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Functions - Lesson 1

Functions - Domains and Ranges

**LI**
- Know what a function is.
- Know what the domain and range of a function are.
- Find the domain and range of a function.

**SC**
- General features of graphs of linear, quadratic, trigonometric, exponential and logarithmic functions.
Functions

A function can be thought of as a machine; something goes in and something comes out. The only requirement is that a specific input cannot give more than one output.

If \( f \) is a function, all the possible inputs taken together is the domain of \( f \) (\( \text{dom} \ f \)) and all the possible outputs taken together is called the range of \( f \) (\( \text{ran} \ f \)).

A function is normally written as an equation, but does not have to be written so.

If a function has input \( x \) and corresponding output \( y \), then we write \( f(x) = y \).

When we specify a function, sometimes the outputs are a smaller part of a bigger collection \( B \) (this bigger collection is called the codomain of \( f \)). If the function has domain \( A \), then we normally write:

\[
 f : A \longrightarrow B
\]
Example 1

Show that the following is a function and state the domain and range.

\[
\text{domain} \quad f \quad \text{codomain}
\]

\[
1 \quad 2 \\
-7 \quad 4 \\
2 \quad 0
\]

Every element of the domain gets sent to a single element in the codomain; so, \( f \) is a function.

\[
\text{dom } f = \{1, -7, 2\} \\
\text{ran } f = \{0, 2\}
\]
Example 2

Show that the following is not a function.

Not every element in the domain gets sent to some element in the codomain; so, \( f \) is not a function.
Example 3

Show that the following is not a function.

3 (in the domain) does not get sent to a unique element in the codomain (3 gets sent to 8 and 6); so, f is not a function.
Finding Domains and Ranges of Functions Written as a Formula

Sketching the graph of a function often helps to find the domain and range of that function.

The domain is all possible $x$ - values for which the function makes sense, i.e. for which a $y$ - value can be worked out.

The only times a $y$ - value cannot be worked out are when:

- a fraction has a 0 denominator.
- a square root has a negative.
- taking the logarithm of 0 or negative number.

The range is all possible $y$ - values that can be worked out.

The range is often easily obtained from the graph of the function.

If a $y$ - value can be worked out for all possible $x$ - values, we say that the function has domain $\mathbb{R}$.

When we write, $x \in \mathbb{R}$, it means that ' $x$ is a real number'.
Example 4

Find the domains and ranges of the following functions:

(a) \( f(x) = 4x - 7 \).

(b) \( g(x) = (x - 3)^2 + 2 \).

(c) \( h(x) = \sin x \).

(d) \( k(x) = 6^x \).

(e) \( L(x) = \log_5(2x + 17) \).

(a) There is no restriction on the \( x \)-values, so all \( x \)-values are possible. From the graph, all \( y \)-values are possible too. Thus,

\[
\text{dom } f = \mathbb{R} \quad \text{allowed to write 'all } x \text{ - values'}
\]
\[
\text{ran } f = \mathbb{R} \quad \text{and 'all } y \text{ - values'}
\]

(b) There is no restriction on the \( x \)-values; however, there is a minimum turning point at \((3, 2)\), so the \( y \)-values cannot go below \( 2 \). So,

\[
\text{dom } g = \mathbb{R} \quad \text{allowed to write 'all } x \text{ - values'}
\]
\[
\text{ran } g = \{ y \in \mathbb{R} : y \geq 2 \} \quad \text{and '} y \geq 2 \text{'}
\]

(c) There is no restriction on the \( x \)-values; however, the \( y \)-values are between \(-1\) and \(1\) (including both). So,

\[
\text{dom } h = \mathbb{R} \quad \text{allowed to write 'all } x \text{ - values'}
\]
\[
\text{ran } h = \{ y \in \mathbb{R} : -1 \leq y \leq 1 \} \quad \text{and '}-1 \leq y \leq 1\text{'}
\]
(d) There is no restriction on the \( x \) - values; however, the \( y \) - values are always above the \( x \) - axis. So,

\[
\text{dom } k = \mathbb{R} \\
\text{ran } k = \{ y \in \mathbb{R} : y > 0 \} \\
\text{allowed to write} \\
'\text{all } x \text{- values}' \\
'\text{and } y > 0' \\
\]

(e) There is no restriction on the \( y \) - values; however, we require \( 2x + 17 > 0 \) (can't take the log of 0 or a negative). So, we require \( x > -17/2 \). Hence,

\[
\text{dom } L = \{ x \in \mathbb{R} : x > -17/2 \} \\
\text{ran } L = \mathbb{R} \\
\text{allowed to write} \\
'x > -17/2' \\
'\text{and all } y \text{- values}' \\
\]
Example 5

Find the largest possible domains of these functions:

(a) \( A (x) = \frac{1}{2x + 5} \).

(b) \( P (x) = \sqrt{9 - 4x} \).

(c) \( n (x) = \frac{1}{\sqrt{x + 7}} \).

(d) \( D (x) = 2 \log_{10} (1 - 9x) \).

(e) \( r (x) = 3 \cos (4x - 5.67) + 0.6 \).

The 'largest possible domain' just means what we call 'the domain'.

(a) We require the denominator to be non-zero. The only time when the denominator is 0 is when \( 2x + 5 = 0 \), i.e. when \( x = -5/2 \). So,

\[ \text{dom } A = \{ x \in \mathbb{R} : x \neq -5/2 \} \]

allowed to write 'all } x \text{-values except } -5/2 \text{' or ' } x \neq -5/2 \text{'.}

(b) We require the root to be non-negative, i.e. we need \( 9 - 4x \geq 0 \). This gives \( x \leq 9/4 \). So,

\[ \text{dom } P = \{ x \in \mathbb{R} : x \leq 9/4 \} \]

allowed to write 'all } x \text{-values less than or equal to } 9/4 \text{' or ' } x \leq 9/4 \text{'.}

(c) We require \( x + 7 > 0 \). This gives \( x > -7 \). So,

\[ \text{dom } n = \{ x \in \mathbb{R} : x > -7 \} \]

allowed to write 'all } x \text{-values greater than } -7 \text{' or ' } x > -7 \text{'.
(d) We require $1 - 9x > 0$. This gives $x < 1/9$.
So,

$$\text{dom } D = \{ x \in \mathbb{R} : x < 1/9 \}$$

allowed to write 'all x - values less than 1/9' or 'x < 1/9'

(e) There is no restriction on the $x$ - values.
So,

$$\text{dom } r = \mathbb{R}$$

allowed to write 'all x - values'
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