Solving Trigonometric Equations - Lesson 2

Solving Trigonometric Equations
(No Rearranging : Mixed Domains)

LI

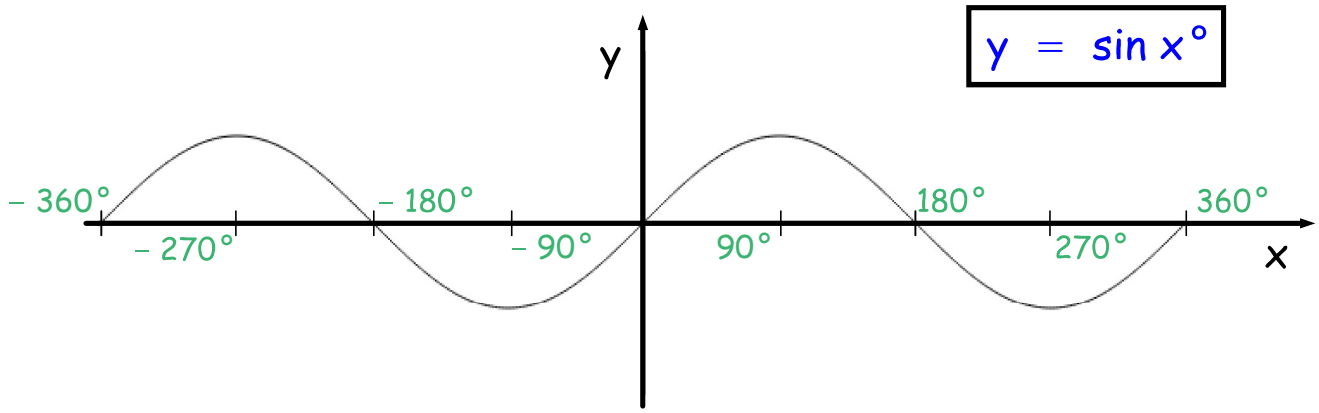
- Solve trigonometric equations of the form :

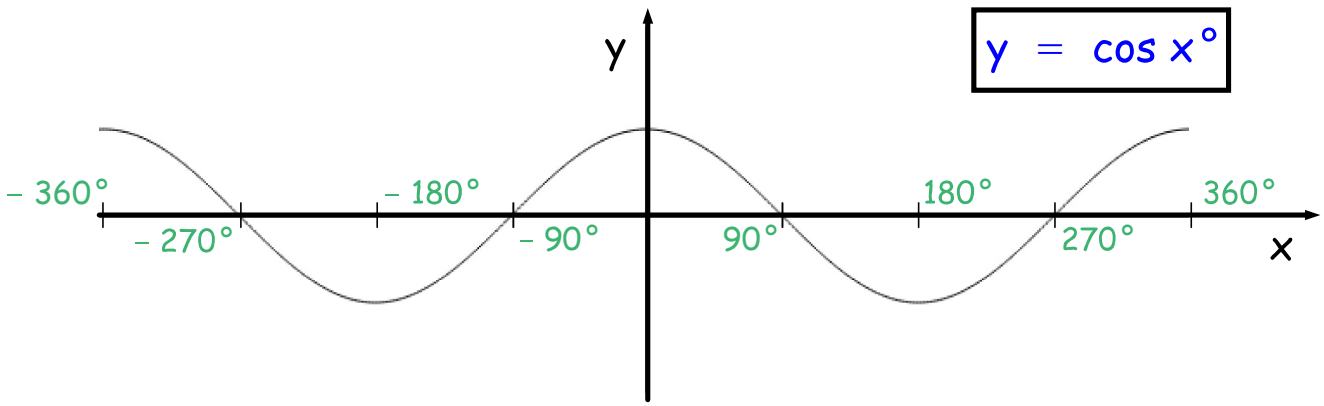
$$\sin x^\circ = a$$
$$\cos x^\circ = a \qquad \text{not necessarily between } 0^\circ \text{ and } 360^\circ$$
$$\tan x^\circ = a$$

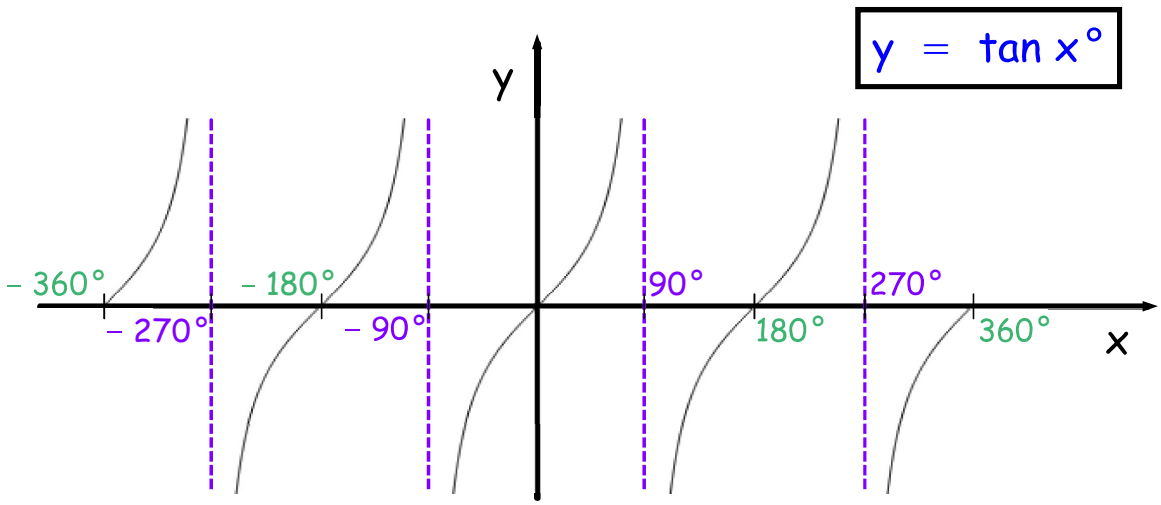
SC

- Know extended graphs of $y = \sin x^\circ$, $y = \cos x^\circ$ and $y = \tan x^\circ$.
- Use the ASTC Diagram .

Trigonometric Graphs Between – 360° and 360°



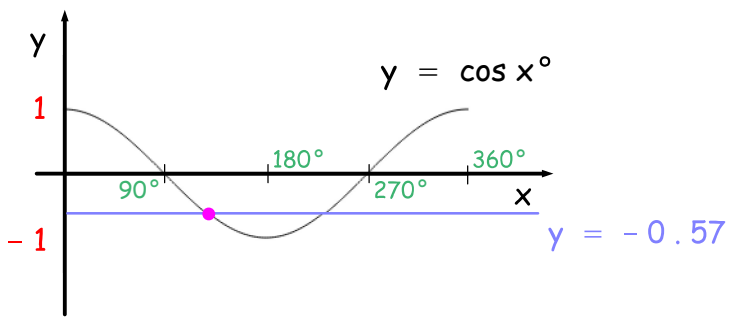




Example 1

Solve $\overset{\text{main equation}}{\cos x^\circ = -0.57}$ for $0^\circ \le x^\circ \le 180^\circ$ (to the nearest degree).

STEP 1



(Expecting 1 answer)

STEP 2

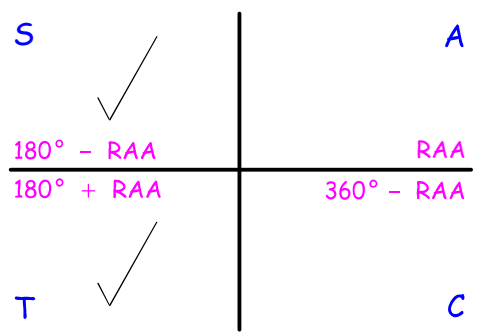
$$RAA = \cos^{-1}(0.57)$$

$$\Rightarrow \underline{RAA = 55.2\ldots^\circ}$$

STEP 3

$$\overset{\text{main equation}}{\cos x^\circ = -0.57} \qquad \boxed{\cos \text{ is } -\text{ve}}$$

STEP 4



$$x^\circ = 180^\circ - RAA,\ 180^\circ + RAA$$

$$\therefore \quad x^\circ = 180^\circ - 55.2^\circ,\ 180^\circ + 55.2^\circ$$

$$\Rightarrow \quad x^\circ = 124.8^\circ, \boxed{235.2^\circ}$$

out of range

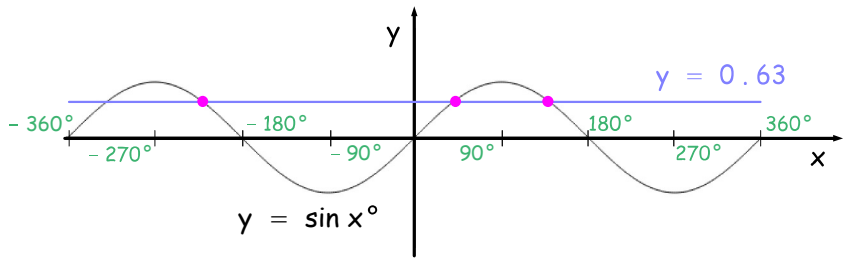
$$\therefore \quad \boxed{x^\circ = 125^\circ}$$

Example 2

Solve $\sin x° = 0.63$ for $270° \le x° \le 360°$ (1 d.p.).

main equation

STEP 1



(Expecting 3 answers)

STEP 2

$$RAA = \sin^{-1}(0.63)$$

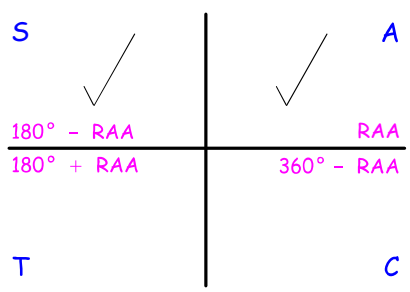
$$\Rightarrow \quad \underline{RAA = 39.05\ldots°}$$

STEP 3

main equation

$$\sin x° = 0.63 \qquad \boxed{\text{sin is } + \text{ve}}$$

STEP 4



$$x° = RAA,\ 180° - RAA$$

$$\therefore \quad x° = 39.05\ldots°,\ 180° - 39.05\ldots°$$

$$\Rightarrow \quad \underline{x° = 39.1°,\ 140.9°}$$

This only gives 2 answers; how do we get the third one ?

The sine graph repeats every 360°, so subtract 360° from the second answer above (see graph).

$$x° = 140.9° - 360°$$

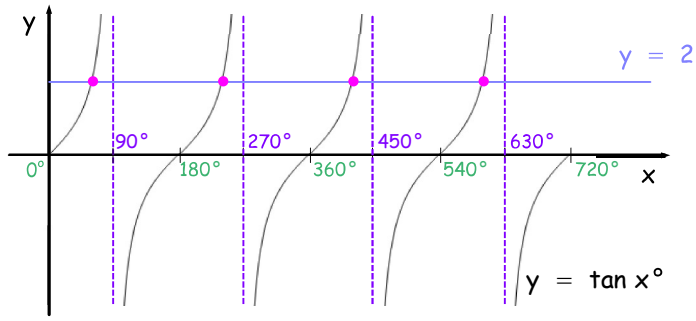
$$\Rightarrow \quad \underline{x° = -219.1°}$$

$$\therefore \quad \boxed{x° = -219.1°,\ 39.1°,\ 140.9°}$$

Example 3

Solve $\overset{\text{main equation}}{\tan x^\circ = 2}$ for $0^\circ \le x^\circ \le 720^\circ$ (nearest degree).

STEP 1



(Expecting 4 answers)

STEP 2

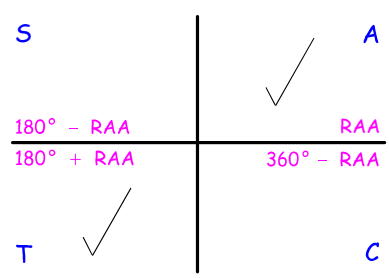
$$RAA = \tan^{-1}(2)$$

$$\Rightarrow \quad \underline{RAA = 63.43\ldots^\circ}$$

STEP 3

$$\overset{\text{main equation}}{\tan x^\circ = 2} \qquad \boxed{\text{tan is } + \text{ve}}$$

STEP 4



$$x^\circ = RAA,\ 180^\circ + RAA$$

$$\therefore \quad x^\circ = 63^\circ,\ 180^\circ + 63^\circ$$

$$\Rightarrow \quad \underline{x^\circ = 63^\circ,\ 243^\circ}$$

This only gives 2 answers; how do we get the other two ?

The tangent graph repeats every 180°, so keep adding 180° to the bigger answer above (look at the graph).

$$x^\circ = 243^\circ + 180^\circ,\ 243^\circ + 180^\circ + 180^\circ$$

$$\Rightarrow \quad \underline{x^\circ = 423^\circ,\ 603^\circ}$$

$$\therefore \quad \boxed{x^\circ = 63^\circ,\ 243^\circ,\ 423^\circ,\ 603^\circ}$$

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Questions

4 Solve the following, giving your answers to the nearest degree.

a $\cos x^\circ = 0.8,\ 0 \leqslant x \leqslant 180$

b $\sin x^\circ = 0.62,\ 0 \leqslant x \leqslant 720$

c $\tan x^\circ = 4,\ 180 \leqslant x \leqslant 360$

d $\sin x^\circ = -0.72,\ 0 \leqslant x \leqslant 720$

e $\cos x^\circ = -0.55,\ 0 \leqslant x \leqslant 180$

f $\tan x^\circ = -6.5,\ 180 \leqslant x \leqslant 540$

Answers

4 **a** $x = 37°$

b $x = 38°$, $x = 142°$, $x = 398°$, $x = 502°$

c $x = 256°$

d $x = 226°$, $x = 314°$, $x = 586°$, $x = 674°$

e $x = 123°$

f $x = 279°, 459°$