21 / 12 / 16

Recurrence Relations - Lesson 1

Investigating Recurrence Relations

LI

- Know what a (linear) recurrence relations is.
- Use a recurrence relation to calculate unknown terms.
- Make a recurrence relation to model a real-life situation.

SC

- Use a calculator.

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A recurrence relation is a rule for obtaining a sequence given a previous value or previous values

A linear recurrence relation is a recurrence relation of the form :

$$u_{n+1} = a\,u_n + b$$

(a, b are constants with $a \neq 0$)

Example 1

Find a recurrence relation for the sequence that starts 7, 11, 15, 19, ...

Any term of the sequence (apart from the first), is obtained by adding 4 to the previous term. So,

$$\boxed{u_{n+1} = u_n + 4 \quad (u_1 = 7)}$$

Example 2

Find a recurrence relation for the sequence that starts 3, 9, 27, 81, ...

Any term of the sequence (apart from the first), is obtained by multiplying the previous term by 3. So,

$$\boxed{u_{n+1} = 3u_n \quad (u_1 = 3)}$$

Example 3

Given the recurrence relation $u_{n+1} = 5u_n - 2$ ($u_0 = 1$), find the 4th term.

$$u_{n+1} = 5u_n - 2 \quad (u_0 = 1)$$

$$u_1 = 5u_0 - 2 = 5 \times 1 - 2 \Rightarrow \underline{u_1 = 3}$$

$$u_2 = 5u_1 - 2 = 5 \times 3 - 2 \Rightarrow \underline{u_2 = 13}$$

$$u_3 = 5u_2 - 2 = 5 \times 13 - 2 \Rightarrow \underline{u_3 = 63}$$

$$u_4 = 5u_3 - 2 = 5 \times 63 - 2 \Rightarrow \boxed{u_4 = 313}$$

Example 4

240 ml of a drug is administered to a patient. Every hour 7 % of the drug passes out of his bloodstream. To compensate, a further 11 ml dose is given every hour.

(a) Form a recurrence relation to model this situation.

(b) Calculate the amount of drug remaining after 4 hours.

(a) Let u_n be the amount of drug in the bloodstream at the end of the n^{th} hour. If 7 % of the original amount passes out, then 93 % of the original remains. So,

$$u_{n+1} = 0.93\,u_n + 11 \quad (u_0 = 240)$$

(b)

$$u_1 = 0.93 \times 240 + 11$$

$$\Rightarrow \underline{u_1 = 234.2}$$

$$u_2 = 0.93 \times 234.2 + 11$$

$$\Rightarrow \underline{u_2 = 228.806}$$

$$u_3 = 0.93 \times 228.806 + 11$$

$$\Rightarrow \underline{u_3 = 223.789\,58}$$

$$u_4 = 0.93 \times 223.789\,58 + 11$$

$$\Rightarrow \underline{u_4 = 219.124\ldots}$$

$\boxed{\text{After 4 hours, 219.1 ml remain.}}$

Example 5

240 ml of a drug is administered to a patient. Every 3 hours 30 % of the drug passes out of her bloodstream. To compensate, a further 10 ml dose is given every 3 hours.

(a) Form a recurrence relation to model this situation.

(b) Calculate the amount of drug remaining after 12 hours.

(a) Let u_n be the amount of drug in the bloodstream at the end of 3 n hours. Then,

$$u_{n+1} = 0.7\,u_n + 10 \quad (u_0 = 240)$$

(b)

$u_1 = 0.7 \times 240 + 10$

$\Rightarrow$ $u_1 = 178$ (at the end of 3 hours)

$u_2 = 0.7 \times 178 + 10$

$\Rightarrow$ $u_2 = 134.6$ (at the end of 6 hours)

$u_3 = 0.7 \times 134.6 + 10$

$\Rightarrow$ $u_3 = 104.22$ (at the end of 9 hours)

$u_4 = 0.7 \times 104.22 + 10$

$\Rightarrow$ $u_4 = 82.954...$ (at the end of 12 hours)

After 12 hours, 82.95 ml remain.

Example 6

A sequence is defined by the recurrence relation $u_{n+1} = m u_n + c$. If $u_1 = 32$, $u_2 = 20$ and $u_3 = 14$, find the values of m and c.

$u_2 = 20$ and $u_1 = 32$ gives,

$$u_2 = m u_1 + c$$

$$20 = 32m + c$$

$u_3 = 14$ and $u_2 = 20$ gives,

$$u_3 = m u_2 + c$$

$$14 = 20m + c$$

Subtracting the two equations gives,

$$6 = 12m$$

$$\boxed{m = 1/2}$$

Substituting this into the first equation gives,

$$20 = 32(1/2) + c$$

$$20 = 16 + c$$

$$\boxed{c = 4}$$

CfE Higher Maths

- pg. 321 - 2 Ex. 15B Q 1 a-g, 2 - 5
- pg. 323 - 4 Ex. 15C Q 1 - 7