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Solving Trigonometric Equations - Lesson 1

Solving Linear Trigonometric Equations

**LI**

- Solve trigonometric equations of the form:

\[
\begin{align*}
    a \sin (b \times + c) + d &= 0 \\
    a \cos (b \times + c) + d &= 0 \\
    a \tan (b \times + c) + d &= 0
\end{align*}
\]

for various ranges of \( x \) (in degrees or radians).

**SC**

- Trig. Graphs.
- Related Acute Angle.
Strategy

- If \( f(x) \) is one of \( \sin(bx + c) \), \( \cos(bx + c) \) or \( \tan(bx + c) \), get equation into the form:

  \[ f(x) = k \]

- Sketch the graphs of \( y = f(x) \) and \( y = k \) for the relevant range of \( x \)-values to see how many times the graphs cross in this range (and roughly where the solutions are); these intersection points give the solutions for \( x \).

- Find related acute angle.

- Use ASTC diagram to find (normally 2 distinct) solutions.

- Use graph (or relevant range of \( x \)-values) to get any other solutions (and possibly eliminating some solutions).
Example 1

Solve \(4 \tan x^\circ - 3 = 0 \) \((0 \leq x \leq 360)\).

\[
4 \tan x^\circ - 3 = 0
\]

\[
\tan x^\circ = 0.75
\]

\[
y = \tan x^\circ
\]

\[
y = 0.75
\]

2 solutions expected

\[
\tan x^\circ = 0.75
\]

\[
\therefore \quad \text{RAA} = \tan^{-1}(0.75)
\]

\[
\Rightarrow \quad \text{RAA} = 36.86\ldots^\circ
\]

\[
\text{tan is + ve}
\]

\[
\begin{array}{c|c|c}
S & \text{RAA} & A \\
180^\circ - \text{RAA} & \checkmark & \\
180^\circ + \text{RAA} & & \\
360^\circ - \text{RAA} & & \\
T & & \\
\end{array}
\]

\[
\therefore \quad x^\circ = 36.86\ldots, 180^\circ + 36.86\ldots^\circ
\]

\[
\Rightarrow \quad x^\circ = 36.9^\circ, 216.9^\circ \quad \text{(to 1 d.p.)}
\]
Example 2 (non-calculator)

Solve \(2 \sin p + 1 = 0\) \((-\pi/2 \leq p \leq 2\pi)\).

\[
2 \sin p + 1 = 0 \\
\sin p = -1/2
\]

\[
\begin{array}{c}
\text{y} \\
-\pi/2 \quad \pi \quad 2\pi \\
\text{y} = \sin p \\
y = -1/2
\end{array}
\]

3 solutions expected

\[
\sin p = -1/2
\]

\[
\therefore \quad \text{RAA} = \sin^{-1} (0.5)
\]

\[
\Rightarrow \quad \text{RAA} = \pi/6
\]

\[
\begin{array}{c|c}
\text{S} & \text{A} \\
\pi - \text{RAA} & \text{RAA} \\
\end{array}
\]

\[
\text{sin is -ve}
\]

\[
\begin{array}{c|c}
\sqrt{1} & \sqrt{1} \\
\pi + \text{RAA} & 2\pi - \text{RAA} \\
\text{T} & \text{C}
\end{array}
\]

\[
\therefore \quad p = \pi + \pi/6, 2\pi - \pi/6
\]

\[
\Rightarrow \quad p = 7\pi/6, 11\pi/6
\]

From the graph, we expect one more solution; as \(\sin p\) is periodic with period \(2\pi\), other solutions are obtained by subtracting \(2\pi\) from the solutions obtained so far. Hence, other possible solutions are,

\[
p = 7\pi/6 - 2\pi, 11\pi/6 - 2\pi
\]

\[
\Rightarrow \quad p = -5\pi/6, -\pi/6
\]

As \(-\pi/2 \leq p \leq 2\pi\), \(-5\pi/6\) must be rejected, as it is out of this range. So,

\[
p = -\pi/6, 7\pi/6, 11\pi/6
\]
Example 3

Solve $3 \cos x - 2 = 0 \ (0 \leq x \leq 2\pi)$.

\[
3 \cos x - 2 = 0 \\
\cos x = 2/3
\]

\[
y = 2/3 \\
y = \cos x
\]

2 solutions expected

\[
\cos x = 2/3 \\
\therefore \quad RAA = \cos^{-1}(2/3) \\
\Rightarrow \quad RAA = 0.841\ldots
\]

\[
\begin{array}{c|c|c}
S & A \\
\pi - RAA & RAA & \checkmark \\
\pi + RAA & 2\pi - RAA & \\
\end{array}
\]

\[
\cos \text{ is } +ve
\]

\[
\therefore \quad x = 0.841\ldots, 2\pi - 0.841\ldots \\
\Rightarrow \quad x = 0.841, 5.442 \ (\text{to } 3 \text{ d.p.})
\]
Example 4

Solve $5 \tan 2x^\circ = 3 \ (0 \leq x \leq 360)$.

$$5 \tan 2x^\circ = 3$$

$$\tan 2x^\circ = \frac{3}{5}$$

4 solutions expected

$$\tan 2x^\circ = \frac{3}{5}$$

∴ $\text{RAA} = \tan^{-1}(3/5)$

⇒ $\text{RAA} = 30.96...^\circ$

<table>
<thead>
<tr>
<th>S</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$180^\circ - \text{RAA}$</td>
<td>$\sqrt{\text{RAA}}$</td>
</tr>
</tbody>
</table>

$tan \text{ is } +ve$

∴ $2x^\circ = 30.96...^\circ, 180^\circ + 30.96...^\circ$

⇒ $2x^\circ = 30.96...^\circ, 210.96...^\circ$

As $\tan$ is periodic with period $180^\circ$, the other 2 solutions are obtained by constantly adding $180^\circ$ to the above two values for $2x^\circ$ until we reach 2 more different values for $2x^\circ$ up to $720^\circ$ (as $0 \leq x \leq 360, 0 \leq 2x \leq 720$).

∴ $2x^\circ = 30.96...^\circ, 210.96...^\circ, 390.96...^\circ, 570.96...^\circ$

⇒ $x^\circ = 15.5^\circ, 105.5^\circ, 195.5^\circ, 285.5^\circ$
Example 5 (non-calculator)

Solve $2 \sin (3\theta + \pi/4) - 1 = 0$ ($0 < \theta \leq 2\pi$).

\[
2 \sin (3\theta + \pi/4) - 1 = 0 \\
\sin (3\theta + \pi/4) = 1/2
\]

6 solutions expected

\[
\sin (3\theta + \pi/4) = 1/2 \\
\therefore \quad \text{RAA} = \sin^{-1} (0.5) \\
\Rightarrow \quad \text{RAA} = \pi/6
\]

S

\[
\begin{array}{c|c}
\text{S} & \text{A} \\
\hline
\pi - \text{RAA} & \checkmark \\
\text{RAA} & \checkmark \\
\pi + \text{RAA} & \\
T & 2\pi - \text{RAA} \\
\hline
\end{array}
\]

\[
\therefore \quad 3\theta + \pi/4 = \pi/6, \pi - \pi/6
\]

\[
\Rightarrow \quad 3\theta = -\pi/12, 7\pi/12
\]

As $\sin$ is periodic with period $2\pi$, the other 4 solutions are obtained by adding $2\pi$ to the above values for $3\theta$ until we reach values for $3\theta$ between 0 and $6\pi$ (as $0 < \theta \leq 2\pi$, $0 < 3\theta \leq 6\pi$). Also, note that $6\pi = 72\pi/12$.

\[
\therefore \quad 3\theta = -\pi/12, 7\pi/12, 23\pi/12, 31\pi/12, 47\pi/12, 55\pi/12, 71\pi/12, 79\pi/12
\]

Taking the values that are in the relevant range ($0 < 3\theta \leq 72\pi/12$), we have,

\[
\theta = 7\pi/36, 23\pi/36, 31\pi/36, 47\pi/36, 55\pi/36, 71\pi/36
\]
CfE Higher Maths

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pg. 182-4  Ex. 8D  All Q