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Solving Trigonometric Equations - Lesson 1

Solving Linear Trigonometric Equations

LI

- Solve trigonometric equations of the form :

$$a \sin(b x + c) + d = 0$$
$$a \cos(b x + c) + d = 0$$
$$a \tan(b x + c) + d = 0$$

for various ranges of x (in degrees or radians).

SC

- Trig. Graphs.
- Related Acute Angle.

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Strategy

- If $f(x)$ is one of $\sin(bx + c)$, $\cos(bx + c)$ or $\tan(bx + c)$, get equation into the form :

$$f(x) = k$$

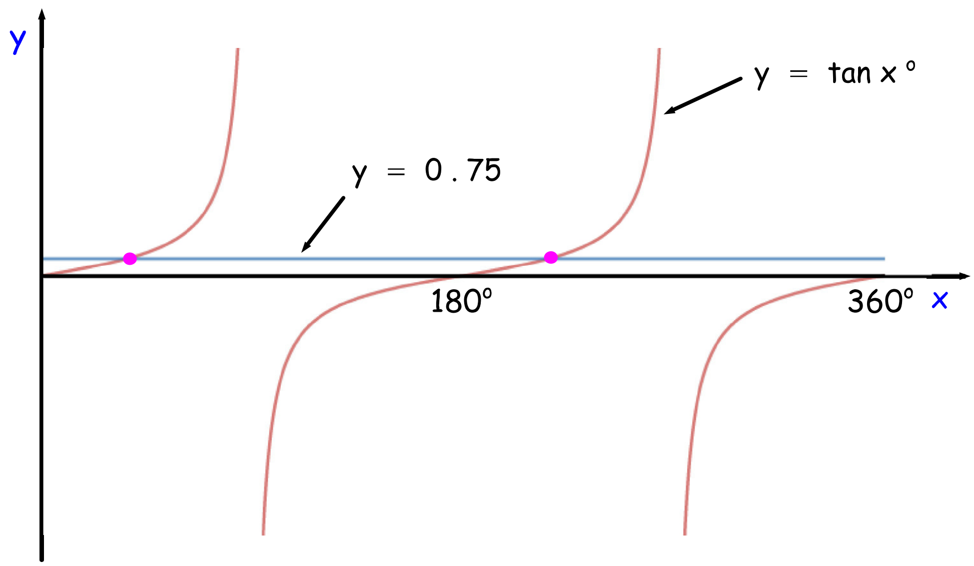
- Sketch the graphs of $y = f(x)$ and $y = k$ for the relevant range of x - values to see how many times the graphs cross in this range (and roughly where the solutions are); these intersection points give the solutions for x.
- Find related acute angle.
- Use ASTC diagram to find (normally 2 distinct) solutions.
- Use graph (or relevant range of x - values) to get any other solutions (and possibly eliminating some solutions).

Example 1

Solve $4\tan x^{\circ} - 3 = 0 \ (0 \leq x \leq 360)$.

$$4\tan x^{\circ} - 3 = 0$$

$$\tan x^{\circ} = 0.75$$



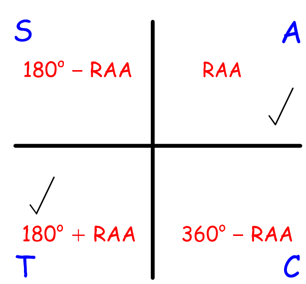
2 solutions expected

$$\tan x^{\circ} = 0.75$$

$$\therefore \quad RAA = \tan^{-1}(0.75)$$

$$\Rightarrow \quad \underline{RAA = 36.86\ldots^{\circ}}$$

tan is + ve



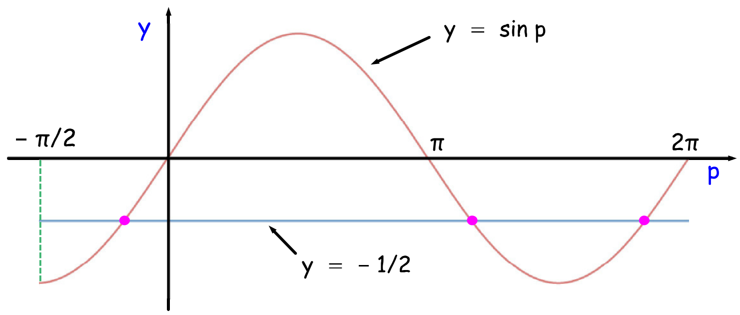
$$\therefore \quad x^{\circ} = 36.86\ldots^{\circ}, 180^{\circ} + 36.86\ldots^{\circ}$$

$$\Rightarrow \quad \boxed{x^{\circ} = 36.9^{\circ}, 216.9^{\circ} \ \text{(to 1 d.p.)}}$$

Example 2 (non-calculator)

Solve $2 \sin p + 1 = 0$ $(-\pi/2 \leq p \leq 2\pi)$.

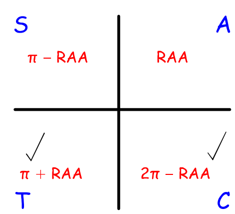
$$2 \sin p + 1 = 0$$
$$\sin p = -1/2$$



3 solutions expected

$$\sin p = -1/2$$
$$\therefore \quad RAA = \sin^{-1}(0.5)$$
$$\Rightarrow \quad \underline{RAA = \pi/6}$$

sin is − ve



$$\therefore \quad p = \pi + \pi/6, \; 2\pi - \pi/6$$
$$\Rightarrow \quad \underline{p = 7\pi/6, \; 11\pi/6}$$

From the graph, we expect one more solution; as $\sin p$ is periodic with period 2π, other solutions are obtained by subtracting 2π from the solutions obtained so far. Hence, other possible solutions are,

$$p = 7\pi/6 - 2\pi, \; 11\pi/6 - 2\pi$$
$$\Rightarrow \quad \underline{p = -5\pi/6, \; -\pi/6}$$

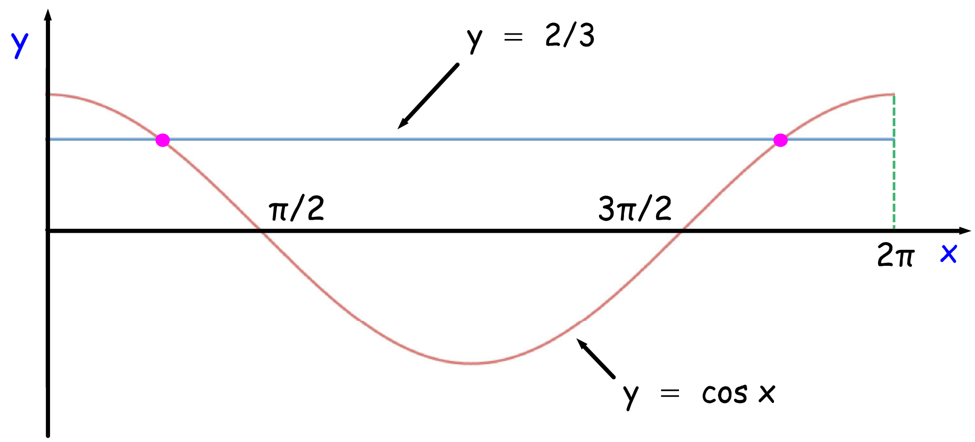
As $-\pi/2 \leq p \leq 2\pi$, $-5\pi/6$ must be rejected, as it is out of this range. So,

$$\boxed{p = -\pi/6, \; 7\pi/6, \; 11\pi/6}$$

Example 3

Solve $3\cos x - 2 = 0 \ (0 \leq x \leq 2\pi)$.

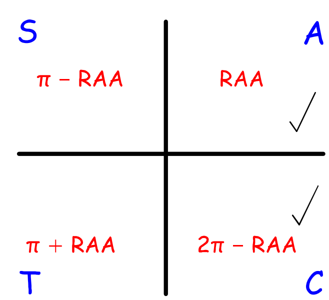
$$3\cos x - 2 = 0$$
$$\cos x = 2/3$$



2 solutions expected

$$\cos x = 2/3$$
$$\therefore \quad RAA = \cos^{-1}(2/3)$$
$$\Rightarrow \quad \underline{RAA = 0.841...}$$

cos is + ve



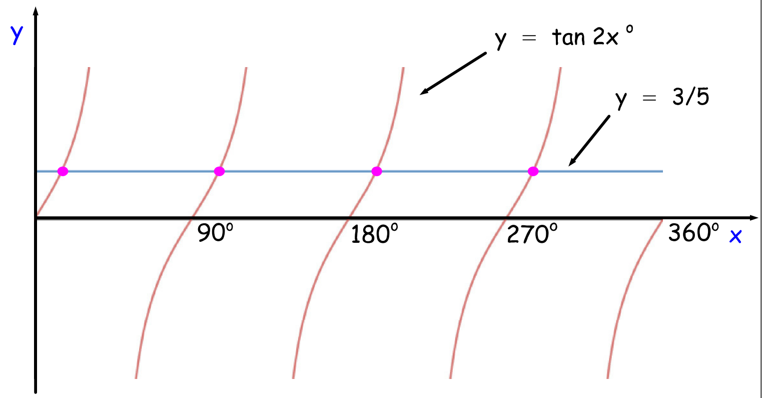
$$\therefore \quad x = 0.841..., 2\pi - 0.841...$$
$$\Rightarrow \quad \boxed{x = 0.841, 5.442 \text{ (to 3 d.p.)}}$$

Example 4

Solve $5 \tan 2x^\circ = 3 \ (0 \leq x \leq 360)$.

$$5 \tan 2x^\circ = 3$$

$$\tan 2x^\circ = 3/5$$



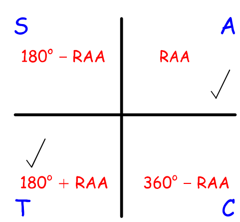
4 solutions expected

$$\tan 2x^\circ = 3/5$$

$$\therefore \quad RAA = \tan^{-1}(3/5)$$

$$\Rightarrow \quad \underline{RAA = 30.96\ldots^\circ}$$

tan is + ve



$$\therefore \quad 2x^\circ = 30.96\ldots^\circ, 180^\circ + 30.96\ldots^\circ$$

$$\Rightarrow \quad 2x^\circ = 30.96\ldots^\circ, 210.96\ldots^\circ$$

As tan is periodic with period 180°, the other 2 solutions are obtained by constantly adding 180° to the above two values for $2x^\circ$ until we reach 2 more different values for $2x^\circ$ up to 720° (as $0 \leq x \leq 360, 0 \leq 2x \leq 720$).

$$\therefore \quad 2x^\circ = 30.96\ldots^\circ, 210.96\ldots^\circ, 390.96^\circ, 570.96\ldots^\circ$$

$$\Rightarrow \quad \boxed{x^\circ = 15.5^\circ, 105.5^\circ, 195.5^\circ, 285.5^\circ}$$

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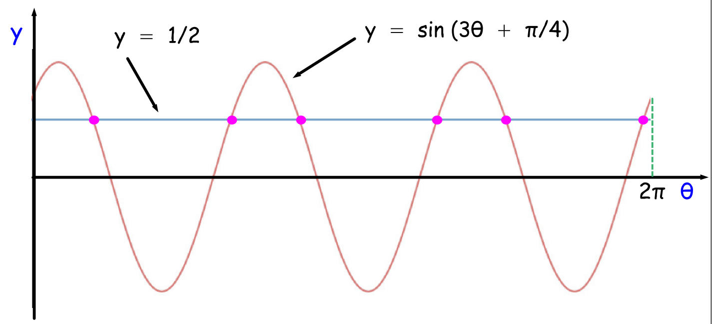
Example 5 (non-calculator)

Solve $2\sin(3\theta + \pi/4) - 1 = 0 \ (0 < \theta \leq 2\pi)$.

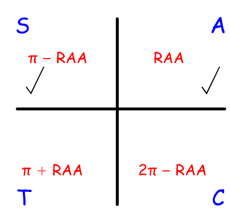
$$2\sin(3\theta + \pi/4) - 1 = 0$$
$$\sin(3\theta + \pi/4) = 1/2$$



6 solutions expected

$$\sin(3\theta + \pi/4) = 1/2$$
$$\therefore \quad RAA = \sin^{-1}(0.5)$$
$$\Rightarrow \quad \underline{RAA = \pi/6}$$

sin is + ve



$$\therefore \quad 3\theta + \pi/4 = \pi/6, \pi - \pi/6$$
$$\Rightarrow \quad 3\theta = -\pi/12, 7\pi/12$$

As sin is periodic with period 2π, the other 4 solutions are obtained by adding 2π to the above values for 3θ until we reach values for 3θ between 0 and 6π (as $0 < \theta \leq 2\pi$, $0 < 3\theta \leq 6\pi$). Also, note that $6\pi = 72\pi/12$.

$$\therefore \quad 3\theta = -\pi/12, 7\pi/12, 23\pi/12, 31\pi/12, 47\pi/12, 55\pi/12, 71\pi/12, 79\pi/12$$

Taking the values that are in the relevant range ($0 < 3\theta \leq 72\pi/12$), we have,

$$\boxed{\theta = 7\pi/36, 23\pi/36, 31\pi/36, 47\pi/36, 55\pi/36, 71\pi/36}$$

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