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Unit 1 : Integral Calculus - Lesson 1

Standard Integrals

LI

- Know standard integrals.

SC

- Integration as the opposite of differentiation.

If we have a derivative of a function, then we automatically obtain an integral.

For example,

$$\frac{d}{dx}x^3 = 3x^2 \Rightarrow \int x^2\,dx = \frac{1}{3}x^3 + C$$

Using known results for standard derivatives, we similarly have the following standard integrals (a, b and n are constants) :

$$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$$

The vertical bars - 'modulus' - make any negatives into positives; it's possible that $ax + b$ may be negative; but for the logarithm to make sense, we need to make sure that it is positive.

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C \qquad (n \neq -1)$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$$

$$\int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + C$$

$$\int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx = -\frac{1}{a}\operatorname{cosec}(ax+b) + C$$

$$\int \operatorname{cosec}^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b) + C$$

All the above integrals can be checked by differentiating the RHS of the stipulated equality and making sure that the integrand on the LHS is obtained.

In this lesson, the first 6 integrals will be the focus.

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Example 1

Integrate $y = \sec^2(4x + 7)$.

$$\int \sec^2(4x + 7)\,dx = \boxed{\frac{1}{4}\tan(4x + 7) + C}$$

Example 2

Integrate $y = e^{8x} + \dfrac{1}{3x}$.

$$\int e^{8x} + \frac{1}{3x}\, dx$$

$$= \int e^{8x}\, dx + \frac{1}{3}\int \frac{1}{x}\, dx$$

$$= \boxed{\frac{1}{8} e^{8x} + \frac{1}{3} \ln|x| + C}$$

Example 3

Integrate $y = 6(3x + 2)^{-1} - e^{5 - x/2}$.

$$\int 6(3x + 2)^{-1} - e^{5 - x/2}\,dx$$

$$= \int \frac{6}{3x + 2}\,dx - \int e^{5 - x/2}\,dx$$

$$= 6\int \frac{1}{3x + 2}\,dx - \int e^{5 - x/2}\,dx$$

$$= 6(1/3)\ln|3x + 2| - 1/(-1/2)\,e^{5 - x/2} + C$$

$$= \boxed{2\ln|3x + 2| + 2e^{5 - x/2} + C}$$

Example 4

Find $\displaystyle\int_1^2 \frac{1}{1-2x}\,dx$ as an exact value.

$$\int_1^2 \frac{1}{1-2x}\,dx$$

$$= \left[-\frac{1}{2}\ln|1-2x|\right]_1^2$$

$$= -\frac{1}{2}\left[\ln|1-2x|\right]_1^2$$

$$= -\frac{1}{2}\left(\ln|-3| - \ln|-1|\right)$$

$$= \boxed{-\frac{1}{2}\ln 3}$$

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Example 5

Find $\int_{\ln 2}^{\ln 3} e^{2x} - e^{-2x}\, dx$ as an exact value.

$$\int_{\ln 2}^{\ln 3} e^{2x} - e^{-2x}\, dx$$

$$= \left[\frac{1}{2} e^{2x} + \frac{1}{2} e^{-2x}\right]_{\ln 2}^{\ln 3}$$

$$= \frac{1}{2}\left[e^{2x} + e^{-2x}\right]_{\ln 2}^{\ln 3}$$

$$= \frac{1}{2}\left(e^{2\ln 3} + e^{-2\ln 3}\right) - \frac{1}{2}\left(e^{2\ln 2} + e^{-2\ln 2}\right)$$

$$= \frac{1}{2}\left(e^{\ln 9} + e^{\ln(1/9)}\right) - \frac{1}{2}\left(e^{\ln 4} + e^{\ln(1/4)}\right)$$

$$= \frac{1}{2}(9 + (1/9)) - \frac{1}{2}(4 + (1/4))$$

$$= \frac{1}{2}(82/9 - 17/4)$$

$$= \boxed{\frac{175}{72}}$$

AH Maths - MiA (2^{nd} Edn.)

- pg. 100-1 Ex. 7.1 Q 1 e - i, 2 b, c, e, f, h, 3, 4 a - e, 5.

Ex. 7.1

1 Find each of these indefinite integrals.

e $\int \frac{2x^3 + 1}{x^2}\,dx$ **f** $\int \frac{x + 1}{\sqrt{x}}\,dx$

g $\int \sec^2(2x + 3)\,dx$ **h** $\int (4 \sin 2x + 1)\,dx$ **i** $\int \cos(1 - 2x)\,dx$

2 Calculate these indefinite integrals.

b $\int (4 - 3x)^{-2}\,dx$ **c** $\int (\sin(5x - 1) - \cos(1 - 5x))\,dx$

e $\int \frac{1 - 2x - x^2}{x}\,dx$ **f** $\int \sec^2(1 - x)\,dx$

h $\int (2 \cos 2x + 3 \sin(1 - x))\,dx$

3 Find these indefinite integrals.

a $\int \frac{1}{2x}\,dx$ **b** $\int \frac{3}{2x + 1}\,dx$ **c** $\int (4x + 5)^{-1}\,dx$

d $\int e^{3x}\,dx$ **e** $\int 3e^{4x-1}\,dx$ **f** $\int 3e^{1-x}\,dx$

g $\int \frac{dx}{e^x}$ **h** $\int \left(\frac{1}{2x + 1} + \frac{1}{3x - 1}\right) dx$ **i** $\int \left(\frac{1}{e^{x+1}} + \frac{3}{3x - 1}\right) dx$

j $\int \frac{x}{2x^2 + 3x}\,dx$ **k** $\int 4^x\,dx$ [Hint: $4^x = e^{x\ln 4}$] **l** $\int 4^{3x-1}\,dx$

[Hint: divide numerator and denominator by x.]

4 Evaluate these definite integrals.

a $\int_1^3 \frac{dx}{x + 1}$ **b** $\int_0^{\frac{1}{2}} e^{2x-1}\,dx$ **c** $\int_0^1 \left(\frac{1}{2x + 1} + \frac{1}{4x + 1}\right) dx$

d $\int_0^{\frac{\pi}{3}} \sin x + \sec^2 x\,dx$ **e** $\int_1^e x + x^{-1}\,dx$

5 Simplify the expression and then evaluate the definite integral.

a $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 + \cos^2 x}{\cos^2 x}\,dx$ **b** $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 + \sin^2 x}{\cos^2 x}\,dx$ **c** $\int_0^{\frac{\pi}{6}} \cos x \sec^3 x\,dx$

d $\int_2^6 \frac{x - 2}{x^2 - 4}\,dx$ **e** $\int_0^3 \frac{x + 2}{2x^2 + 7x + 6}\,dx$ **f** $\int_0^{\pi} \frac{\sin 2x}{\sin x}\,dx$

Answers to AH Maths (MiA), pg. 100-1, Ex. 7.1

1 **e** $x^2 - x^{-1} + c$ **f** $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

g $\frac{1}{2}\tan(2x+3) + c$ **h** $-2\cos 2x + x + c$

i $-\frac{1}{2}\sin(1-2x) + c$

2 **b** $\frac{1}{3}(4-3x)^{-1} + c$

c $-\frac{1}{5}\cos(5x-1) + \frac{1}{5}\sin(1-5x) + c$

e $\ln|x| - 2x - \dfrac{x^2}{2} + c$ **f** $-\tan(1-x) + c$

h $\sin 2x + 3\cos(1-x) + c$

3 **a** $\frac{1}{2}\ln|x| + c = \ln|A\sqrt{x}|$ or $\frac{1}{2}\ln|2x| + c = \ln|A\sqrt{2x}|$

Note: Answers really only differ in the form and the value of the constant.

b $\frac{3}{2}\ln|2x+1| + c$ **c** $\frac{1}{4}\ln|4x+5| + c$

d $\frac{1}{3}e^{3x} + c$ **e** $\frac{3}{4}e^{4x-1} + c$

f $-3e^{1-x} + c$ **g** $-e^{-x} + c$

h $\frac{1}{2}\ln|2x+1| + \frac{1}{3}\ln|3x-1|$

i $-\dfrac{1}{e^{x+1}} + \ln|3x-1| + c$ **j** $\frac{1}{2}\ln|2x+3| + c$

k $\dfrac{4^x}{\ln 4}$ **l** $\dfrac{4^{3x-1}}{3\ln 4}$

4 **a** $\ln 2$ **b** $\frac{1}{2} - \frac{1}{2e}$ **c** $\frac{1}{4}\ln 45$

d $\frac{1}{2} + \sqrt{3}$ **e** $\frac{1}{2}(e^2+1)$

5 **a** $2 + \frac{\pi}{2}$ **b** $4 - \frac{\pi}{2}$ **c** $\frac{1}{\sqrt{3}}$

d $\ln 2$ **e** $\ln\sqrt{3}$ **f** 0

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