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Unit 1 : Differential Calculus - Lesson 1

Chain Rule Revision

LI

- Differentiating a composition of two or more functions.

SC

- Chain Rule in different forms.

The Chain Rule

The Chain Rule tells us how to differentiate a composition of functions

Lagrange Form

If $y = f(g(x))$, then,

$$y' = f'(g(x)) \times g'(x)$$

(y dashed equals f dashed of g (x) multiplied by g dashed of x)

Leibniz Form

If $y = f(g(x))$, then letting $u = g(x)$, we have that $y = f(u)$ and $u = g(x)$ (i. e., y is a function of u and u is a function of x); then,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Example 1

Differentiate $y = (x^3 - 6x + 2)^9$.

Lagrange Style

$$y = (x^3 - 6x + 2)^9$$

$$\therefore \quad y' = 9(x^3 - 6x + 2)^8 . (3x^2 - 6)$$

$$\Rightarrow \quad \boxed{\begin{array}{l} y' = 9(3x^2 - 6)(x^3 - 6x + 2)^8 \\ \left(y' = 27(x^2 - 2)(x^3 - 6x + 2)^8\right) \end{array}}$$

Leibniz Style

Let $u = x^3 - 6x + 2$. Then,

$$y = u^9 \quad , \quad u = x^3 - 6x + 2$$

$$\therefore \quad \frac{dy}{du} = 9u^8 \quad , \quad \frac{du}{dx} = 3x^2 - 6$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\therefore \quad \frac{dy}{dx} = 9u^8 \times (3x^2 - 6)$$

$$\Rightarrow \quad \boxed{\frac{dy}{dx} = 9(3x^2 - 6)(x^3 - 6x + 2)^8}$$

Example 2

Differentiate $y = \dfrac{1}{14(6x - 5)^4}$.

$$y = \frac{1}{14(6x - 5)^4}$$

$$y = \frac{1}{14}(6x - 5)^{-4}$$

$$\therefore \quad y' = -\frac{4}{14}(6x - 5)^{-4-1} . 6$$

$$\Rightarrow \quad \boxed{\begin{array}{l} y' = -\dfrac{12}{7}(6x - 5)^{-5} \\ \left(y' = -\dfrac{12}{7(6x - 5)^5} \right) \end{array}}$$

Example 3

Differentiate $y = \sin^5 2x$.

$$y = \sin^5 2x$$

$$y = (\sin 2x)^5$$

$$\therefore \quad y' = 5(\sin 2x)^{5-1} \cdot \frac{d}{dx}(\sin 2x)$$

$$\Rightarrow \quad y' = 5(\sin 2x)^4 \cdot 2\cos 2x$$

$$\Rightarrow \quad \boxed{y' = 10\cos 2x \sin^4 2x}$$

Example 4

Differentiate $y = \dfrac{1}{\cos(x^2)}$.

$$y = \frac{1}{\cos(x^2)}$$

$$y = \left[\cos(x^2)\right]^{-1}$$

$$\therefore \quad y' = -\left[\cos(x^2)\right]^{-2} . \frac{d}{dx}\left[\cos(x^2)\right]$$

$$\Rightarrow \quad y' = -\left[\cos(x^2)\right]^{-2} . \left[-\sin(x^2)\right] . 2x$$

$$\Rightarrow \quad \boxed{y' = 2x\sin(x^2) . \left[\cos(x^2)\right]^{-2} \quad \left(y' = \frac{2x\sin(x^2)}{\cos^2(x^2)}\right)}$$

AH Maths - MiA (2^{nd} Edn.)

- pg. 48 Ex. 4.3 Q 1 - 5.
- pg. 49 Ex. 4.4 Q 1 - 3.

Ex. 4.3

1 Find the derivative of each of these expressions.

a $(3x + 4)^6$ b $3(2x - 5)^4$ c $(3x^2 + 2x - 1)^5$ d $\sin(x^3)$

2 Find $f'(x)$ given

a $f(x) = \cos 7x$ b $f(x) = (2x^3 + 4x^2 - 1)^4$ c $f(x) = \sin(2x^2 - 5x)$

3 Find $\frac{dy}{dx}$ given that

a $y = \frac{1}{3x + 1}$ b $y = \frac{1}{(3x + 1)^2}$ c $y = \frac{3}{(3x + 2)^3}$ d $y = \frac{1}{\sin x}$

4 Use the fact that $x° = \frac{\pi}{180}x$ radians to help you differentiate these expressions.

a $\sin x°$ b $\cos x°$ c $\sin(2x + 30)°$

5 Differentiate these expressions.

a $\sin(\cos x)$ b $\cos(\cos x)$ c $\sin(\sin x)$ d $\cos(\sin x)$

Ex. 4.4

1 Find $\frac{dy}{dx}$ for each of these.

a $y = \sin^2 3x$ b $y = \cos^2(\sin x)$ c $y = (x + \sin 3x)^2$ d $y = \cos(\sin^2 x)$

2 Differentiate

a $\cos^3(2x + 4)$ b $\frac{1}{\sin^2(3x + 1)}$ c $\cos\left(\frac{1}{x^2 + 2x + 1}\right)$

3 Find the derivative of

a $\frac{1}{\cos(x^2 + x)}$ b $\frac{1}{\sin(\cos x)}$ c $\frac{1}{\sqrt{\sin(3x + 2)}}$

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Answers to AH Maths (MiA), pg. 48, Ex. 4.3

1 **a** $18(3x+4)^5$ **b** $24(2x-5)^3$

c $5(6x+2)(3x^2+2x-1)^4$ **d** $3x^2\cos(x^3)$

e $3\cos x\sin^2 x$

2 **a** $-7\sin 7x$

b $4(6x^2+8x)(2x^3+4x^2-1)^3$

c $(4x-5)\cos(2x^2-5x)$

3 **a** $-\dfrac{3}{(3x+1)^2}$ **b** $-\dfrac{6}{(3x+1)^3}$ **c** $-\dfrac{27}{(3x+2)^4}$

d $-\dfrac{\cos x}{\sin^2 x}$ **e** $\dfrac{\sin x}{\cos^2 x}$

4 **a** $\frac{\pi}{180}\cos x°$ **b** $-\frac{\pi}{180}\sin x°$

c $\frac{\pi}{90}\cos(2x+30)°$

5 **a** $-\sin x\cos(\cos x)$ **b** $\sin x\sin(\cos x)$

c $\cos x\cos(\sin x)$ **d** $-\cos x\sin(\sin x)$

Answers to AH Maths (MiA), pg. 49, Ex. 4.4

1 **a** $6\sin 3x\cos 3x$

b $-2\cos(\sin x)\,.\,\sin(\sin x)\,.\,\cos x$

c $2(x+\sin 3x)(1+3\cos 3x)$

d $-2\sin(\sin^2 x)\sin x\cos x$

2 **a** $-6\cos^2(2x+4)\sin(2x+4)$

b $-\dfrac{6\cos(3x+1)}{\sin^3(3x+1)}$

c $\sin\left(\dfrac{1}{x^2+2x+1}\right)\dfrac{2x+2}{(x^2+2x+1)^2}$

3 **a** $\dfrac{(2x+1)\sin(x^2+x)}{\cos^2(x^2+x)}$ **b** $\dfrac{\sin x\cos(\cos x)}{\sin^2(\cos x)}$

c $-\dfrac{3\cos(3x+2)}{2(\sin(3x+2))^{\frac{3}{2}}}$