2 / 11 / 17

*Unit 2 : Sequences and Series - Lesson 2*

# Arithmetic Series

LI

- Know what an Arithmetic Series is.
- Find the $n^{th}$ term formula for an arithmetic series.
- Solve problems involving arithmetic series.

SC

- Arithmetic of real numbers.

Jan 10-12:11

A series is obtained by adding the terms of a sequence

An arithmetic series (up to n terms) is obtained by adding the first n terms of an arithmetic sequence

The sum to n terms of an arithmetic sequence is :

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Oct 9-15:09

## Example 1

Find the sum of the first 9 terms of the arithmetic sequence which starts 4, 10, 16, . . . .

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore \quad S_n = \frac{9}{2}[2(4) + (9 - 1)6]$$

$$\Rightarrow \quad S_n = \frac{9}{2}(56)$$

$$\Rightarrow \quad \boxed{S_n = 252}$$

Oct 2-23:10

Example 2

The sum of the first five terms of an arithmetic sequence is 85; the sum of the first 10 terms is 295.

Show that $S_n = R n^2 + T n$, stating the values of the constants R and T; also determine $S_{20}$.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$S_5 = 85$ $\qquad$ $S_{10} = 295$

$$85 = \frac{5}{2}[2a + (5 - 1)d] \qquad 295 = \frac{10}{2}[2a + (10 - 1)d]$$

$$\Rightarrow \underline{34 = 2a + 4d} \qquad \Rightarrow \underline{59 = 2a + 9d}$$

$$2a + 4d = 34$$
$$2a + 9d = 59$$
$$\therefore \quad 5d = 25$$
$$\Rightarrow \quad \underline{d = 5}$$

$$2a + 4d = 34$$
$$\Rightarrow \quad 2a = 34 - 4(5)$$
$$\Rightarrow \quad 2a = 14$$
$$\Rightarrow \quad \underline{a = 7}$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$
$$\therefore \quad S_n = \frac{n}{2}[2(7) + (n - 1)5]$$
$$\Rightarrow \quad S_n = \frac{n}{2}(5n + 9)$$
$$\Rightarrow \quad \boxed{S_n = \frac{5}{2}n^2 + \frac{9}{2}n \quad (R = 5/2, T = 9/2)}$$

$$S_{20} = \frac{5}{2}(20)^2 + \frac{9}{2}(20)$$
$$\Rightarrow \quad \boxed{S_{20} = 1090}$$

Oct 2-23:10

Example 3

When does the sum of the arithmetic sequence which starts 1, 7, 13, 19, . . . first exceed 100 ?

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore \quad S_n = \frac{n}{2}[2(1) + (n - 1)6]$$

$$\Rightarrow \quad S_n = \frac{n}{2}(6n - 4)$$

$$\Rightarrow \quad \underline{S_n = 3n^2 - 2n}$$

$$S_n > 100$$

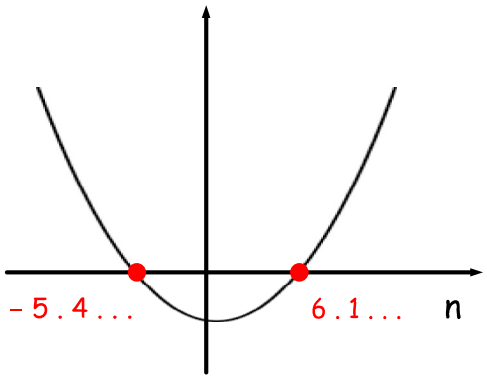
$$\therefore \quad 3n^2 - 2n > 100$$

$$\Rightarrow \quad 3n^2 - 2n - 100 > 0$$

This quadratic inequation is solved in the usual way; solving the associated quadratic equation gives the solutions,

$$n = 6.1\ldots, -5.4\ldots$$

As $n > 0, n \neq -5.4\ldots$.




$$\boxed{n = 7}$$

Oct 2-23:10

## AH Maths - MiA (2nd Edn.)

- pg. 153-5 Ex. 9.2 Q 1 - 4, 6, 7, 11, 13.

Feb 10-15:47

# Ex. 9.2

**1** a Calculate the sum to 10 terms of the arithmetic series which starts $2 + 8 + 14 + \ldots$.

b Find $S_{16}$ for an arithmetic series when $u_1 = 7$, $u_2 = 28$ and $u_3 = 49$.

c Find the required sum when each of these is an AP.

i $4 + 9 + 14 + \ldots : S_{20}$ ii $6 + 20 + 34 + \ldots : S_{50}$

iii $(-1) + (-8) + (-15) + \ldots : S_{15}$ iv $-9 - 7 - 5 - \ldots : S_{10}$

**2** The first two terms of an arithmetic sequence are 14 and 25 in that order.

a Find the sum of the first 15 terms and the first 16 terms.

b Hence calculate the 16th term.

c Repeat this process for if the first two terms are 25 and 14 in that order.

**3** Find these sums, given that each is an arithmetic series.

a $7 + 8 + 9 + \ldots + 40$ b $17 + 24 + 31 + \ldots + 360$

c $-16 + (-18) + \ldots + (-54)$ d $5 - 3 - 11 - \ldots - 83$

e $0.3 + 0.7 + 1.1 + \ldots + 2.3$ f $\frac{1}{12} + \frac{1}{6} + \frac{1}{4} + \frac{1}{3} + \ldots + 5$

**4** a The sum of the first 80 terms of an arithmetic series is 25 680.
The common difference is 8. What is the first term?

b The sum of the first 41 terms of an arithmetic series is 0.
The common difference is 7. What is the first term?

**6** a An arithmetic series starts with 8, has 20 terms and totals 2440.
Calculate the common difference.

b The first 60 terms of an arithmetic progression sum to 891.
If the first term is 0.1, what is the common difference?

**7** a The first three terms of an arithmetic sequence total 30. The next three total 69. What is the sum of the three after that?

b The sum of the first four terms of an arithmetic sequence is $-2$.
The next three total 51. What is the 16th term?

**11** A roll of sticky tape is wound round a spindle of radius 5 cm.
The tape is 0.05 cm thick.

a Taking each complete winding as approximately circular, and keeping $\pi$ in your answer, find the circumference of each of the first five windings.

b If there are 200 windings on the roll calculate the total length of tape on the roll.

**13** An arithmetic sequence starts $a_1, a_2, a_3, a_4, a_5, a_6, \ldots$
Show that the terms $(a_1 + a_2 + a_3), (a_4 + a_5 + a_6), (a_7 + a_8 + a_9), \ldots$ also form an arithmetic sequence.

Jun 19-11:45

## Answers to AH Maths (MiA), pg. 153-5, Ex. 9.2

**1** **a** 290 **b** 2632

**c** **i** 1030 **ii** 17 450 **iii** −750 **iv** 0

**2** **a** 1365, 1544 **b** 179 **c** −780, −920; −140

**3** **a** 799 **b** 9 425 **c** −700

**d** −468 **e** 7·8 **f** 152·5

**4** **a** 5 **b** −140

**6** **a** 12 **b** 0·5

**7** **a** 108 **b** 67

**11** **a** $10{\cdot}1\pi$, $10{\cdot}2\pi$, $10{\cdot}3\pi$, $10{\cdot}4\pi$, $10{\cdot}5\pi$

**b** $4010\,\pi$ cm

**13** The terms are $3a + 3d$, $3a + 12d$, $3a + 21d$, ... which is an arithmetic sequence with first term $3a$ and common difference $9d$. Proof by induction.

Jun 1-11:51