

An example (aka instance) is a specific case that satisfies a general statement

WARNING : A general statement is never proven by verifying a specific case.

A direct proof is a way of proving that a certain conclusion follows directly from a set of assumptions using a logically convincing chain of reasoning

Notation

- $\mathbb N\,$ set of all natural numbers.
- $\mathbb W$ set of all whole numbers.
- $\ensuremath{\mathbb{Z}}$ set of all integers.
- ${\mathbb Q}\,$ set of all rational numbers.
- $\mathbb R$ set of all real numbers (all rational and irrational numbers).
- $\mathbb C$ set of all complex numbers.

Example 1

Prove that the square of any even number is even.

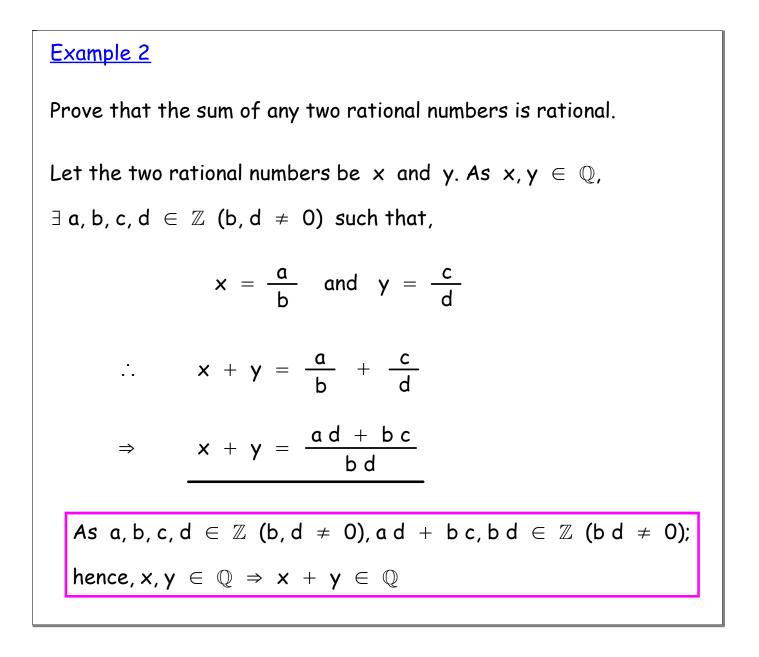
Let n be any even number. Then $\exists k \in \mathbb{Z}$ such that,

$$n = 2 k$$

$$\therefore n^{2} = (2 k)^{2}$$

$$\Rightarrow n^{2} = 4 k^{2}$$

$$\Rightarrow n^{2} = 2 (2 k^{2})$$
As $k \in \mathbb{Z}, 2 k^{2} \in \mathbb{Z}$; hence, n even $\Rightarrow n^{2}$ even



Example 3

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Prove that the square of any odd number plus the cube of any even number is odd.

Let n be any odd number and let m be any even number.

Then $\exists k, p \in \mathbb{Z}$ such that,

n = 2 k + 1 and m = 2 p

$$n^{2} + m^{3} = (2 k + 1)^{2} + (2 p)^{3}$$

$$\Rightarrow n^{2} + m^{3} = 4 k^{2} + 4 k + 1 + 8 p^{3}$$

$$\Rightarrow \qquad n^{2} + m^{3} = 2(2k^{2} + 2k + 4p^{3}) + 1$$

As k, p $\in \mathbb{Z}$, 2 k² + 2 k + 4 p³ $\in \mathbb{Z}$; hence, n odd and m even \Rightarrow n² + m³ is odd

Example 4

Prove that the product of any odd function and any even function is odd.

Let f be any odd function and let g be any even function. Then,

$$f(-x) = -f(x) \quad (\forall x \in \text{dom } f)$$

and

$$g(-x) = g(x) \quad (\forall x \in \text{dom } g)$$

Let h(x) = f(x)g(x). We must show that h is an odd function, i.e., that h(-x) = -h(x) ($\forall x \in \text{dom } h$). Let $x \in \text{dom } h$. Then,

$$h(-x) = f(-x)g(-x)$$

$$\Rightarrow h(-x) = (-f(x))g(x)$$

$$\Rightarrow h(-x) = -f(x)g(x)$$

$$\Rightarrow h(-x) = -h(x)$$

As h(-x) = -h(x) ($\forall x \in dom h$), the product of an odd and an even function is odd

Questions

Prove the following using direct proof :

- 1) The product of any two rational numbers is rational.
- 2) The difference of any two rational numbers is rational.
- 3) The quotient of any two non-zero rational numbers is rational.
- 4) The square of any odd number is odd.
- 5) The cube of any even number is even.
- 6) The sum of the squares of any two odd numbers is even.
- 7) The composition of any two linear functions is linear.
- 8) The product of any two exponential functions is exponential.
- 9) The derivative of any cubic function is quadratic.
- 10) The integral of any quadratic function is cubic.
- 11) The sum of any two even functions is even.
- 12) The product of any two odd functions is even.
- 13) The conjugate of the sum of any two complex numbers equals the sum of the conjugates of those two complex numbers.