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*Functions - Lesson 1*

# Functions - Domains and Ranges

LI

- Know what a function is.
- Know what the domain and range of a function are.
- Find the domain and range of a function.

SC

- General features of graphs of linear, quadratic, trigonometric, exponential and logarithmic functions.

Jan 10-12:11

## Functions

A function can be thought of as a machine; something goes in and something comes out. The only requirement is that a specific input cannot give more than one output.

If $f$ is a function, all the possible inputs taken together is the domain of $f$ (dom $f$) and all the possible outputs taken together is called the range of $f$ (ran $f$).

A function is normally written as an equation, but does not have to be written so.

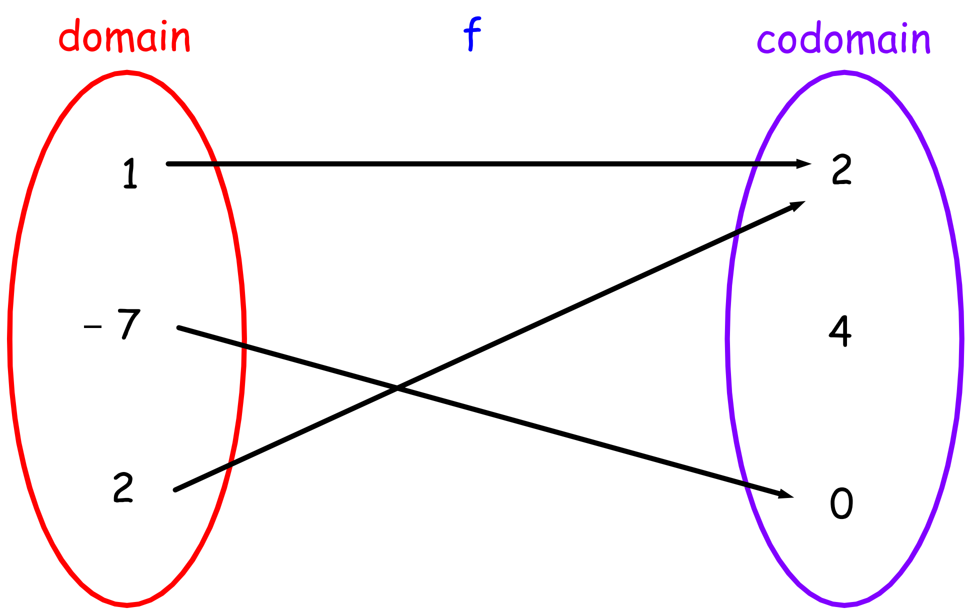
If a function has input $x$ and corresponding output $y$, then we write $f(x) = y$.

When we specify a function, sometimes the outputs are a smaller part of a bigger collection $B$ (this bigger collection is called the codomain of $f$). If the function has domain $A$, then we normally write :

$$f : A \longrightarrow B$$

Example 1

Show that the following is a function and state the domain and range.



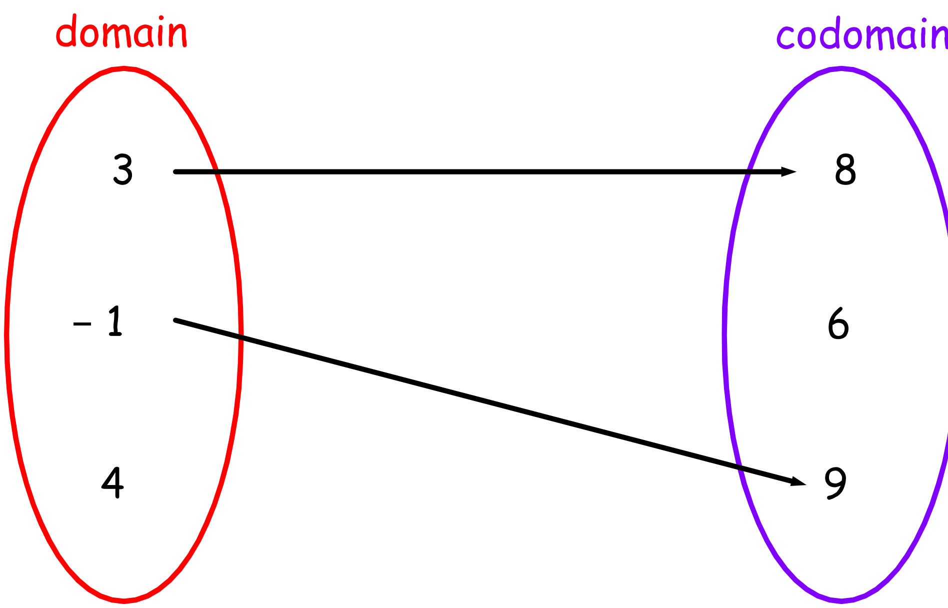

Every element of the domain gets sent to a single element in the codomain; so, f is a function.

$$\text{dom } f = \{1, -7, 2\}$$

$$\text{ran } f = \{0, 2\}$$

Example 2

Show that the following is not a function.




Not every element in the domain gets sent to some element in the codomain; so, $f$ is not a function.

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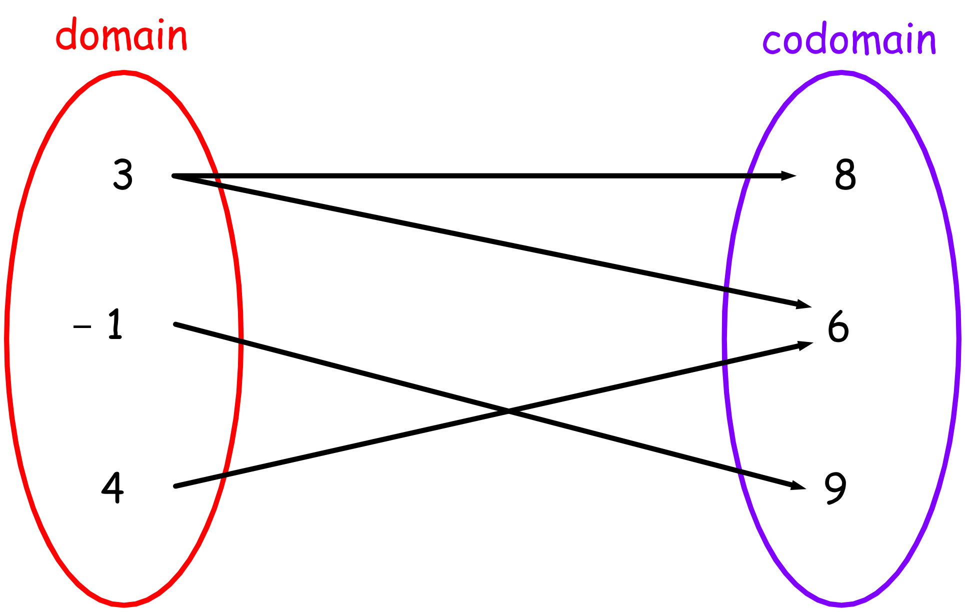
Example 3

Show that the following is not a function.




3 (in the domain) does not get sent to a unique element in the codomain (3 gets sent to 8 and 6); so, f is not a function.

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## Finding Domains and Ranges of Functions Written as a Formula

Sketching the graph of a function often helps to find the domain and range of that function

The domain is all possible $x$ - values for which the function makes sense, i.e. for which a $y$ - value can be worked out

The only times a $y$ - value cannot be worked out are when :

- a fraction has a 0 denominator.
- a square root has a negative.
- taking the logarithm of 0 or negative number.

The range is all possible $y$ - values that can be worked out

The range is often easily obtained from the graph of the function.

If a $y$ - value can be worked out for all possible $x$ - values, we say that the function has domain $\mathbb{R}$ .

When we write, $x \in \mathbb{R}$, it means that '$x$ is a real number'.

Example 4

Find the domains and ranges of the following functions :

(a) $f(x) = 4x - 7$.

(b) $g(x) = (x - 3)^2 + 2$.

(c) $h(x) = \sin x$.

(d) $k(x) = 6^x$.

(e) $L(x) = \log_5(2x + 17)$.

(a) There is no restriction on the $x$ - values, so all $x$ - values are possible. From the graph, all $y$ - values are possible too. Thus,

$\text{dom } f = \mathbb{R}$

$\text{ran } f = \mathbb{R}$

allowed to write 'all x - values' and 'all y - values'

(b) There is no restriction on the $x$ - values; however, there is a minimum turning point at (3, 2), so the $y$ - values cannot go below 2. So,

$\text{dom } g = \mathbb{R}$

$\text{ran } g = \{y \in \mathbb{R} : y \geq 2\}$

allowed to write 'all x - values' and '$y \geq 2$'

(c) There is no restriction on the $x$ - values; however, the $y$ - values are between $-1$ and 1 (including both). So,

$\text{dom } h = \mathbb{R}$

$\text{ran } h = \{y \in \mathbb{R} : -1 \leq y \leq 1\}$

allowed to write 'all x - values' and '$-1 \leq y \leq 1$'

(d) There is no restriction on the $x$ - values; however, the $y$ - values are always above the $x$ - axis. So,

$$\text{dom } k = \mathbb{R}$$

$$\text{ran } k = \{y \in \mathbb{R} : y > 0\}$$

allowed to write
'all $x$ - values'
and '$y > 0$'

(e) There is no restriction on the $y$ - values; however, we require $2x + 17 > 0$ (can't take the log of $0$ or a negative). So, we require $x > -17/2$. Hence,

$$\text{dom } L = \{x \in \mathbb{R} : x > -17/2\}$$

$$\text{ran } L = \mathbb{R}$$

allowed to write
'$x > -17/2$'
and 'all $y$ - values'

Example 5

Find the largest possible domains of these functions :

(a) $A(x) = \dfrac{1}{2x + 5}$ .

(b) $P(x) = \sqrt{9 - 4x}$ .

(c) $n(x) = \dfrac{1}{\sqrt{x + 7}}$ .

(d) $D(x) = 2 \log_{17}(1 - 9x)$.

(e) $r(x) = 3\cos(4x - 5.67) + 0.6$.

The 'largest possible domain' just means what we call 'the domain'.

(a) We require the denominator to be non-zero. The only time when the denominator is 0 is when $2x + 5 = 0$, i.e. when $x = -5/2$. So,

$$\boxed{\text{dom } A = \{x \in \mathbb{R} : x \neq -5/2\}}$$

allowed to write 'all x - values except $-5/2$' or '$x \neq -5/2$'

(b) We require the root to be non-negative, i.e. we need $9 - 4x \geq 0$. This gives $x \leq 9/4$. So,

$$\boxed{\text{dom } P = \{x \in \mathbb{R} : x \leq 9/4\}}$$

allowed to write 'all x - values less than or equal to $9/4$' or '$x \leq 9/4$'

(c) We require $x + 7 > 0$. This gives $x > -7$. So,

$$\boxed{\text{dom } n = \{x \in \mathbb{R} : x > -7\}}$$

allowed to write 'all x - values greater than $-7$' or '$x > -7$'

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(d) We require $1 - 9x > 0$. This gives $x < 1/9$. So,

$$\text{dom } D = \{ x \in \mathbb{R} : x < 1/9 \}$$

allowed to write 'all x - values less than 1/9' or '$x < 1/9$'

(e) There is no restriction on the $x$ - values. So,

$$\text{dom } r = \mathbb{R}$$

allowed to write 'all x - values'

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