

If we have a derivative of a function, then we automatically obtain an integral.

For example,

$$\frac{d}{dx} x^{3} = 3 x^{2} \Rightarrow \int x^{2} dx = \frac{1}{3} x^{3} + C$$

Using known results for standard derivatives, we similarly have the following standard integrals (a, b and n are constants):

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| + C$$
The vertical bars - 'modulus' - make any magnitude into particles, if possible that  $a + b + C$ 

$$\int \sec^2(ax+b) dx = \frac{1}{a} \ln |ax+b| + C$$

$$\int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + C$$

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + C$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + C$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C \quad (n \neq -1)$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + C$$

$$\int \sec(ax+b) \tan(ax+b) dx = \frac{1}{a} \sec(ax+b) + C$$

$$\int \sec(ax+b) \tan(ax+b) dx = -\frac{1}{a} \csc(ax+b) + C$$

$$\int \sec(ax+b) \tan(ax+b) dx = -\frac{1}{a} \csc(ax+b) + C$$

All the above integrals can be checked by differentiating the RHS of the stipulated equality and making sure that the integrand on the LHS is obtained.

In this lesson, the first 6 integrals will be the focus.

Example 1  
Integrate 
$$y = \sec^2(4x + 7)$$
.  
 $\int \sec^2(4x + 7) \, dx = \frac{1}{4} \tan(4x + 7) + C$ 

Example 2  
Integrate 
$$\gamma = e^{8x} + \frac{1}{3x}$$
.  

$$\int e^{8x} + \frac{1}{3x} dx$$

$$= \int e^{8x} dx + \frac{1}{3} \int \frac{1}{x} dx$$

$$= \frac{1}{8} e^{8x} + \frac{1}{3} \ln |x| + C$$

Example 3  
Integrate 
$$y = 6 (3 x + 2)^{-1} - e^{5 - x/2}$$
.  

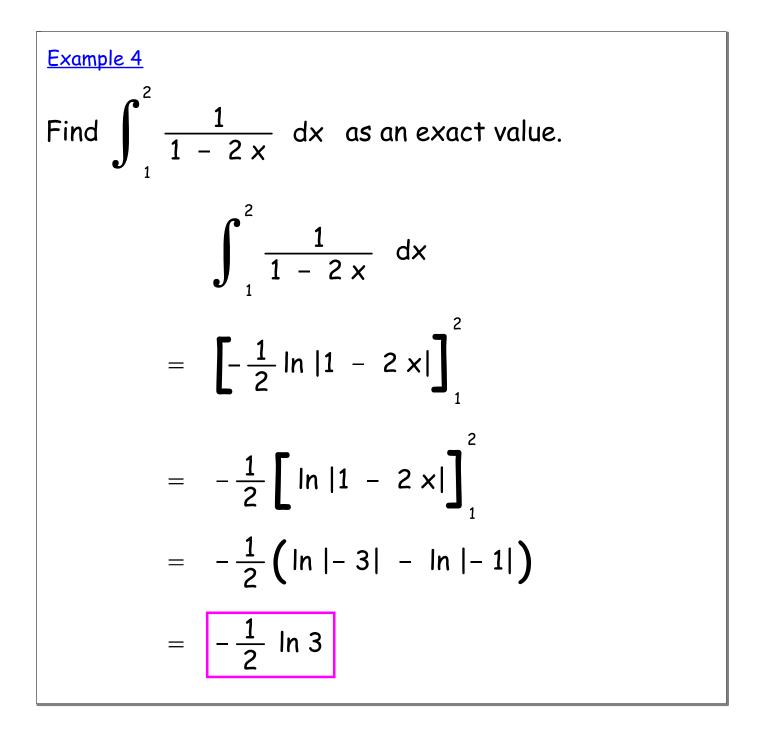
$$\int 6 (3 x + 2)^{-1} - e^{5 - x/2} dx$$

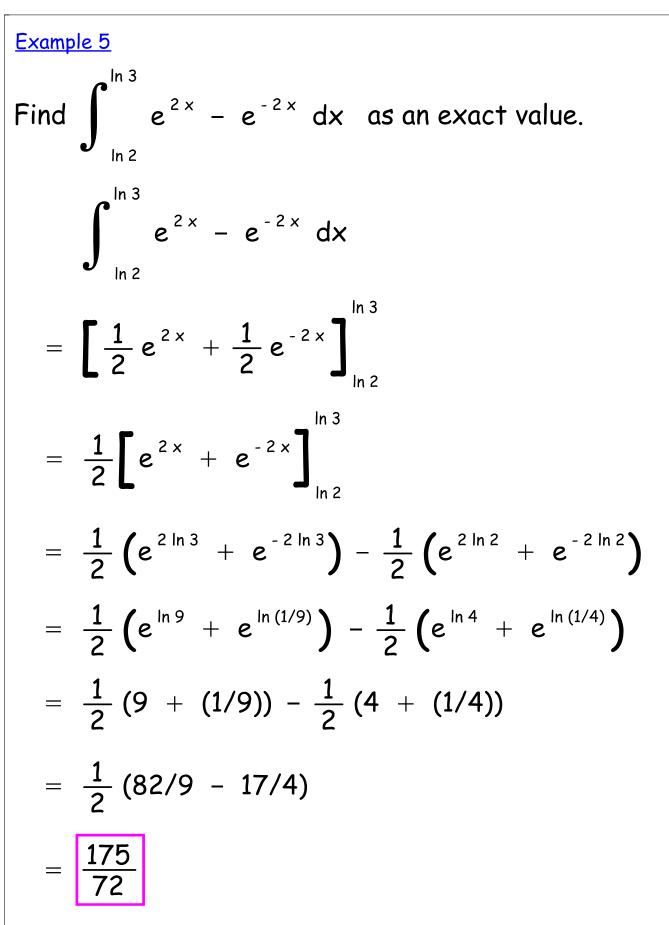
$$= \int \frac{6}{3 x + 2} dx - \int e^{5 - x/2} dx$$

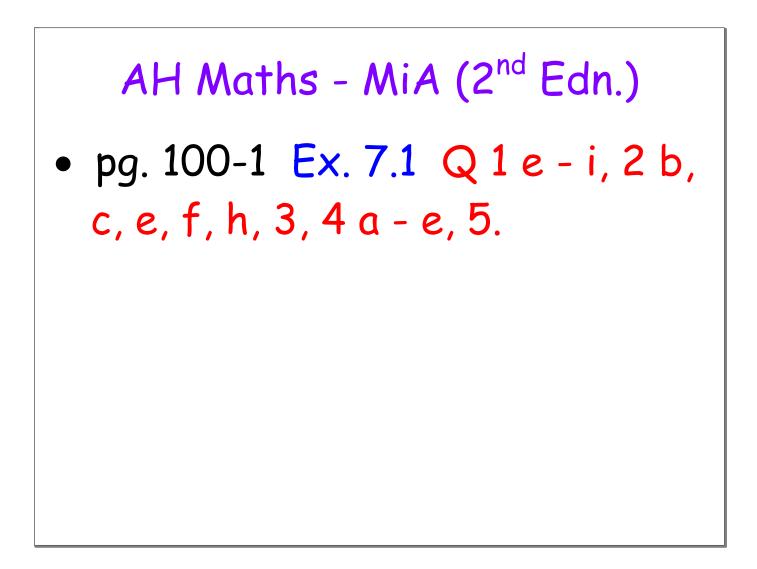
$$= 6 \int \frac{1}{3 x + 2} dx - \int e^{5 - x/2} dx$$

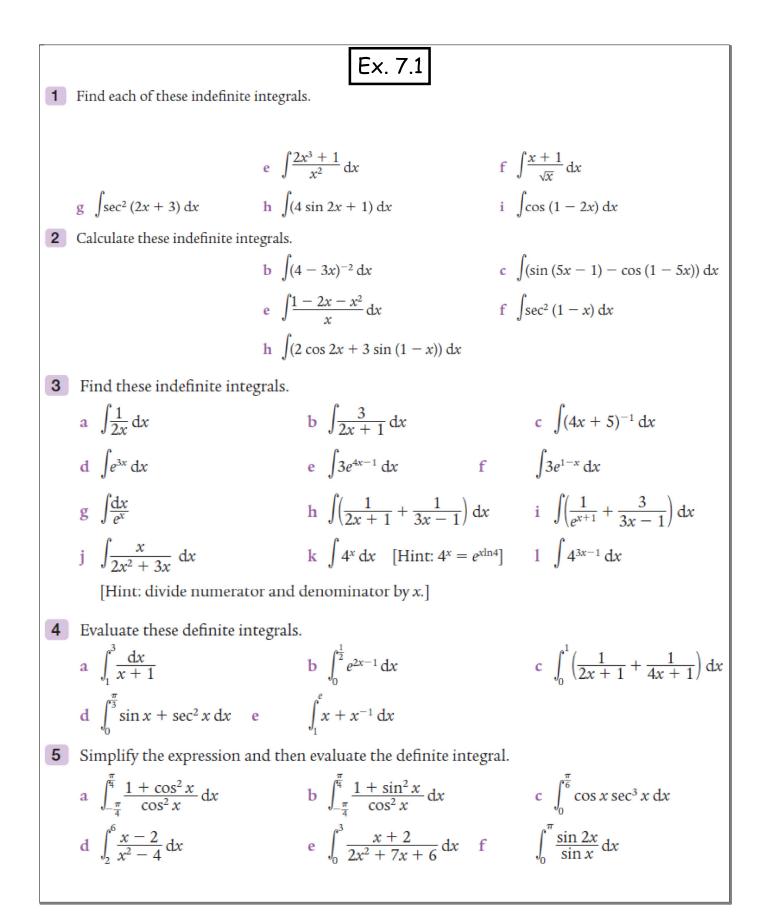
$$= 6 (1/3) \ln |3 x + 2| - 1/(-1/2) e^{5 - x/2} + C$$

$$= 2 \ln |3 x + 2| + 2 e^{5 - x/2} + C$$









Answers to AH Maths (MiA), pg. 100-1, Ex. 7.1							
1	e	$x^2 - x^{-1} + c$			f	$\frac{2}{3}\chi^{\frac{3}{2}}.$	$+2x^{\frac{1}{2}}+c$
	g	$\frac{1}{2}\tan(2x+3)$ -	+ c		h	-20	$\cos 2x + x + c$
	i	$-\frac{1}{2}\sin\left(1-2x\right)$	+	С			
2	b	$\frac{1}{3}(4-3x)^{-1}+$	С				
	с	$-\frac{1}{5}\cos(5x-1) + \frac{1}{5}\sin(1-5x) + c$					
	e	$\ln x  - 2x - \frac{x^2}{2}$	2 - +	С	f	-ta	n(1-x) + c
	h	$\sin 2x + 3\cos \left(1 - x\right) + c$					
3	a	$\frac{1}{2}\ln x  + c = \ln A\sqrt{x}  \text{ or } \frac{1}{2}\ln 2x  + c = \ln A\sqrt{2x} $					
		Note: Answers really only differ in the form and the					
		value of the constant.					
	b	$\frac{3}{2}\ln 2x+1  +$	С		С	$\frac{1}{4}$ ln	4x + 5  + c
	d	$\frac{1}{3}e^{3x} + c$			e	$\frac{3}{4}e^{4x}$	$c^{-1} + c$
	f	$-3e^{1-x}+c$			g	-e <sup>-</sup>	-x + c
	h	$\frac{1}{2}\ln 2x+1  + \frac{1}{3}\ln 3x-1 $					
	i	$-\frac{1}{e^{x+1}} + \ln 3x $	—	1  + <i>c</i>	j	$\frac{1}{2}\ln$	2x + 3  + c
	k	$\frac{4^x}{\ln 4}$			1	$\frac{4^{3x-3}}{3\ln x}$	- <u>1</u> 4
4	a	ln 2 l	0	$\frac{1}{2} - \frac{1}{2e}$		с	$\frac{1}{4}\ln 45$
	d	$\frac{1}{2} + \sqrt{3}$	9	$\frac{1}{2}(e^2+1)$			
5	a	$2 + \frac{\pi}{2}$ 1	0	$4 - \frac{\pi}{2}$		с	$\frac{1}{\sqrt{3}}$
	d	ln 2 e	2	$\ln \sqrt{3}$		f	0