24 / 5 / 17

Linear and Parabolic Motion - Lesson 1

Displacement, Velocity and Acceleration

LI

- Know the links between displacement, velocity and acceleration.
- Perform rectilinear motion calculations involving these 3 physical quantities.

SC

- Differentiate and integrate powers and trig. functions.
- Substitution.

Basic Definitions Regarding Objects

<u>Definition:</u>

A **body** is a physical object.

<u>Definition:</u>

A **(point) particle** (aka **pointlike particle** or **ideal particle**) is an idealization of a body, represented mathematically as zero-dimensional (no spatial extent) but still retaining physical properties such as mass.

<u>Definition:</u>

A physical property of a body is **uniform** if that property does not change over time.

<u>Definition:</u>

A **reference frame** is a system of coordinates used to describe the motion of a body.

Rectilinear means in a straight line

Definition:

The **displacement (from the origin)** of a particle is a vector function of time, where **u** is a unit vector (normally taken to be **i**, **j** or **k**) in the direction of motion,

$$\mathbf{s}(t) \stackrel{def}{=} s(t)\,\mathbf{u} \equiv x(t)\,\mathbf{u} \stackrel{def}{=} \mathbf{x}(t)$$

The **distance** a particle travels is the magnitude of displacement,

$$s(t) \stackrel{def}{=} |\mathbf{s}| = x(t)$$

Displacement is a vector, whereas distance is a scalar. They both have SI unit the metre (m).

Sometimes 'r' instead of 's' is used for displacement

Definition:

The **velocity** of a particle is the time-derivative of its displacement,

$$\mathbf{v}(t) \stackrel{def}{=} \frac{d\mathbf{s}}{dt} \equiv \dot{\mathbf{s}}(t) \stackrel{def}{=} \dot{s}(t)\,\mathbf{u}$$

The **speed** of a particle is the magnitude of its velocity,

$$v(t) \stackrel{def}{=} |\mathbf{v}| = \dot{s}(t) = \dot{x}(t)$$

Velocity is a vector, whereas speed is a scalar. They both have SI unit the metre per second (m s^{-1}).

Corollary:

Displacement is the integral of velocity,

$$\mathbf{s}(t) = \int \mathbf{v}(t)\,dt \stackrel{def}{=} \int v(t)\,dt\;\mathbf{u}$$

Corollary:

An object with zero velocity has constant displacement.

Definition:

The **acceleration vector** of a particle is the first derivative of velocity (equivalently, the second derivative of displacement),

$$\mathbf{a}(t) \overset{def}{=} \frac{d\mathbf{v}}{dt} \equiv \dot{\mathbf{v}}(t) = \ddot{\mathbf{s}}(t) \equiv \frac{d^2\mathbf{s}}{dt^2}$$

The **magnitude of acceleration** of a particle is the magnitude of its acceleration vector,

$$a(t) \overset{def}{=} |\mathbf{a}| = \dot{v}(t) = \ddot{s}(t) = \ddot{x}(t)$$

Acceleration has SI unit the metre per second squared (m s^{-2}).

Corollary:

Velocity is the integral of acceleration,

$$\mathbf{v}(t) = \int \mathbf{a}(t)\ dt \overset{def}{=} \int a(t)\ dt\ \mathbf{u}$$

Corollary:

An object with zero acceleration has constant velocity.

Useful Terminology

Constant - unchanging

At rest - velocity = 0

Initially - time starts at 0

At origin - displacement = 0

Connections Between Displacement, Velocity and Acceleration

$$\begin{tikzcd} \underline{r}\,(t) \arrow[r, shift left, "\frac{d}{dt}"] & \underline{v}\,(t) \arrow[l, shift left, "\int"] \arrow[r, shift left, "\frac{d}{dt}"] & \underline{a}\,(t) \arrow[l, shift left, "\int"] \end{tikzcd}$$

Example 1

A particle has velocity $\underline{v}(t) = (3t^2 - 4t)\underline{i}$. Initially, the particle is at a distance of -3 m from the origin of the coordinate system.

Calculate the particle's :

(a) displacement at time 4 s.
(b) distance from the origin at $t = 1$ s.
(c) acceleration vector at $t = 2/3$ s.

(a) $$\underline{r}(t) = \int \underline{v}(t)\, dt$$

$$\therefore \quad \underline{r}(t) = \int (3t^2 - 4t)\underline{i}\, dt$$

$$\Rightarrow \quad \underline{r}(t) = (t^3 - 2t^2)\underline{i} + \underline{C}$$

$\underline{r}(0) = -3\underline{i}$ gives,

$$-3\underline{i} = (0^3 - 2 \cdot 0^2)\underline{i} + \underline{C}$$

$$\Rightarrow \quad \underline{\underline{C} = -3\underline{i}}$$

$$\therefore \quad \underline{\underline{r}(t) = (t^3 - 2t^2 - 3)\underline{i}}$$

$$\therefore \quad \underline{r}(4) = ((4)^3 - 2(4)^2 - 3)\underline{i}$$

$$\Rightarrow \quad \boxed{\underline{r}(4) = 29\underline{i}\ \text{m}}$$

(b) $$\underline{r}(1) = ((1)^3 - 2(1)^2 - 3)\underline{i}$$

$$\Rightarrow \quad \underline{r}(1) = -4\underline{i}$$

$$\therefore \quad \boxed{r(1) = 4\ \text{m}}$$

(c) $$\underline{a}(t) = \frac{d}{dt}\underline{v}(t)$$

$$\Rightarrow \quad \underline{a}(t) = \frac{d}{dt}(3t^2 - 4t)\underline{i}$$

$$\Rightarrow \quad \underline{\underline{a}(t) = (6t - 4)\underline{i}}$$

$$\therefore \quad \underline{a}(2/3) = (6(2/3) - 4)\underline{i}$$

$$\Rightarrow \quad \boxed{\underline{a}(2/3) = \underline{0}\ \text{m s}^{-2}}$$

Jun 8-12:56

Example 2

A particle has acceleration $\underline{a}(t) = (\sin 2t)\,\underline{k}$. Initially, the particle is at rest at the origin of the coordinate system.

Calculate the :

(a) particle's speed at time $\pi/2$ s.
(b) particle's distance from the origin at $t = \pi/2$ s.
(c) times at which the acceleration is zero.

(a) $$\underline{v}(t) = \int \underline{a}(t)\ dt$$

$$\therefore \quad \underline{v}(t) = \int (\sin 2t)\,\underline{k}\ dt$$

$$\Rightarrow \quad \underline{v}(t) = (-1/2 \cos 2t)\,\underline{k} + \underline{C}$$

$\underline{v}(0) = \underline{0}$ gives,

$$\underline{0} = (-1/2\,.\,1)\,\underline{k} + \underline{C}$$

$$\Rightarrow \quad \underline{C} = 1/2\,\underline{k}$$

$$\therefore \quad \underline{v}(t) = (1/2 - 1/2 \cos 2t)\,\underline{k}$$

$$\therefore \quad \underline{v}(\pi/2) = (1/2 - 1/2 \cos \pi)\,\underline{k}$$

$$\Rightarrow \quad \underline{v}(\pi/2) = \underline{k}\ \text{m s}^{-1}$$

$$\therefore \quad \boxed{v(\pi/2) = 1\ \text{m s}^{-1}}$$

(b) $$\underline{r}(t) = \int \underline{v}(t)\ dt$$

$$\therefore \quad \underline{r}(t) = \int (1/2 - 1/2 \cos 2t)\,\underline{k}\ dt$$

$$\Rightarrow \quad \underline{r}(t) = (t/2 - 1/4 \sin 2t)\,\underline{k} + \underline{D}$$

$\underline{r}(0) = \underline{0}$ gives,

$$\underline{0} = (0/2 - 1/4\,.\,0)\,\underline{k} + \underline{D}$$

$$\Rightarrow \quad \underline{D} = \underline{0}$$

$$\therefore \quad \underline{r}(t) = (t/2 - 1/4 \sin 2t)\,\underline{k}$$

$$\therefore \quad \underline{r}(\pi/2) = (\pi/4 - 1/4 \sin \pi)\,\underline{k}$$

$$\Rightarrow \quad \underline{r}(\pi/2) = \pi/4\ \underline{k}$$

$$\therefore \quad \boxed{r(\pi/2) = \pi/4\ \text{m}}$$

(c) $$\underline{a}(t) = \underline{0}$$

$$\therefore \quad (\sin 2t)\,\underline{k} = \underline{0}$$

$$\therefore \quad \sin 2t = 0$$

$$\Rightarrow \quad \boxed{t = (n\pi/2)\ \text{s} \quad (n = 0, 1, 2, 3, \ldots)}$$

Jun 8-12:56

Blue Book

- pg. 397 - 400 Ex. 16 A Q 10 - 14, 44 - 48.