

9 / 3 / 16

Trigonometric Phenomena - Lesson 2

Addition Formulae

LI

- Use the 4 Addition Formulae.

SC

- Exact Trig. Values.
- Manipulating Fractions.

Addition Formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

These are sometimes known as **expansions**

Example 1

Expand $\sin(2W + 3b)$.

$$\sin(2W + 3b) = \boxed{\sin 2W \cos 3b + \cos 2W \sin 3b}$$

Example 2

Find the exact value of $\cos 105^\circ$.

$$\begin{aligned}\cos 105^\circ &= \cos (60^\circ + 45^\circ) \\&= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\&= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \\&= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \\&= \boxed{\frac{1 - \sqrt{3}}{2\sqrt{2}}}\end{aligned}$$

Example 3

By writing $\frac{5\pi}{12}$ as $\frac{\pi}{4} + \frac{\pi}{6}$, find the exact value of $\sin \frac{5\pi}{12}$.

$$\begin{aligned}
 \sin \frac{5\pi}{12} &= \sin \left(\frac{\pi}{4} + \frac{\pi}{6} \right) \\
 &= \sin \frac{\pi}{4} \cos \frac{\pi}{6} + \cos \frac{\pi}{4} \sin \frac{\pi}{6} \\
 &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\
 &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\
 &= \boxed{\frac{1 + \sqrt{3}}{2\sqrt{2}}}
 \end{aligned}$$

Example 4

Given that A and B are acute angles with

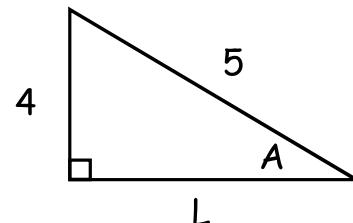
$\sin A = \frac{4}{5}$ and $\tan B = \frac{8}{15}$, find the exact value of $\cos(A - B)$.

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$L = \sqrt{5^2 - 4^2}$$

$$L = \sqrt{9}$$

$$L = 3$$



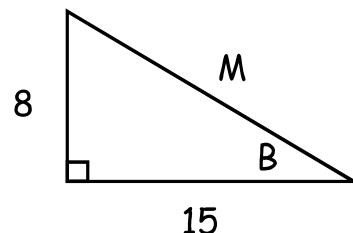
$$\sin A = \frac{4}{5}$$

$$\cos A = \frac{3}{5}$$

$$M = \sqrt{8^2 + 15^2}$$

$$M = \sqrt{289}$$

$$M = 17$$



$$\sin B = \frac{8}{17}$$

$$\cos B = \frac{15}{17}$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{3}{5} \times \frac{15}{17} + \frac{4}{5} \times \frac{8}{17}$$

$$= \frac{45}{85} + \frac{32}{85}$$

$$= \boxed{\frac{77}{85}}$$

CfE Higher Maths

pg. 33 Ex. 2C All Q

pg. 36 - 7 Ex. 2D All Q