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Exponentials and Logarithms - Lesson 1

Exponentials, Logarithms and Logarithmic Rules

LI

- Know what a logarithm is and how it is related to an exponential.
- Know and use the Logarithmic Rules.

SC

- Rules of indices.

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$$a^b = c$$

$$b = \log_a c$$

'a to the power of b equals c'

'b equals the base a logarithm of c'

A logarithm is a power

Examples

$$2^3 = 8 \longleftrightarrow 3 = \log_2 8$$

$$p^5 = 6 \longleftrightarrow 5 = \log_p 6$$

$$4^x = y \longleftrightarrow x = \log_4 y$$

$$5 = \log_n 7 \longleftrightarrow n^5 = 7$$

$$4 = \log_2 16 \longleftrightarrow 2^4 = 16$$

$$w = \log_a x \longleftrightarrow a^w = x$$

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Rules of Logarithms

$\log_b (p \times q) = \log_b p + \log_b q$

'multiplication goes to addition'

WARNING :
$\log_b (p + q) \neq \log_b p + \log_b q$

$\log_b (p \div q) = \log_b p - \log_b q$

'division goes to subtraction'

WARNING :
$\log_b (p - q) \neq \log_b p - \log_b q$

$\log_b p^n = n \log_b p$

'power goes down'

$\log_b 1 = 0$

'log one is zero'

$\log_b b = 1$

'log b b is one'

These 5 rules are derived from the corresponding rules of indices

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Example 1

Simplify $\log_7 3 + \log_7 21$.

$$\log_7 3 + \log_7 21 = \log_7 3 + \log_7 (3 \times 7)$$
$$= \log_7 3 + \log_7 3 + \log_7 7$$
$$= \boxed{2 \log_7 3 + 1}$$

Example 2

Simplify $(1/3)\log_2 8 - (1/5)\log_2 32$.

$$(1/3)\log_2 8 - (1/5)\log_2 32$$

$$= (1/3)\log_2 2^3 - (1/5)\log_2 2^5$$

$$= (1/3 \times 3)\log_2 2 - (1/5 \times 5)\log_2 2$$

$$= \log_2 2 - \log_2 2$$

$$= \boxed{0}$$

Example 3

Simplify $2\log_3 9 + \log_3 27 - 4\log_3 81$.

$$\begin{aligned} & 2\log_3 9 + \log_3 27 - 4\log_3 81 \\ &= 2\log_3 3^2 + \log_3 3^3 - 4\log_3 3^4 \\ &= 4\log_3 3 + 3\log_3 3 - 16\log_3 3 \\ &= -9\log_3 3 \\ &= -9(1) \\ &= \boxed{-9} \end{aligned}$$

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Example 4

If $\log_a y = \log_a 7 + 4\log_a x$,
express y in terms of x.

$$\log_a y = \log_a 7 + 4\log_a x$$

$$\Rightarrow \log_a y = \log_a 7 + \log_a x^4$$

$$\Rightarrow \log_a y = \log_a (7x^4)$$

$$\therefore \boxed{y = 7x^4}$$

Example 5

If $\log_F w = \log_F r - 17 \log_F N$,
express w in terms of r and N.

$$\log_F w = \log_F r - 17 \log_F N$$

$$\Rightarrow \log_F w = \log_F r - \log_F N^{17}$$

$$\Rightarrow \log_F w = \log_F (r / N^{17})$$

$$\therefore \quad \boxed{w = r / N^{17}}$$

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CfE Higher Maths

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pg. 11 Ex. 1D All Q

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