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Unit 2 : Sequences and Series - Lesson 1

Arithmetic Sequences

LI

- Know what an Arithmetic Sequence is.
- Find the n^{th} term formula for an arithmetic sequence.
- Solve problems involving arithmetic sequences.

SC

- Arithmetic of real numbers.

The first term of a sequence is usually denoted by the letter 'a'

An arithmetic sequence is a sequence where the differences between any two successive terms are constant :

$$u_{n+1} - u_n = d \quad (\text{for all } n \in \mathbb{N})$$

(d is called the common difference)

The n^{th} term of an arithmetic sequence is :

$$u_n = a + (n - 1)\,d$$

Example 1

Show that 3, 7, 10, . . . cannot be the first three terms of an arithmetic sequence.

$$7 - 3 = 4$$

$$10 - 7 = 3$$

As successive differences are not constant, these 3 numbers cannot form the start of an arithmetic sequence.

Example 2

An arithmetic sequence has first term 8 and common difference 4.

Find a formula for the n^{th} term.

$$u_n = a + (n - 1)d$$

$$\therefore \quad u_n = 8 + (n - 1)4$$

$$\Rightarrow \quad u_n = 8 + 4n - 4$$

$$\Rightarrow \quad \boxed{u_n = 4n + 4}$$

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Example 3

An arithmetic sequence has third term 10.5 and seventh term 30.5.

Find the n^{th} term formula and also the 20^{th} term.

$$u_n = a + (n-1)d$$

$$u_3 = 10.5 \qquad\qquad u_7 = 30.5$$

$$10.5 = a + (3-1)d \qquad\qquad 30.5 = a + (7-1)d$$

$$\Rightarrow \underline{10.5 = a + 2d} \qquad\qquad \Rightarrow \underline{30.5 = a + 6d}$$

$$a + 2d = 10.5$$

$$\underline{a + 6d = 30.5}$$

$$\therefore \quad 4d = 20$$

$$\Rightarrow \quad \underline{d = 5}$$

$$a + 2d = 10.5$$

$$\Rightarrow \quad a = 10.5 - 2(5)$$

$$\Rightarrow \quad \underline{a = \frac{1}{2}}$$

$$u_n = a + (n-1)d$$

$$\therefore \quad u_n = \frac{1}{2} + (n-1)5$$

$$\Rightarrow \quad \boxed{u_n = 5n - \frac{9}{2}}$$

$$u_{20} = 5(20) - \frac{9}{2}$$

$$\Rightarrow \quad \boxed{u_{20} = 95.5}$$

Example 4

Show that ln 3, ln 4, ln (16/3), . . . could be the first three terms of an arithmetic sequence.

Hence show that $u_n = Pn + Q$, stating the values of the constants P and Q.

$$\ln 4 - \ln 3 = \ln(4/3)$$

$$\ln(16/3) - \ln 4 = \ln(16/(3 \times 4)) = \ln(4/3)$$

As successive differences are constant ($d = \ln(4/3)$), these 3 numbers could be the start of an arithmetic sequence.

$$u_n = a + (n - 1)d$$

$$\therefore \quad u_n = \ln 3 + (n - 1)\ln(4/3)$$

$$\Rightarrow \quad u_n = \ln 3 + n\ln(4/3) - \ln(4/3)$$

$$\Rightarrow \quad u_n = n\ln(4/3) + \ln(3/(4/3))$$

$$\Rightarrow \quad \boxed{\begin{array}{l} u_n = \ln(4/3)\,.\,n + \ln(9/4) \\ (P = \ln(4/3),\ Q = \ln(9/4)) \end{array}}$$

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AH Maths - MiA (2^{nd} Edn.)

- pg. 151-2 Ex. 9.1 Q 1 - 8, 10.

Ex. 9.1

1 Identify a and d in each of these arithmetic sequences.

a $5, 8, 11, \ldots$ b $3, 4, 5, \ldots$ c $0, -3, -6, \ldots$

d $-1, 4, 9, \ldots$ e $-3, -7, -11, \ldots$ f $0, 7, 14, \ldots$

g $3, 2.8, 2.6, \ldots$ h $\frac{1}{15}, \frac{1}{5}, \frac{1}{3}, \ldots$ i $\frac{1}{4}, \frac{1}{7}, \frac{1}{28}, \ldots$

2 Find the nth term for each of these arithmetic sequences.

a $2, 3, 4, \ldots$ b $16, 12, 8, \ldots$ c $5, 3, 1, \ldots$

d $-8, -5, -2, \ldots$ e $14, 10, 6, \ldots$ f $-4, -9, -14, \ldots$

g $-0.5, -1.1, -1.7, \ldots$ h $\frac{3}{20}, \frac{1}{5}, \frac{1}{4}, \ldots$ i $\frac{1}{36}, -\frac{1}{9}, -\frac{1}{4}, \ldots$

3 a Find the value of n when $a = 2, d = 3$ and $u_n = 14$.

b Which term in the sequence $12, 7, 2, \ldots$ is -18?

c If, in an arithmetic sequence $u_1 = -2$ and $u_2 = -6$, which term has the value -46?

4 a Find the value of d when $a = 7$ and $u_{26} = 107$.

b An arithmetic sequence starts with 10. Its 50th term is -382.
What is the common difference?

c Find terms u_2 to u_5 of the arithmetic sequence when the first term is 3 and the 15th term is -4.

5 a For a particular sequence, $u_{51} = 98$. If $d = 2$, find a.

b In an arithmetic sequence the 13th term is 65 and the common difference is 4. Find the first term.

c A man starts with a pile of pebbles. He repeatedly removes three pebbles.
What was the original size of the pile if the 18th term of the sequence he creates is 9?
[Treat the original pile as the first term.]

6 a Identify the arithmetic sequence in each case by listing the first four terms.

i $u_6 = 22$ and $u_8 = 30$ ii $u_5 = -3$ and $u_{11} = -15$ iii $u_{11} = 10$ and $u_{21} = 12$

b The 16th term of an arithmetic sequence is -38. The 25th term is -65.

i Identify the sequence.

ii What is the largest number smaller than -300 that is a term in the sequence?

7 An arithmetic sequence is defined by $u_n = 3n + 9$.

a Find analytically the value of x such that $3u_x = u_{4x}$.

b If a sequence is defined by $u_n = mn + c$, where m and c are integer constants, show that if $3u_x = u_{4x}$ then m is a factor of $2c$.

8 a Show that ln 5, ln 10, ln 20 could be the first three terms of an arithmetic sequence and state the value of d, the common difference.

b Find u_n, expressing your answer in the form $u_n = \ln(A)$.

c For what *value* of x does u_x first exceed 50?

10 In an athletics stadium the seats surround the field, rising in terraces.
The number of seats in the rows form an arithmetic sequence.
There are 880 seats in row 6 and 1200 seats in row 26.

a How many seats are in i the front row ii the 10th row?

b A neighbouring stadium has 500 seats in the front row and 800 seats in row 10. Show that the number of seats in the rows does not form an arithmetic sequence.

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Answers to AH Maths (MiA), pg. 151-2, Ex. 9.1

1 **a** $5, 3$ **b** $3, 1$ **c** $0, -3$
d $-1, 5$ **e** $-3, -4$ **f** $0, 7$
g $3, -0{\cdot}2$ **h** $\frac{1}{15}, \frac{2}{15}$ **i** $\frac{1}{4}, -\frac{3}{28}$

2 **a** $n + 1$ **b** $-4n + 20$ **c** $-2n + 7$
d $3n - 11$ **e** $-4n + 18$ **f** $-5n + 1$
g $-0{\cdot}6n + 0{\cdot}1$ **h** $\frac{n}{20} + \frac{1}{10}$
i $-\frac{5}{36}n + \frac{1}{6}$

3 **a** 5 **b** 7 **c** 12

4 **a** 4 **b** -8 **c** $2{\cdot}5, 2, 1{\cdot}5, 1$

5 **a** -2 **b** 17 **c** 60

6 **a** **i** $2, 6, 10, 14$ **ii** $5, 3, 1, -1$ **iii** $8, 8{\cdot}2, 8{\cdot}4, 8{\cdot}6$
b **i** $7, 4, 1, -2, \ldots\ (a = 7, d = -3)$ **ii** -302

7 **a** $x = 6$ **b** $x = \dfrac{2c}{m} \in N$

8 **a** $d = \ln 10 - \ln 5 = \ln 20 - \ln 10 = \ln 2$
b $u_n = \ln(5 \times 2^{n-1})$ **c** 71

10 **a** **i** 800 **ii** 944
b Proof by contradiction.
Assume arithmetic sequence: $u_{10} = u_1 + (10 - 1)d$
$\Rightarrow d = \dfrac{u_{10} - u_1}{9}, d \in N$
$\dfrac{800 - 500}{9} \notin N$ Hence result.