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Trigonometric Phenomena - Lesson 1

Degrees, Radians and Exact Values

**LT**
- Convert angles from degrees to Radians (and vice versa).
- Know and work out Exact Values of sin, cos and tan.

**SC**
- Exact Value Table or Triangles.
- Trig. Graphs.
1 radian is the angle made by an arc of length equal to the radius of the circle

\[ \pi \text{ radians} = 180^\circ \]

1 radian \( \approx 57.3^\circ \)

Radians are used to make equations in maths and physics look simpler

**Must know these 8 equivalents:**

<table>
<thead>
<tr>
<th>Degrees</th>
<th>Radians</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
</tr>
<tr>
<td>30°</td>
<td>( \frac{\pi}{6} )</td>
</tr>
<tr>
<td>45°</td>
<td>( \frac{\pi}{4} )</td>
</tr>
<tr>
<td>60°</td>
<td>( \frac{\pi}{3} )</td>
</tr>
<tr>
<td>90°</td>
<td>( \frac{\pi}{2} )</td>
</tr>
<tr>
<td>180°</td>
<td>( \pi )</td>
</tr>
<tr>
<td>270°</td>
<td>( \frac{3\pi}{2} )</td>
</tr>
<tr>
<td>360°</td>
<td>2\pi</td>
</tr>
</tbody>
</table>
Exact Values are numerical answers that are not evaluated with a calculator, but left as surds, fractions, expressions with \( \pi \) etc.

Must know standard sin, cos and tan exact values:

\[
\begin{align*}
\sin 30^\circ &= \sin \frac{\pi}{6} = \frac{1}{2} \\
\cos 30^\circ &= \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \\
\tan 30^\circ &= \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} \\
\sin 60^\circ &= \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \\
\cos 60^\circ &= \cos \frac{\pi}{3} = \frac{1}{2} \\
\tan 60^\circ &= \tan \frac{\pi}{3} = \sqrt{3}
\end{align*}
\]

Patel’s Alternative Memory Aid:

<table>
<thead>
<tr>
<th>S</th>
<th>Angle</th>
<th>C</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0°</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>30°</td>
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<tr>
<td></td>
<td>90°</td>
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<td></td>
</tr>
</tbody>
</table>
Example 1

Change $225^\circ$ to radians.

$225^\circ = 180^\circ + 45^\circ$

$= \pi + \frac{\pi}{4}$

$= \frac{5\pi}{4}$

Alternative

$225^\circ = 5 \times 45^\circ$

$= 5 \times \frac{\pi}{4}$

$= \frac{5\pi}{4}$
Example 2

Change \( \frac{4\pi}{3} \) radians into degrees.

\[
\frac{4\pi}{3} = \frac{3\pi + \pi}{3} = \pi + \frac{\pi}{3}
\]

\[= 180^\circ + 60^\circ = 240^\circ\]

Alternative

\[
\frac{4\pi}{3} = 4 \times \frac{\pi}{3} = 4 \times 60^\circ = 240^\circ
\]
Example 3

If \( P \) is an acute angle with \( \tan P = \frac{2}{3} \), find the exact values of \( \sin P \) and \( \cos P \).

\[
L = \sqrt{2^2 + 3^2} = \sqrt{13}
\]

\[
\sin P = \frac{O}{H} = \frac{2}{\sqrt{13}}
\]

\[
\cos P = \frac{A}{H} = \frac{3}{\sqrt{13}}
\]
Example 4

Find the exact value of \( \sin 240^\circ \).

\[
\sin 240^\circ = -\sin 120^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}
\]
CfE Higher Maths

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pg. 31  Ex. 2B  All Q