 53 = 0$$

$$x = 11 - 3y$$

$$(11 - 3y)^2 + y^2 + 2(11 - 3y) + 12y - 53 = 0$$

$$121 - 66y + 10y^2 + 22 - 6y + 12y - 53 = 0$$

$$10y^2 - 60y + 90 = 0$$

$$y^2 - 6y + 9 = 0$$

Either the discriminant can be evaluated or the quadratic can be solved (with only 1 solution) to show that the line is a tangent to the circle; we adopt the former method here, as the point of tangency is required :

$$y^2 - 6y + 9 = 0$$

$$(y - 3)^2 = 0$$

$$\underline{y = 3}$$

As there is only solution for y, the line is a tangent to the circle.

$$x = 11 - 3y$$

$$x = 11 - 3(3)$$

$$\underline{x = 2}$$

Point of contact : (2, 3)

Example 3

Show that the line $y + x = 0$ does not intersect the circle $x^2 + y^2 - 4x - 8y + 11 = 0$.

$$x^2 + y^2 - 4x - 8y + 11 = 0$$

$$y = -x$$

$$x^2 + (-x)^2 - 4x - 8(-x) + 11 = 0$$

$$\underline{2x^2 + 4x + 11 = 0}$$

$$a = 2, b = 4, c = 11$$

$$D = b^2 - 4ac$$

$$D = 4^2 - 4(2)(11)$$

$$D = 16 - 88$$

$$\underline{D = -72}$$

$\boxed{\text{As } D < 0\text{, there are no intersection points.}}$

CfE Higher Maths

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