



The Chain Rule tells us how to differentiate a composition of functions

Lagrange Form

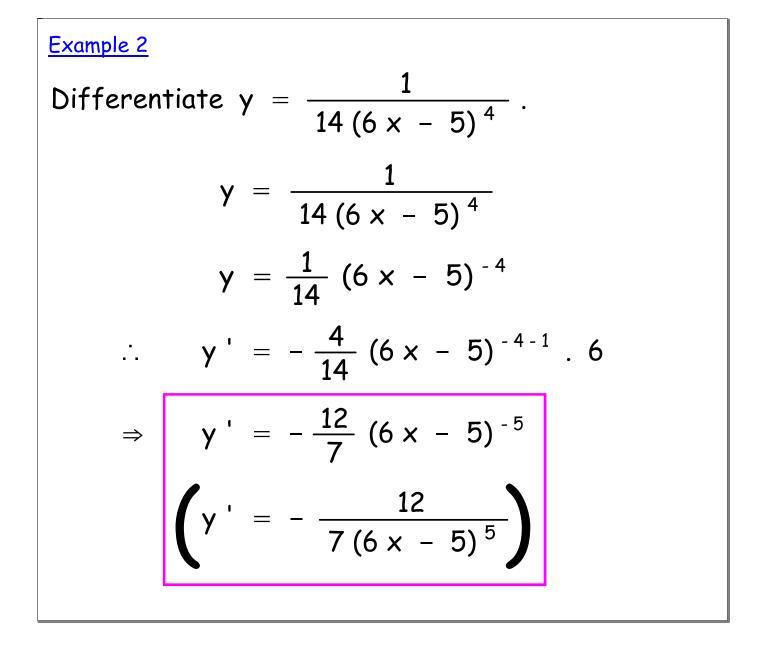
If 
$$y = f(g(x))$$
, then,

(y dashed equals f dashed of g(x) multiplied by g dashed of x)

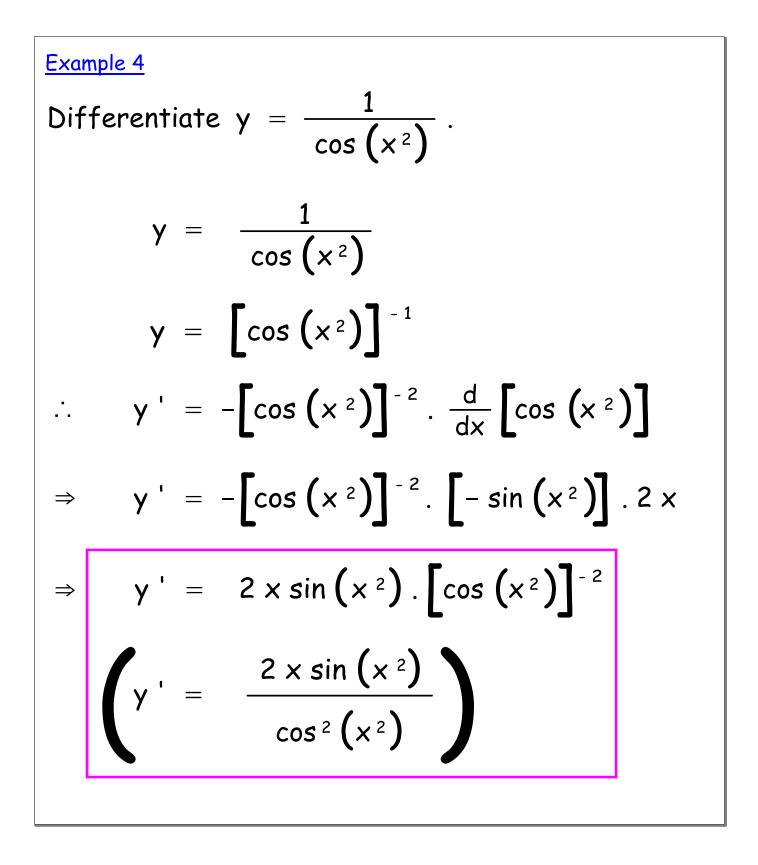
Leibniz Form

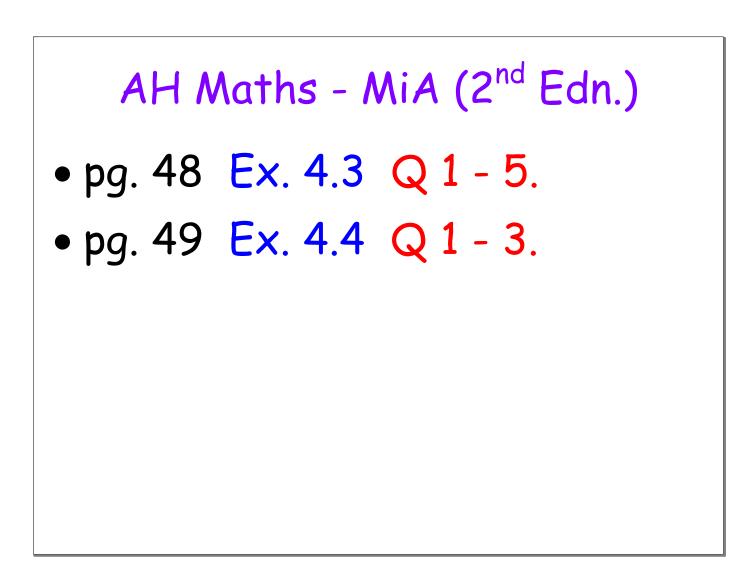
If y = f(g(x)), then letting u = g(x), we have that y = f(u) and u = g(x) (i. e., y is a function of u and u is a function of x); then,  $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ 

Example 1  
Differentiate 
$$y = (x^{3} - 6x + 2)^{9}$$
.  
Lagrange Style  
 $y = (x^{3} - 6x + 2)^{9}$   
 $\therefore y' = 9(x^{3} - 6x + 2)^{8} \cdot (3x^{2} - 6)$   
 $\Rightarrow y' = 9(3x^{2} - 6)(x^{3} - 6x + 2)^{8}$   
 $(y' = 27(x^{2} - 2)(x^{3} - 6x + 2)^{8})$   
Leibniz Style  
Let  $u = x^{3} - 6x + 2$ . Then,  
 $y = u^{9}$ ,  $u = x^{3} - 6x + 2$   
 $\therefore \frac{dy}{du} = 9u^{8}$ ,  $\frac{du}{dx} = 3x^{2} - 6$   
 $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$   
 $\therefore \frac{dy}{dx} = 9u^{8} \times (3x^{2} - 6)$   
 $\Rightarrow \frac{dy}{dx} = 9(3x^{2} - 6)(x^{3} - 6x + 2)^{8}$ 



## Example 3 Differentiate $y = \sin^5 2x$ . $y = \sin^5 2x$ $y = (\sin 2x)^5$ $\therefore \quad y' = 5 (\sin 2x)^{5-1} \cdot \frac{d}{dx} (\sin 2x)$ $\Rightarrow \quad y' = 5 (\sin 2x)^4 \cdot 2 \cos 2x$ $\Rightarrow \quad y' = 10 \cos 2x \sin^4 2x$





Ex. 4.31Find the derivative of each of these expressions.  
a 
$$(3x + 4)^6$$
 b  $3(2x - 5)^4$  c  $(3x^2 + 2x - 1)^5$  d  $\sin(x^3)$ 2Find  $f'(x)$  given  
a  $f(x) = \cos 7x$  b  $f(x) = (2x^3 + 4x^2 - 1)^4$  c  $f(x) = \sin(2x^2 - 5x)$ 3Find  $\frac{dy}{dx}$  given that  
a  $y = \frac{1}{3x + 1}$  b  $y = \frac{1}{(3x + 1)^2}$  c  $y = \frac{3}{(3x + 2)^3}$  d  $y = \frac{1}{\sin x}$ 4Use the fact that  $x^\circ = \frac{\pi}{180}x$  radians to help you differentiate these expressions.  
a  $\sin x^\circ$  b  $\cos x^\circ$  c  $\sin(2x + 30)^\circ$ 5Differentiate these expressions.  
a  $\sin(\cos x)$  b  $\cos(\cos x)$  c  $\sin(\sin x)$  d  $\cos(\sin x)$ **Ex. 4.4**1Find  $\frac{dy}{dx}$  for each of these.  
a  $y = \sin^2 3x$  b  $y = \cos^2(\sin x)$  c  $y = (x + \sin 3x)^2$  d  $y = \cos(\sin^2 x)$ 2Differentiate  
a  $\cos^3(2x + 4)$  b  $\frac{1}{\sin^2(3x + 1)}$  c  $\cos\left(\frac{1}{x^2 + 2x + 1}\right)$ 3Find the derivative of  
a  $\frac{1}{\cos(x^2 + x)}$  b  $\frac{1}{\sin(\cos x)}$  c  $\frac{1}{\sqrt{\sin(3x + 2)}}$ 

Answers to AH Maths (MiA), pg. 48, Ex. 4.3 1 a  $18(3x + 4)^5$  b  $24(2x - 5)^3$ c  $5(6x + 2)(3x^2 + 2x - 1)^4$  d  $3x^2 \cos (x^3)$ e  $3 \cos x \sin^2 x$ 2 a  $-7 \sin 7x$ b  $4(6x^2 + 8x)(2x^3 + 4x^2 - 1)^3$ c  $(4x - 5) \cos (2x^2 - 5x)$ 3 a  $-\frac{3}{(3x + 1)^2}$  b  $-\frac{6}{(3x + 1)^3}$  c  $-\frac{27}{(3x + 2)^4}$ d  $-\frac{\cos x}{\sin^2 x}$  e  $\frac{\sin x}{\cos^2 x}$ 4 a  $\frac{\pi}{180} \cos x^\circ$  b  $-\frac{\pi}{180} \sin x^\circ$ c  $\frac{\pi}{90} \cos (2x + 30)^\circ$ 5 a  $-\sin x \cos (\cos x)$  b  $\sin x \sin (\cos x)$ c  $\cos x \cos (\sin x)$  d  $-\cos x \sin (\sin x)$ 

## Answers to AH Maths (MiA), pg. 49, Ex. 4.4

1 a  $6 \sin 3x \cos 3x$ b  $-2 \cos (\sin x) \cdot \sin (\sin x) \cdot \cos x$ c  $2(x + \sin 3x)(1 + 3 \cos 3x)$ d  $-2 \sin (\sin^2 x) \sin x \cos x$ 2 a  $-6 \cos^2 (2x + 4) \sin(2x + 4)$ b  $-\frac{6 \cos (3x + 1)}{\sin^3 (3x + 1)}$ c  $\sin \left(\frac{1}{x^2 + 2x + 1}\right) \frac{2x + 2}{(x^2 + 2x + 1)^2}$ 3 a  $\frac{(2x + 1) \sin (x^2 + x)}{\cos^2 (x^2 + x)}$  b  $\frac{\sin x \cos (\cos x)}{\sin^2 (\cos x)}$ c  $-\frac{3 \cos (3x + 2)}{2 (\sin (3x + 2))^{\frac{3}{2}}}$