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Trigonometric Phenomena - Lesson 2

Addition Formulae

LI
• Use the 4 Addition Formulae.

SC
• Exact Trig. Values.
• Manipulating Fractions.
Addition Formulae

\[
\begin{align*}
\sin (A + B) &= \sin A \cos B + \cos A \sin B \\
\sin (A - B) &= \sin A \cos B - \cos A \sin B
\end{align*}
\]

\[
\begin{align*}
\cos (A + B) &= \cos A \cos B - \sin A \sin B \\
\cos (A - B) &= \cos A \cos B + \sin A \sin B
\end{align*}
\]

These are sometimes known as expansions
Example 1

Expand \( \sin (2W + 3b) \).

\[
\sin (2W + 3b) = \sin 2W \cos 3b + \cos 2W \sin 3b
\]
Example 2

Find the exact value of $\cos 105^\circ$.

$$\cos 105^\circ = \cos (60^\circ + 45^\circ)$$

$$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$$
Example 3

By writing $\frac{5\pi}{12}$ as $\frac{\pi}{4} + \frac{\pi}{6}$, find the exact value of $\sin \frac{5\pi}{12}$.

\[
\sin \frac{5\pi}{12} = \sin \left( \frac{\pi}{4} + \frac{\pi}{6} \right)
\]

\[
= \sin \frac{\pi}{4} \cos \frac{\pi}{6} + \cos \frac{\pi}{4} \sin \frac{\pi}{6}
\]

\[
= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}
\]

\[
= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}
\]

\[
= \frac{1 + \sqrt{3}}{2\sqrt{2}}
\]
Example 4

Given that \( A \) and \( B \) are acute angles with

\[
\sin A = \frac{4}{5} \quad \text{and} \quad \tan B = \frac{8}{15},
\]

find the exact value of \( \cos (A - B) \).

\[
\cos (A - B) = \cos A \cos B + \sin A \sin B
\]

\[
L = \sqrt{5^2 - 4^2}
\]

\[
L = \sqrt{9}
\]

\[
L = 3
\]

\[
\sin A = \frac{4}{5}, \quad \cos A = \frac{3}{5}
\]

\[
M = \sqrt{8^2 + 15^2}
\]

\[
M = \sqrt{289}
\]

\[
M = 17
\]

\[
\sin B = \frac{8}{17}, \quad \cos B = \frac{15}{17}
\]

\[
\cos (A - B) = \cos A \cos B + \sin A \sin B
\]

\[
= \frac{3}{5} \times \frac{15}{17} + \frac{4}{5} \times \frac{8}{17}
\]

\[
= \frac{45}{85} + \frac{32}{85}
\]

\[
= \frac{77}{85}
\]
CfE Higher Maths

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