19 / 9 / 16

Vectors - Lesson 1

Vector Components, Addition and Subtraction, Scalar Multiplication, Equality, Parallel Vectors, Magnitude of a Vector and Unit Vectors

LI

- Know what a Vector is (compared to a Scalar).
- $+$, $-$ and Scalar Multiply vectors.
- Know when vectors are equal.
- Know when vectors are Parallel.
- Work out the Magnitude of a vector.
- Find a Unit Vector parallel to a given vector.

SC

- Arithmetic.

Vector - quantity with magnitude (aka size) and direction.

Scalar - quantity with magnitude only. (these are just numbers)

Examples of Vectors

Displacement, velocity, force, electric field.

Examples of Scalars

Distance, speed, frequency, temperature, mass, volume.

Vectors can exist in n dimensions, but we will only study 2D and 3D vectors

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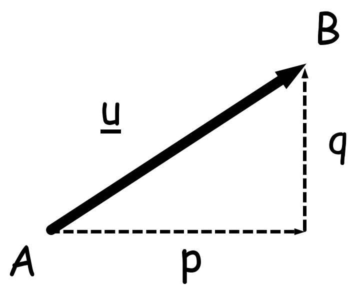
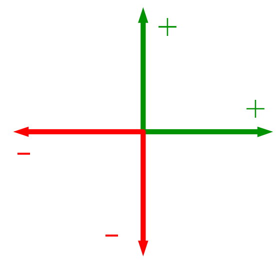
Basic Vectors Terminology and Notation

2D Vectors





Components of **u**

$$\mathbf{u} = \overrightarrow{AB} = \begin{pmatrix} p \\ q \end{pmatrix}$$

p is the x - component of **u**

q is the y - component of **u**

3D Vectors

Components of **v**

$$\mathbf{v} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$$

p is the x - component of **v**

q is the y - component of **v**

r is the z - component of **v**

Equality of Vectors

Two vectors are equal if all their respective components are equal, i.e. x - components equal to each other, y - components equal to each other (and z for 3D)

Addition and Subtraction of Vectors

The sum (aka resultant) of two vectors is obtained by adding their respective components

The difference of vectors **u** and **w** (denoted **u** − **w**) is the vector obtained by subtracting the components of **w** from the components of **u**

Scalar Multiplication of a Vector (by a Number)

The scalar multiple of **u** by a number d is the vector obtained by multiplying each component of **u** by d.

> Two vectors **u** and **v** are parallel if one is a scalar multiple of the other, i.e. if,
>
> $$\mathbf{u} = d\,\mathbf{v}$$

Magnitude of a Vector

If $\mathbf{u} = \overrightarrow{AB} = \begin{pmatrix} p \\ q \end{pmatrix}$, the magnitude of **u** is the scalar :

$$\boxed{|\mathbf{u}| = \sqrt{p^2 + q^2}}$$

If $\mathbf{v} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$, the magnitude of **v** is the scalar :

$$\boxed{|\mathbf{v}| = \sqrt{p^2 + q^2 + r^2}}$$

Sep 19-12:12

Special Vectors

The Zero Vector

The zero vector is the vector with all components equal to 0

Unit Vector

A unit vector is a vector that has magnitude equal to 1

In 3 dimensions :

- the unit vector in the x - direction is denoted by **i**.
- the unit vector in the y - direction is denoted by **j**.
- the unit vector in the z - direction is denoted by **k**.

In 2D, there is no **k**.

Every 3D vector $\mathbf{v} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ can be written as :

$\mathbf{v} = p\,\mathbf{i} + q\,\mathbf{j} + r\,\mathbf{k}$.

In 3D,

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

In 2D,

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Every 2D vector $\mathbf{u} = \begin{pmatrix} p \\ q \end{pmatrix}$ can be written as :

$\mathbf{u} = p\,\mathbf{i} + q\,\mathbf{j}$.

Any vector (that's not already a unit vector) can be made into a unit vector by dividing the original vector by its magnitude

Example 1

If $\mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$, find :

(a) $\mathbf{u} + 2\mathbf{v}$.

(b) $|\mathbf{u}|$.

(c) $|2\mathbf{u} - 3\mathbf{v}|$.

(a) $$\mathbf{u} + 2\mathbf{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 2\begin{pmatrix} -3 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} 2 + (-6) \\ 1 + 10 \end{pmatrix}$$
$$= \boxed{\begin{pmatrix} -4 \\ 11 \end{pmatrix}}$$

(b) $$|\mathbf{u}| = \sqrt{2^2 + 1^2}$$
$$= \boxed{\sqrt{5}}$$

(c) $$2\mathbf{u} - 3\mathbf{v} = 2\begin{pmatrix} 2 \\ 1 \end{pmatrix} - 3\begin{pmatrix} -3 \\ 5 \end{pmatrix}$$
$$= \begin{pmatrix} 4 - 9 \\ 2 - 15 \end{pmatrix}$$
$$= \underline{\begin{pmatrix} -5 \\ -13 \end{pmatrix}}$$

$$\therefore \quad |2\mathbf{u} - 3\mathbf{v}| = \sqrt{(-5)^2 + (-13)^2}$$
$$= \sqrt{25 + 169}$$
$$= \boxed{\sqrt{194}}$$

Example 2

If $\mathbf{w} = 3\,\mathbf{i} + 6\,\mathbf{j} - \mathbf{k}$, find :

(a) the magnitude of **w**.

(b) a unit vector parallel to **w**.

(a) $$|\,\mathbf{w}\,| = \sqrt{3^2 + 6^2 + (-1)^2} = \sqrt{9 + 36 + 1} = \boxed{\sqrt{46}}$$

(b) A unit vector parallel to **w** is obtained by taking the vector **w** and dividing each of its components by $|\,\mathbf{w}\,|$. So,

$$\boxed{\text{A unit vector parallel to } \mathbf{w} \text{ is } \frac{1}{\sqrt{46}}\begin{pmatrix} 3 \\ 6 \\ -1 \end{pmatrix}}$$

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CfE Higher Maths

pg. 98-9 Ex. 5A All Q

Oct 29-09:14