Polynomials - Lesson 1

Evaluating Polynomials and Nested Form

LI

- Know the difference between a Polynomial Expression, a Polynomial Equation and a Polynomial Function.
- Use the Nested Form to evaluate a polynomial function.

SC

- +, − integers.

Polynomial

many term

A Polynomial Expression is an expression of the form :

$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x^1 + a_0 x^0$

where n is a whole number, $a_0, \ldots, a_n$ are called coefficients

a_n is the leading coefficient

A Polynomial Equation is an equation obtained by putting a polynomial expression equal to 0

A Polynomial Function is a function obtained by putting a polynomial expression equal to f (x)

The degree of a polynomial is the value of the highest power

- Degree 0 polynomial - constant number (e. g. $4 = 4x^0$)
- Degree 1 polynomial - linear (e. g. $3 - 6x$)
- Degree 2 polynomial - quadratic (e. g. $x^2 - 6 + 5x$)
- Degree 3 polynomial - cubic (e. g. $x^2 - 2x^3 + 9x - 8$)
- Degree 4 polynomial - quartic (e. g. $x^4 + x$)

Examples of Polynomials

$4x^3 - 3.7x^2 + x$

$x^4 + 0.5x^2 + x^3 - \pi$

Examples of Non - Polynomials

$x^{2/3} + 5x^2 - 3$

$x^{0.6} + 8x^3 + 2$

In Higher Maths, polynomials tend to have integer coefficients.

A polynomial function is evaluated by
substituting in an x - value

Quick way of doing this : Nested Form (aka Horner's Method)

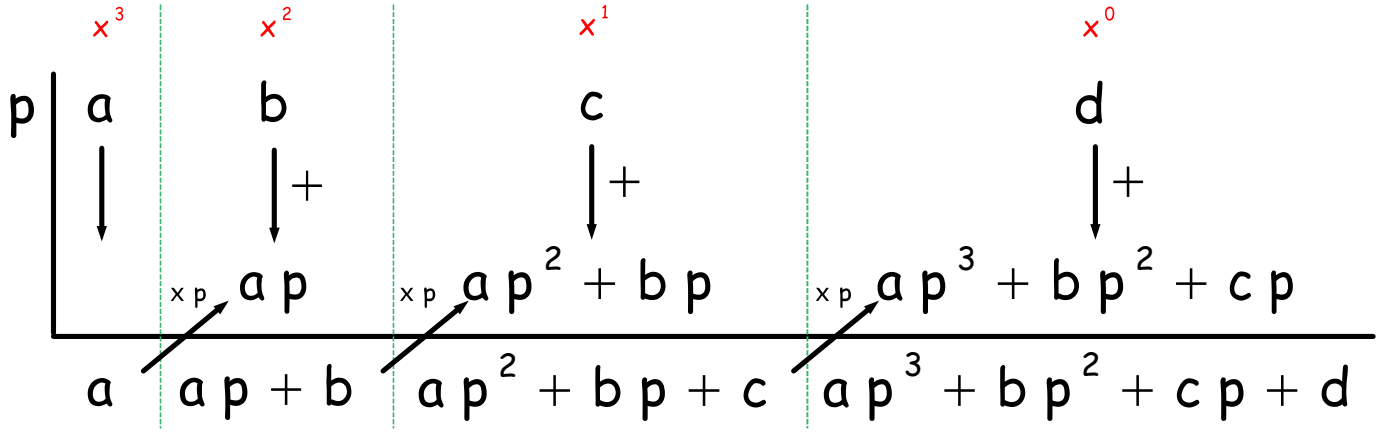
$$f(x) = a x^3 + b x^2 + c x + d$$

$$f(x) = x(a x^2 + b x + c) + d$$

$$f(x) = x(x(a x + b) + c) + d$$

This can be done without mentioning the variable (x) explicitly.

For example, to work out $f(x)$ at a specific value $x = p$,
i. e. to calculate $f(p)$, we use a shorthand :



Example 1

If $f(x) = 2x^3 - x^2 + 3x + 7$, calculate $f(2)$.

	x^3	x^2	x^1	x^0
2	2	−1	3	7
		4	6	18
	2	3	9	25

$\therefore$ $\boxed{f(2) = 25}$

Example 2

Given that $g(x) = 2x^3 - 5x^2 + 3$, evaluate $g(-3)$.

	x^3	x^2	x^1	x^0
-3	2	-5	0	3
		-6	33	-99
	2	-11	33	-96

$\therefore$ $\boxed{g(-3) = -96}$

Questions

Evaluate (all non-calculator) :

1) $f(2)$ when $f(x) = 2x^3 - x^2 + 4x - 5$.

2) $g(1)$ when $g(x) = 13x^3 + 8x^2 - 5$.

3) $h(-1)$ when $h(x) = 12x^3 - 8x^2 - 6x + 12$.

4) $p(-2)$ when $p(x) = -2x^3 + 3x^2 - 11x + 39$.

5) $k(-3)$ when $k(x) = -5x^3 - 10x + 1$.

6) $w(-5)$ when $w(x) = -x^3 - 3x^2 - 4x - 20$.

7) $j(-6)$ when $j(x) = -2x^3 + 2x^2 - 2x$.

Answers

1) $f(2) = 15.$

2) $g(1) = 16.$

3) $h(-1) = -2.$

4) $p(-2) = 89.$

5) $k(-3) = 166.$

6) $w(-5) = 50.$

7) $j(-6) = 516.$