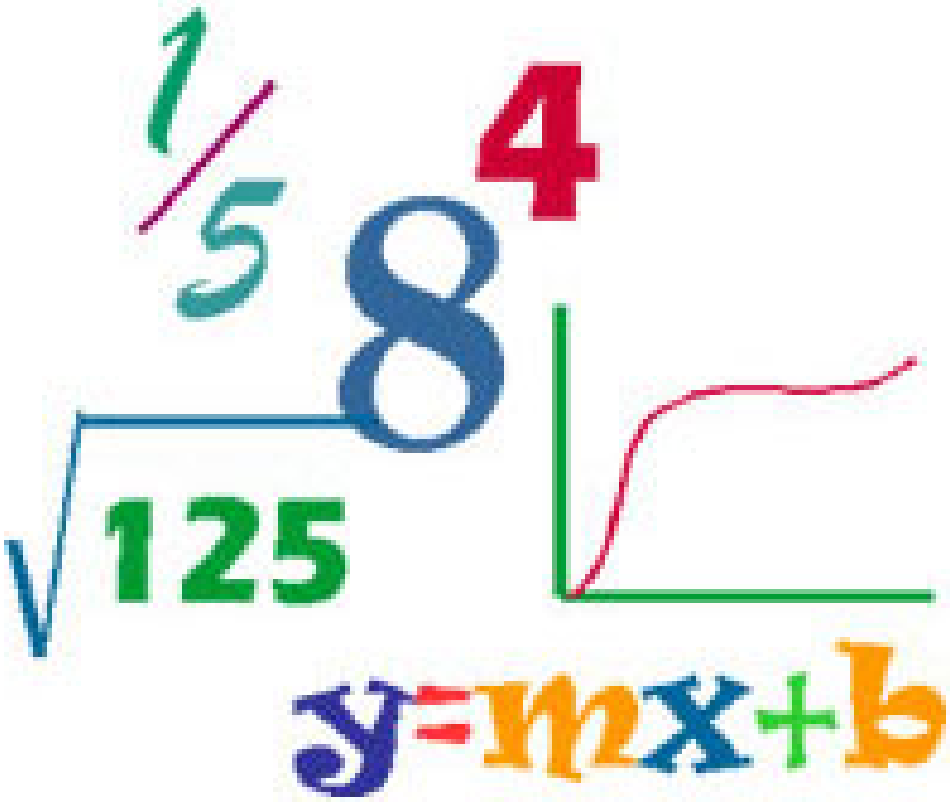
Linwood High School

S3 CREDIT NOTES



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CHAPTER 1: CALCULATIONS AND THE CALCULATOR

CALCULATORS AND ROUNDING

The number of significant figures indicates the accuracy of a **measurement**.

For example, 3400 centimetres = 34 metres = 0·034 kilometres
same measurement, same accuracy, each 2 significant figures.

significant figures: count the number of figures used, but
do **not** count **zeros** at the **end** of a number **without** a decimal point
do **not** count **zeros** at the **start** of a number **with** a decimal point.

These zeros simply give the place-value[1] of the figures and do not indicate accuracy.

rounding:

For example,

5713·4	has 5 significant figures
5700	to 2 significant figures (this case, the nearest Hundred)
0·057134	has 5 significant figures
0·057	to 2 significant figures (this case, the nearest Thousandth)

(note 0·057000 would be wrong)

Example involving the use of a calculator:

Evaluate $\dfrac{3\cdot 24\times 8\cdot 47-25\cdot 64}{2\cdot 4+0\cdot 84}$, giving your answer correct to **2 significant figures**.

$= \dfrac{1\cdot 8028}{3\cdot 24}$ ***evaluate*** *'top' and 'bottom' by entering in the calculator and write these answers*

$= 0\cdot 5564...$ *divide 'top' by 'bottom' and write the unrounded answer*

$\approx 0\cdot 56$ *write the rounded answer*

[1]place-value meaning **H**undreds, **T**ens , **U**nits, **t**enths, **h**undredths etc.

CALCULATORS AND SCIENTIFIC NOTATION
(STANDARD FORM)

Writing numbers in the form $a \times 10^n$

where $1 \le a < 10$ and n is an integer — for a, place the decimal point after the first non-zero digit numbers ...-3, -2, -1, 0, 1, 2, 3....

For example,

$$\mathbf{32800} = 3 \cdot 28 \times 10 \times 10 \times 10 \times 10 = \mathbf{3 \cdot 28 \times 10^4}$$

$$\mathbf{0 \cdot 000328} = 3 \cdot 28 \div 10 \div 10 \div 10 \div 10 = 3 \cdot 28 \div 10^4 = \mathbf{3 \cdot 28 \times 10^{-4}}$$

Notice for numbers starting $0\cdot$ the power of 10 is negative (same as $\div 10$).

Examples involving the use of a calculator:

(1) One **milligram** of hydrogen gas contans $2 \cdot 987 \times 10^{20}$ molecules.
Calculate the number of molecules in 5 **grams** of hydrogen gas,
giving you answer correct to 3 significant figures.

$5000 \times 2 \cdot 987 \times 10^{20}$ — *learn to enter standard form in the calculator using the appropriate button* EE *or* EXP *or* x10^n *eg. 2·987* EXP *20*

$= 1 \cdot 4935 \times 10^{24}$

$\approx 1 \cdot 49 \times 10^{24}$ molecules

(2) The total mass of argon in a flask is $4 \cdot 15 \times 10^{-2}$ grams.
The mass of a single atom of argon is $6 \cdot 63 \times 10^{-23}$ grams.
Find, correct to 3 significant figures, the number of argon atoms in the flask.

$\dfrac{4 \cdot 15 \times 10^{-2}}{6 \cdot 63 \times 10^{-23}}$ — *use the* (-) *button for a minus* *eg. 4·15* EXP (-) *2*

$= 6 \cdot 259... \times 10^{20}$ — *divide 'top' by 'bottom' and write the unrounded answer*

$\approx 6 \cdot 26 \times 10^{20}$ atoms — *write the rounded answer*

FRACTIONS No calculators!

Addition and subtraction requires a common denominator:

Example:

$$11\frac{5}{6} - 3\frac{2}{9}$$

least common multiple *of 6 and 9 is 18 for the common denominator*

$$\frac{5 \times 3}{6 \times 3} = \frac{15}{18} \qquad \frac{2 \times 2}{9 \times 2} = \frac{4}{18}$$

both fractions now 18ths

$$= \mathbf{11}\frac{15}{18} - \mathbf{3}\frac{4}{18}$$

*subtract whole numbers **11 - 3 = 8***

subtract fractions by subtracting the 'top' 15 - 4 = 11
while the 'bottom' stays the same at 18

$$= \mathbf{8}\frac{11}{18}$$

Multiplication and division requires **no** mixed numbers:

Examples:

(1)

$$\frac{3}{10} \text{ of } 2\frac{3}{4}$$

of** means **multiply

***no** mixed numbers, go 'top-heavy'*

$$\mathbf{2}\frac{3}{4} = \mathbf{2} + \frac{3}{4} = \frac{\mathbf{8}}{\mathbf{4}} + \frac{3}{4} = \frac{11}{4}$$

*or for the 'top' 4 x **2** +3= 11*
'bottom' stays as 4

$$= \frac{3}{10} \times \frac{11}{4}$$

multiply the 'top' numbers 3 x 11 = 33
multiply the 'bottom' numbers 10 x 4 = 40

$$= \frac{33}{40}$$

(2)

$$\frac{4}{5} \div \frac{7}{8}$$

$$= \frac{4}{5} \times \frac{8}{7}$$

*division becomes multiply by the **reciprocal***

$$\div \frac{a}{b} \text{ becomes } \times \frac{b}{a}$$

$$= \frac{32}{35}$$

ORDER OF CALCULATION

Rule: **x and ÷ before + or -**

to change the order use brackets **BRACKETS FIRST**

Carry out separate calculations:

Examples:

(1) $\frac{5}{8} - \frac{2}{9} \times \frac{3}{4}$

$\frac{2}{9} \times \frac{3}{4}$ *multiply first*

$= \frac{6}{36}$ *fully simplify*

$= \frac{1}{6}$

$\frac{5}{8} - \frac{1}{6}$ *subtraction last*

$= \frac{15}{24} - \frac{4}{24}$ *common denominator*

$= \frac{11}{24}$

(2) $\left(\frac{5}{6} - \frac{1}{2}\right) \div 1\frac{3}{4}$

$\frac{5}{6} - \frac{1}{2}$ *brackets first*

$= \frac{5}{6} - \frac{3}{6}$

$= \frac{2}{6}$ *fully simplify*

$= \frac{1}{3}$

$\frac{1}{3} \div 1\frac{3}{4}$

$= \frac{1}{3} \div \frac{7}{4}$ *'top-heavy' first*

$= \frac{1}{3} \times \frac{4}{7}$ *multiply by reciprocal*

$= \frac{4}{21}$

CHAPTER 2: SIMILAR SHAPES

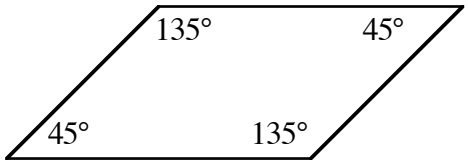
Shapes are similar if they are enlargement or reductions of each other.

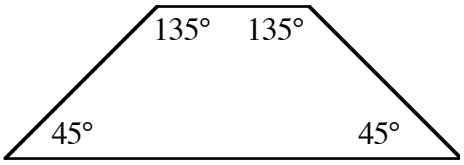
(1) the angles remain unchanged - the shapes are **equiangular**.

and (2) the sides are enlarged or reduced by some **scale factor** (SF).

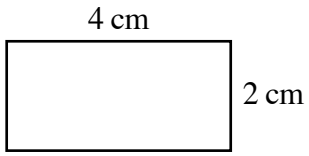
For example:

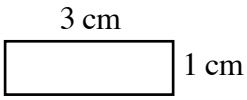
corresponding angles not equal (wrong order): shapes not equiangular so not similar.





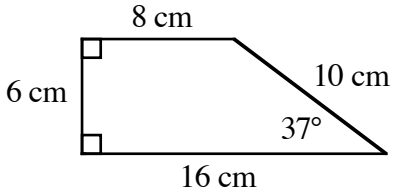
ratios of corresponding sides not equal (no SF): shapes equiangular but not similar.

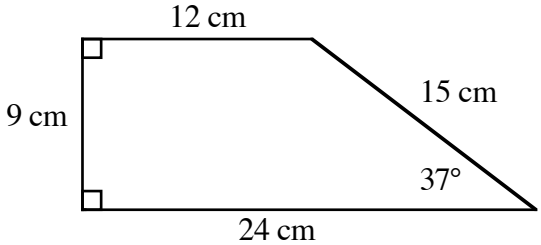




$$\frac{4}{3} \neq \frac{2}{1}$$

shapes equiangular **and** ratios of corresponding sides equal (has SF): shapes are similar.





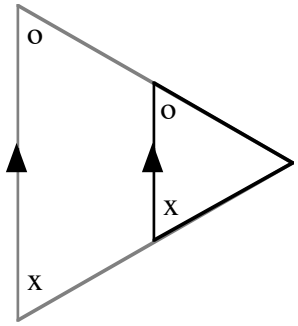
$$\frac{9}{6} = \frac{12}{8} = \frac{15}{10} = \frac{24}{16}\ ,\ \text{scale factor} = \frac{3}{2}$$

Triangles are special:

Enlarge or reduce a triangle by some scale factor and the two triangles will be equiangular.

Rule:

If triangles are equiangular then they are similar and so the sides have been scaled.

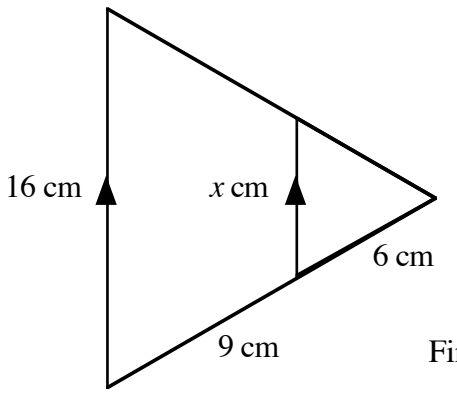


SCALING SIDES

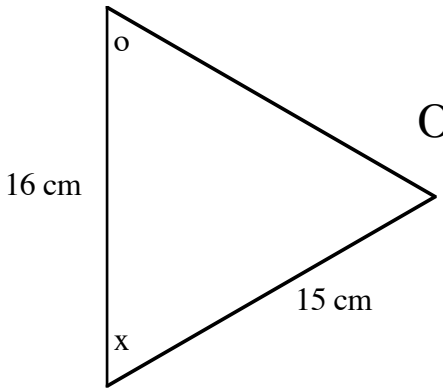
length scale factor, $SF = \dfrac{\text{image side}}{\text{original side}}$

enlargement if $SF > 1$
reduction if $0 < SF < 1$

Example:



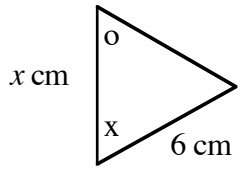
Find the value of x.



Original

shape with the unknown side

Image



triangles are equiangular and so similar

$$SF = \frac{image}{original} = \frac{6}{15} = \frac{2}{5}$$

0 < SF < 1 as expected for a reduction

$$x = \frac{2}{5} \times 16 = 6 \cdot 4$$

smaller than 16 as expected for a reduction

SCALING AREAS

To enlarge a 2D shape both dimensions, length and breadth, must be scaled.
For example,

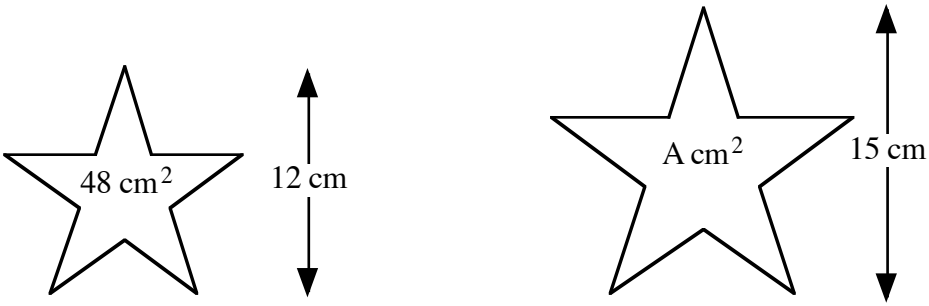
original

sides doubled
image contains 4 of
the original rectangles

sides tripled
image contains 9 of
the original rectangles

Rule: if the **length** of the sides are made **n** times bigger, **length SF = n**
the **area** of the enlarged shape is **n x n** bigger. **area SF = n^2**

Example:



Given that the two shapes shown are similar, find the area of the larger shape.

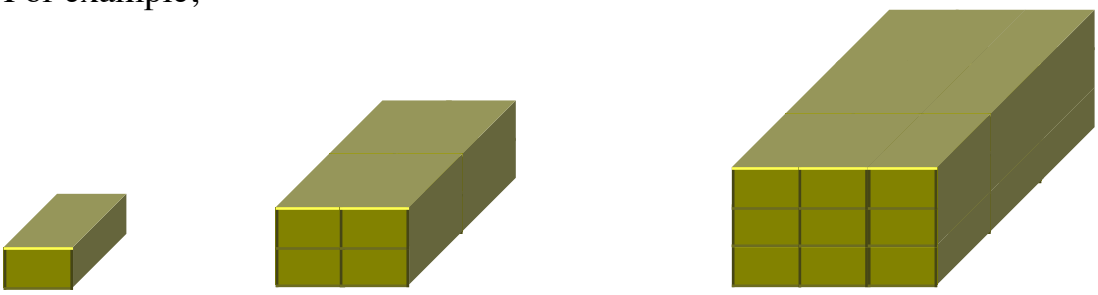
$$\textit{length SF} = \frac{\textit{image}}{\textit{original}} = \frac{15}{12} = \frac{5}{4}$$ *SF > 1 as expected for an enlargement*

$$\textit{area SF} = \frac{5}{4} \times \frac{5}{4} = \frac{25}{16}$$

$$A = \frac{25}{16} \times 48 = 75$$ *bigger than 48 as expected for an enlargement*

SCALING VOLUMES

To enlarge a 3D solid the dimensions length, breadth and height, must be scaled.
For example,



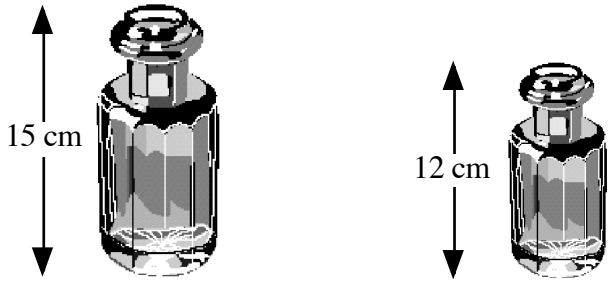
original

sides doubled
image contains 8 of the original cuboids

sides tripled
image contains 27 of the original cuboids

Rule: if the **length** of the sides are made **n** times bigger, **length SF = n**
the **volume** of the enlarged solid is **n x n x n** bigger. **volume SF = n^3**

Example:



250 ml V ml

Given that the two solids shown are similar, find the volume of the smaller solid.

$$\textit{length SF} = \frac{\textit{image}}{\textit{original}} = \frac{12}{15} = \frac{4}{5}$$ *0 < SF < 1 as expected for a reduction*

$$\textit{volume SF} = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{64}{125}$$

$$V = \frac{64}{125} \times 250 = 128$$ *smaller than 250 as expected for a reduction*

CHAPTER 3: GOING PLACES

Speed is a rate, the change of distance with time.
It is given as the distance travelled in a unit of time.

For example, 20 km/h: 20 kilometres per hour means travelling 20 kilometres in 1 hour.

Average speed only considers the change in distance and time between the start and finish of a journey. It does not take into account the varying speed during the journey.

$$average\ speed = \frac{total\ distance\ travelled}{total\ time\ taken}$$

Example:

Travel 23 km in 30 minutes, then 30 km in 45 minutes and finally 13 km in 15 minutes.

total distance 66 km , total time 90 minutes (1·5 hours)

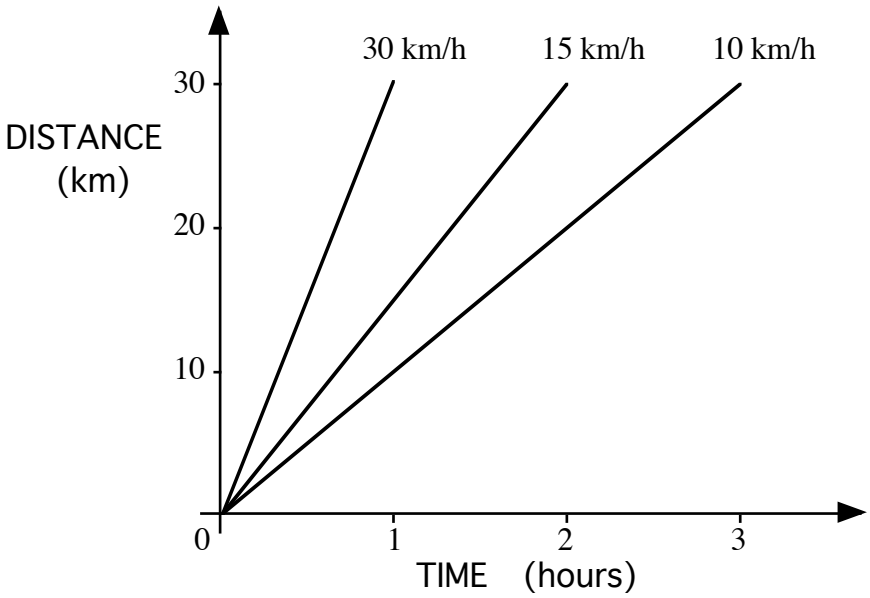
$$average\ speed = \frac{66\ km}{1 \cdot 5\ hours} = 44\ km/h$$

NOTE: The speeds at each stage 46 km/h , 40 km/h and 52 km/h average 46 km/h.
None of these are involved in calculating the average speed.

Distance/Time Graphs

The slope of the graph is the speed. The steeper the graph the greater the speed.

For example,



FORMULAE

WATCH UNITS! The distance and time units must match the speed units.

For example,

speed **km/h** requires using time in **hours** and distance in **kilometres**
cm/s requires using time in **seconds** and distance in **centimetres**

Examples:

(1) Find the speed if:
travel 30 km in 1 hour 12 mins

(2) Find the time if:
travel 50 km at 12 km/h

(3) Find the distance if:
travel for 2 mins at 8 cm/s

units:

12 min = 12 ÷ 60 = 0·2 hours
so 1 hour 12 mins = 1·2 hours

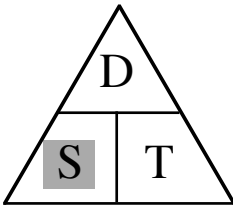
2 min = 2 x 60 = 120 secs

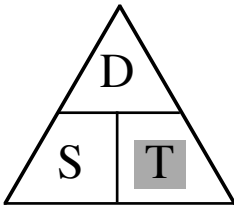
km and hours
results in speed in km/h

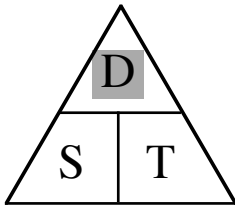
***km** matches **km**/h*
results in time in hours

seconds** matches cm/**s
results in distance in cm

calculations:







$$S = \frac{D}{T}$$
$$= 30 \div 1 \cdot 2$$
$$= 25\ km/h$$

$$T = \frac{D}{S}$$
$$= 50 \div 12$$
$$= 4 \cdot 1666....\ hours$$
$$= 4\ hours\ 10\ mins$$

$$D = ST$$
$$= 8 \times 120$$
$$= 960\ cm$$

time units: 0·1666... hours = 0·1666... x 60 = 10 min
so 4·1666... hours = 4 hour 10 mins

CHAPTER 4: SAVING AND SPENDING

Examples:

(1) **earnings**

GROSS PAY = BASIC PAY + OVERTIME , BONUS , COMMISSION etc

NET PAY = GROSS PAY - DEDUCTIONS (tax , national insurance etc)

(i) 40 hours at basic rate £8·40 per hour.
6 hours overtime at 'time and a half'.

overtime rate = £8·40 x 1·5 = £12·60

basic pay = £8·40 x 40 = £336

overtime pay = £12·60 x 6 = £75·60

gross pay = £336 + £75·60 = £411·60

(ii) basic pay £118·40 a week plus 12·5% commission on sales over £2000.

Weekly sales of £4200 were achieved.

extra sales = £4200 - £2000 = £2200

commission = £2200 ÷ 100 x 12·5 = £275

gross pay = £118·40 + £275 = £393·40

(2) **simple interest**

Invest £12000 for 8 months at 6% pa
(pa = per annum, per year)

for 1 year £12000 ÷ 100 x 6 = £720

for 8 months £720 ÷ 12 x 8 = £480

(3) **VAT**

Radio costs £60 excluding VAT at 20%.
Find the cost inclusive of VAT.

VAT = £60 ÷ 100 x 20 = £12

cost = £60 + £12 = £72

(4) **HP**

To purchase a TV requires a £60 deposit and 24 monthly instalments of £12·50

instalments = 24 x £12·50 = £300

HP cost = £60 + £300 = £360

(5) **foreign exchange**

Exchange rate is $1·50 to the £.

Change: (i) £30 to $

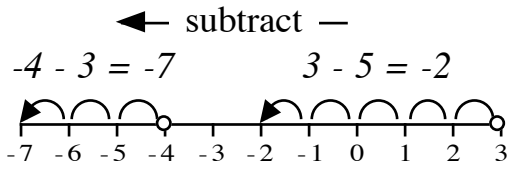
(ii) $60 to £

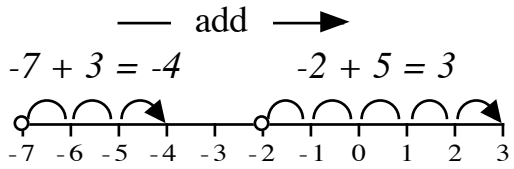
(i) £30 x 1·50 = $45

(ii) $60 ÷ 1·50 = £40

CHAPTER 5: POSITIVE AND NEGATIVE NUMBERS
(DIRECTED NUMBERS)

ADD AND SUBTRACT





add negative = subtract

a **+** (**-**b) = a **-** b

subtract negative = add

a **-** (**-**b) = a **+** b

For example,

-4 **+** (**-**3)	3 **+** (**-**5)
= -4 **-** 3	= 3 **-** 5
= -7	= -2

-7 **-** (**-**3)	-2 **-** (**-**5)
= -7 **+** 3	= -2 **+** 5
= -4	= 3

MULTIPLY AND DIVIDE

Rules:

x or ÷	+	-
+	+	-
-	-	+

For example,

2 x 7 = **+**14	15 ÷ 5 = **+**3
-2 x (-7) = **+**14	-15 ÷ (-5) = **+**3
2 x (-7) = **-**14	15 ÷ (-5) = **-**3
-2 x 7 = **-**14	-15 ÷ 5 = **-**3

CALCULATIONS

Examples:

If a = -3 , b = 2 and c = 5 , evaluate:

(1) $3c - ab$

$$\begin{aligned} & 3 \times c - a \times b \\ &= 3 \times 5 - (-3) \times 2 \\ &= 15 - (-6) \\ &= 15 + 6 \\ &= 21 \end{aligned}$$

(2) $2a^2$

$$\begin{aligned} & 2 \times a \times a \\ &= 2 \times (-3) \times (-3) \\ &= 2 \times 9 \\ &= 18 \end{aligned}$$

(3) $\dfrac{a-b}{c}$

$$\begin{aligned} & \frac{-3-2}{5} \\ &= \frac{-5}{5} \\ &= -1 \end{aligned}$$

EQUATIONS

Examples:

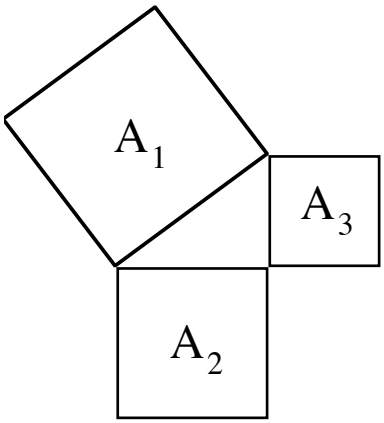
(1) Solve: $5x - 4 = 2x - 19$

$$\begin{aligned} 5x - 4 &= 2x - 19 & \\ 3x - 4 &= \quad -19 & \textit{subtracted } 2x \textit{ from each side} \\ 3x \quad &= \quad -15 & \textit{added } 4 \textit{ to each side} \\ x &= -5 & \textit{divided each side by } 3 \end{aligned}$$

(2) Solve: $2 - 5t = 10 - 3t$

$$\begin{aligned} 4 - 5t &= 10 - 3t & \\ 4 \quad &= 10 + 2t & \textit{added } 5t \textit{ to each side} \\ -6 \quad &= 2t & \textit{subtracted } 10 \textit{ from each side} \\ -3 &= t & \textit{divided each side by } 2 \\ t &= -3 & \end{aligned}$$

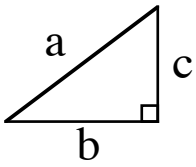
CHAPTER 6: PYTHAGORAS' THEOREM



Squares sit on the sides of a triangle;
the biggest square, area A_1, is opposite
the biggest angle.

For **right-angled triangles only**:

$$A_1 = A_2 + A_3$$



Side length **a** units is opposite the right angle.
It is the longest side, the **hypotenuse**.

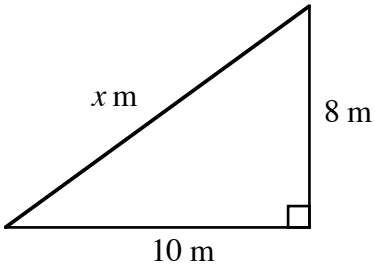
For **right-angled triangles**:

$$a^2 = b^2 + c^2$$

FINDING AN UNKNOWN SIDE

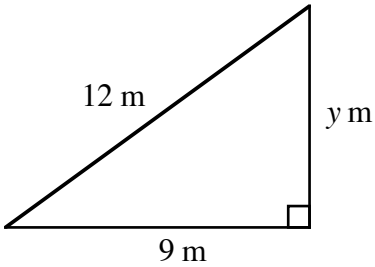
Examples:

(1) **finding the longest side:**



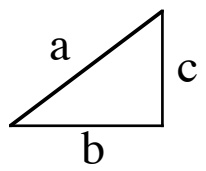
$$x^2 = 10^2 + 8^2$$
$$= 100 + 64$$
$$x^2 = 164$$
$$x = \sqrt{164}$$
$$= 12 \cdot 806...$$
$$x = 12 \cdot 8$$

(2) **finding a short side:**



$$y^2 = 12^2 - 9^2 \qquad (\text{since } y^2 + 9^2 = 12^2)$$
$$= 144 - 81$$
$$y^2 = 63$$
$$y = \sqrt{63}$$
$$= 7 \cdot 937...$$
$$y = 7 \cdot 9$$

CONVERSE OF PYTHAGORAS' THEOREM



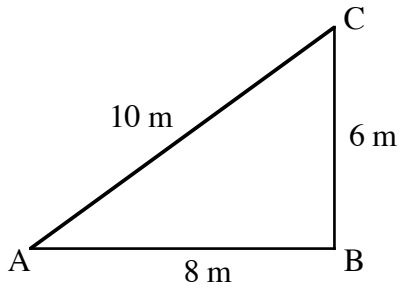
Side length **a** units, opposite the biggest angle, is the longest side.

If $a^2 = b^2 + c^2$

then the triangle is right angled

Example:

Show that triangle ABC is right angled.



$$AB^2 + BC^2 = 8^2 + 6^2 = 100$$
$$AC^2 = 10^2 = 100$$

since $AB^2 + BC^2 = AC^2$

by the Converse of Pyth. Thm.

ΔABC *is right angled at* B (*ie.* $\angle ABC = 90^\circ$)

CHAPTER 7: BRACKETS AND EQUATIONS

Examples:

SINGLE BRACKETS

Each term in the brackets is multiplied by the term outside the brackets.

(1) $3x\left(2x-y+7\right)$

$3x \times 2x = 6x^2$
$3x \times -y = -3xy$
$3x \times +7 = +21x$

$= 6x^2 - 3xy + 21x$

(2) $-2\left(3t+5\right)$

$-2 \times 3t = -6t$
$-2 \times +5 = -10$

$= -6t - 10$

(3) $-3w\left(w^2-4\right)$

$-3w \times w^2 = -3w^3$
$-3w \times -4 = +12w$

$= -3w^3 + 12w$

Fully simplify:

(4) $2\text{t}\left(3-t\right)+5t^2$

$= 6t - 2t^2 + 5t^2$
$= 6t + 3t^2$

(5) $5-3\left(n-2\right)$

$= 5 - 3n + 6$
$= 5 + 6 - 3n$
$= 11 - 3n$

DOUBLE BRACKETS

(1) $\left(3x+2\right)\left(2x-5\right)$

$\left(3x+2\right)\left(2x-5\right)$
$= 3x\left(2x-5\right) + 2\left(2x-5\right)$
$= 6x^2 - 15x + 4x - 10$
$= 6x^2 - 11x - 10$

or

"FOIL"

$\left(3x+2\right)\left(2x-5\right)$

$= 6x^2 - 15x + 4x - 10$
$= 6x^2 - 11x - 10$

"FOIL" pairs multiplied between the two brackets.

$\left(3x+2\right)\left(2x-5\right)$ **F**irst $6x^2$

$\left(3x+2\right)\left(2x-5\right)$ **O**uter $-15x$

$\left(3x+2\right)\left(2x-5\right)$ **I**nner $+4x$

$\left(3x+2\right)\left(2x-5\right)$ **L**ast -10

(2) $(2t-3)^2$

or

$$(2t-3)(2t-3)$$
$$= 2t(2t-3) - 3(2t-3)$$
$$= 4t^2 - 6t - 6t + 9$$
$$= 4t^2 - 12t + 9$$

$$(2t-3)(2t-3)$$
$$= 4t^2 - 6t - 6t + 9$$
$$= 4t^2 - 12t + 9$$

(3) $(w+2)(w^2-3w+5)$

$$(w+2)(w^2-3w+5)$$
$$= w(w^2-3w+5) + 2(w^2-3w+5)$$
$$= w^3 - 3w^2 + 5w + 2w^2 - 6w + 10$$
$$= w^3 - 3w^2 + 2w^2 + 5w - 6w + 10$$
$$= w^3 - w^2 - w + 10$$

EQUATIONS

Remove the brackets, fully simplifying, then follow the rules for solving equations.

Example:

Solve: $(4x+3)(x-2) = (2x-3)^2$

$4x^2 - 5x - 6 =$	$4x^2 - 12x + 9$	*removed brackets, fully simplifying*
$-5x - 6 =$	$-12x + 9$	*subtracted* $4x^2$ *from each side*
$7x - 6 =$	$+9$	*added* $12x$ *to each side*
$7x =$	15	*added* 6 *to each side*
$x = \frac{15}{7}$		*divided each side by* 7

CHAPTER 8: STATISTICS SUPPLEMENT

Studying statistical information, it is useful to consider: (1) typical result: **average**
(2) distribution of results: **spread**

AVERAGES:

$$mean = \frac{total\ of\ all\ results}{number\ of\ results}$$

$$median = middle\ result\ of\ the\ ordered\ results$$

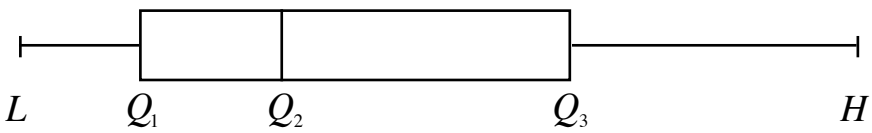
$$mode = most\ frequent\ result$$

SPREAD:

Ordered results are split into 4 equal groups so each contains 25% of the results.

The **5 figure summary** identifies: L , Q_1 , Q_2 , Q_3 , H
(lowest result , 1st , 2nd and 3rd quartiles , highest result)

A **Box Plot** is a statistical diagram that displays the 5 figure summary:



$$range,\ R = H - L$$

$$interquartile\ range,\ IQR = Q_3 - Q_1$$

$$semi-interquartile\ range,\ SIQR = \frac{Q_3 - Q_1}{2}$$

NOTE: If Q_1 , Q_2 or Q_3 fall between two results, the mean of the two results is taken.
For example,
12 ordered results: split into 4 equal groups of 3 results

Q_1 Q_2 Q_3

10 11 13 | 17 18 20 | 20 23 25 | 26 27 29

$$Q_1 = \frac{13+17}{2} = 15 \quad , \quad Q_2 = \frac{20+20}{2} = 20 \quad , \quad Q_3 = \frac{25+26}{2} = 25 \cdot 5$$

Example:

Pulse rates: $66, 64, 71, 56, 60, 79, 77, 75, 69, 73, 75, 62, 66, 71, 66$ beats per minute.

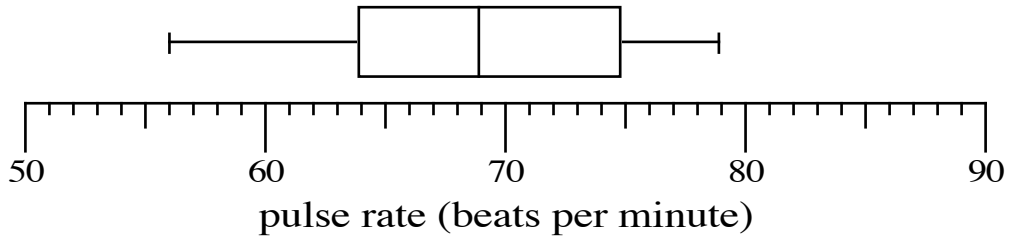
15 ordered results:

			Q_1				Q_2				Q_3			
56	60	62	**64**	66	66	66	**69**	71	71	73	**75**	75	77	79

5 Figure Summary:

$$L = 56 \quad , \quad Q_1 = 64 \quad , \quad Q_2 = 69 \quad , \quad Q_3 = 75 \quad , \quad H = 79$$

Box Plot:



Spread:

$$R = H - L = 79 - 56 = 23$$

$$IQR = Q_3 - Q_1 = 75 - 64 = 11$$

$$SIQR = \frac{Q_3 - Q_1}{2} = \frac{75 - 64}{2} = \frac{11}{2} = 5 \cdot 5$$

Averages: $(total = 66 + 64 + 71 + ... + 66 = 1030)$

$$MEAN = \frac{1030}{15} = 68 \cdot 666... = 68 \cdot 7$$

$$(Q_2)\, MEDIAN = 69$$

$$MODE = 66$$

OTHER STATISTICAL DIAGRAMS

Examples:

ordered stem-and-leaf:

Race Times (seconds):

10·4 , 10·2 , 9·9 , 12·1 , 11·7 , 10·9 , 9·9 , 11·4 , 10·6 , 11·5 , 10·1 , 9·8 , 10·2 , 11·3 , 11·0

unordered first

9	*9*	*9*	*8*			
10	*4*	*2*	*9*	*6*	*1*	*2*
11	*7*	*4*	*5*	*3*	*0*	
12	*1*					

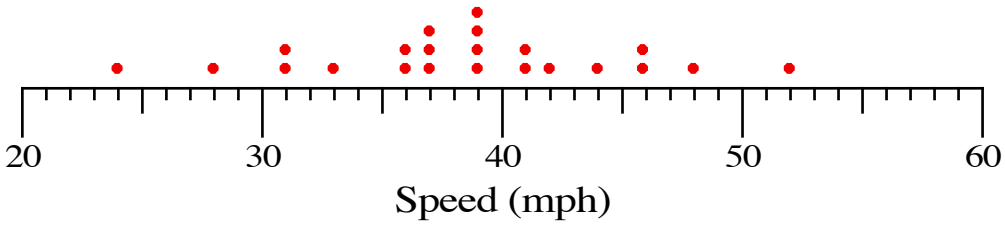
Race Time (seconds)

9	8	9	9			
10	1	2	2	4	6	9
11	0	3	4	5	7	
12	1					

n = 15 9 | 8 = 9.8

dot plot:

Car speeds (mph):



FREQUENCY DISTRIBUTION TABLES

Useful for dealing with a large number of results.

In a competition 50 people take part. The table shows the distribution of points scored.

Scores (points)

result	frequency
10	4
11	5
12	9
13	12
14	10
15	7
16	3

mean:

result	frequency	result x frequency
10	4	40
11	5	55
12	9	108
13	12	156
14	10	140
15	7	105
16	3	48
TOTALS	50	652

$$MEAN = \frac{652}{50} = 13 \cdot 04$$

cummulative frequency:

result	frequency	cummulative frequency
10	4	4
11	5	9
12	9	18
13	12	30
14	10	40
15	7	47
16	3	50

quarter the results: 50 ÷ 4 = 12 R 2

12 1 12 12 1 12

13th 25 / 26th 38th

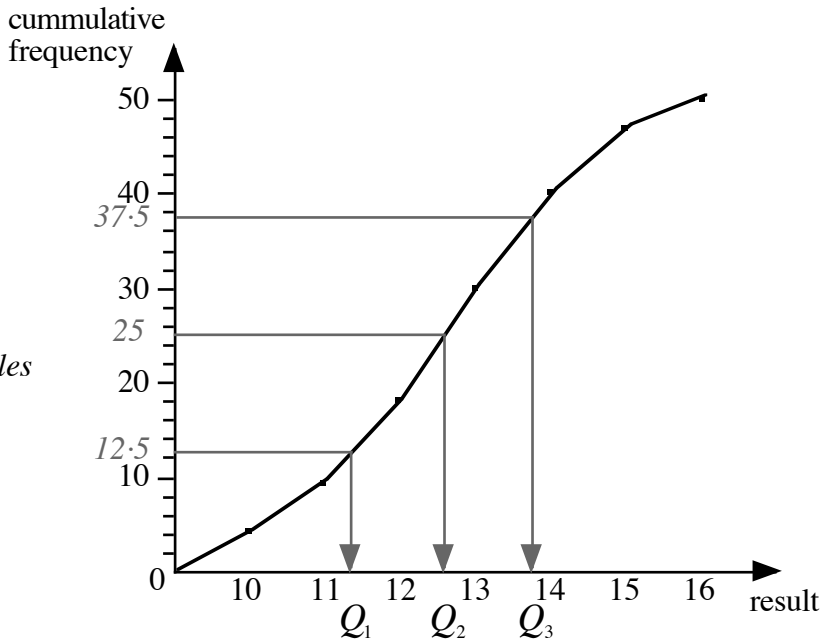
$Q_1 = 12$, *13th result included here*

$Q_2 = 13$, *25th / 26th result included here*

$Q_3 = 14$, *38th result included here*

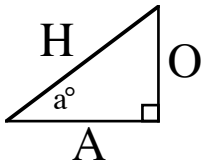
divide the total frequency into 4 quarter intervals to ***estimate*** *quartiles*

50 ÷ 4 = 12·5



CHAPTER 9: TRIGONOMETRY

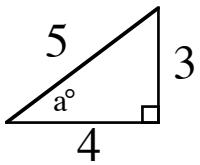
SOH-CAH-TOA The sides of a right-angled triangle are labelled:



Opposite: opposite the angle a°.
Adjacent: next to the angle a°.
Hypotenuse: opposite the right angle.

The ratios of sides $\frac{O}{H}, \frac{A}{H}$ and $\frac{O}{A}$ have values which depend on the size of angle a°.
These are called the sine, cosine and tangents of a°, written sin a° , cos a° and tan a°.

For example,



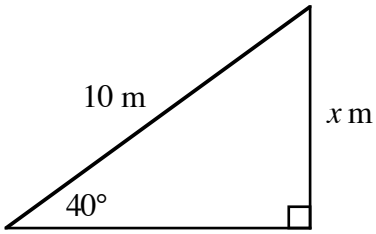
$$S = \frac{O}{H} \qquad C = \frac{A}{H} \qquad T = \frac{O}{A}$$

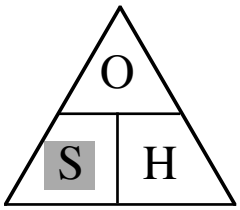
$$\sin a^\circ = \frac{3}{5} \qquad \cos a^\circ = \frac{4}{5} \qquad \tan a^\circ = \frac{3}{4}$$

FINDING AN UNKNOWN SIDE

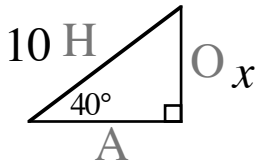
Examples:

(1) Find x.





$$S = \frac{O}{H}$$



know H , find O

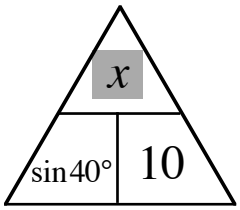
✓✓ ✓ ✓

SOH-CAH-TOA

sine ratio uses O and H

$$\sin 40^\circ = \frac{x}{10}$$

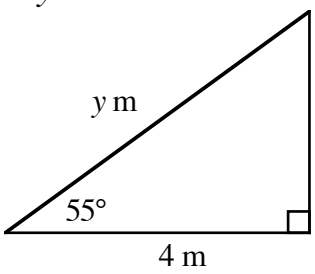
rearrange for x

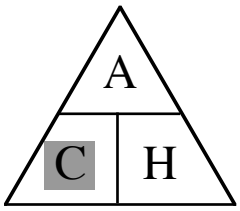


*ensure calculator set to **DEGREES***

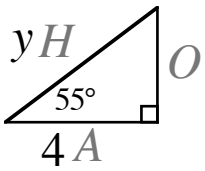
$$x = 10 \times \sin 40^\circ$$
$$= 6 \cdot 427....$$
$$x = 6 \cdot 4$$

(2) Find y.





$$C = \frac{A}{H}$$



know A , find H

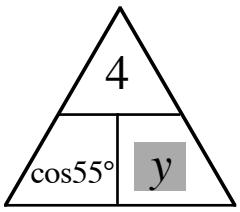
✓ ✓✓ ✓

SOH-CAH-TOA

cosine ratio uses A and H

$$\cos 55^\circ = \frac{4}{y}$$

rearrange for y



$$y = \frac{4}{\cos 55^\circ}$$

4 ÷ cos 55°, calculator set to DEGREES

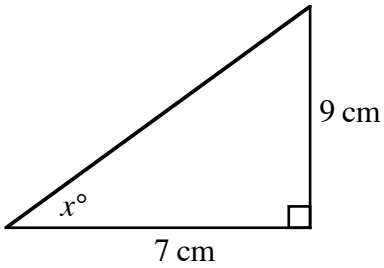
$$= 6 \cdot 973....$$

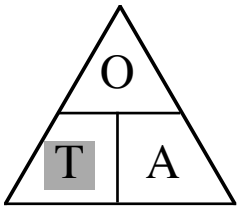
$$y = 7 \cdot 0$$

FINDING AN UNKNOWN ANGLE

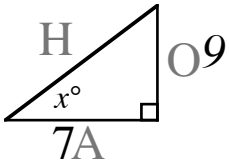
Example:

Find x.





$$T = \frac{O}{A}$$



know O , know A

✓ ✓ ✓✓

SOH-CAH-TOA

tangent ratio uses O and A

$$\tan x^\circ = \frac{9}{7}$$

$$x = \tan^{-1}\left(\frac{9}{7}\right)$$

use brackets for (9 ÷ 7), calculator set to DEGREES

$$= 52 \cdot 125....$$

$$x = 52 \cdot 1$$

CHAPTER 10: SIMULTANEOUS EQUATIONS

EQUATION OF A LINE

The equation gives a rule connecting the x and y coordinates of any point on the line.

For example, $2x + y = 6$

sample points:

$(0,6)$	$x = 0 \;,\; y = 6$	$2 \times 0 + 6 = 6$
$(3,0)$	$x = 3 \;,\; y = 0$	$2 \times 3 + 0 = 6$
$(-2,10)$	$x = -2 \;,\; y = 10$	$2 \times (-2) + 10 = 6$
$(\frac{1}{2}, 5)$	$x = \frac{1}{2} \;,\; y = 5$	$2 \times \frac{1}{2} + 5 = 6$

Infinite points cannot be listed but can be shown as a graph.

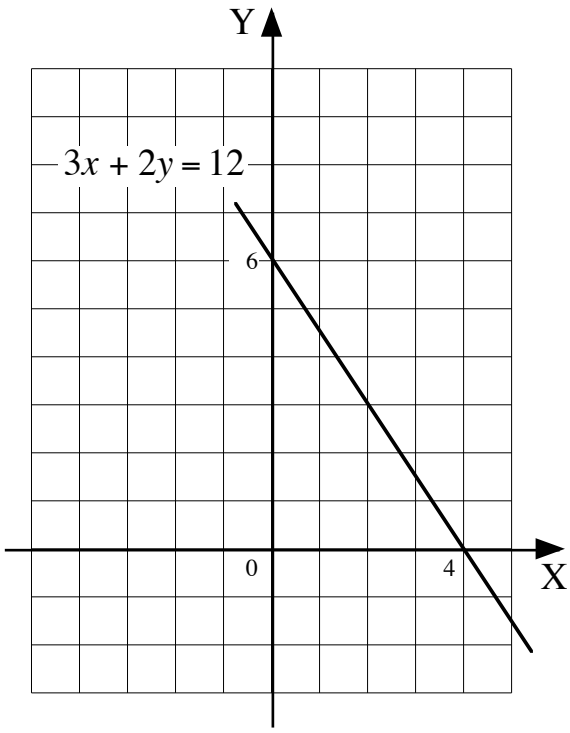
SKETCHING STRAIGHT LINES

Show where the line meets the axes.

Example:
Sketch the graph with equation $3x + 2y = 12$.

$$3x + 2y = 12$$
$$3 \times 0 + 2y = 12 \quad \textit{substituted for } x = 0$$
$$2y = 12$$
$$y = 6 \quad \textit{plot } (0,6)$$

$$3x + 2y = 12$$
$$3x + 2 \times 0 = 12 \quad \textit{substituted for } y = 0$$
$$3x = 12$$
$$x = 4 \quad \textit{plot } (4,0)$$



SOLVE SIMULTANEOUS EQUATIONS: GRAPHICAL METHOD

Sketch the two lines and the point of intersection is the solution.

Example:

Solve **graphically** the system of equations: $y + 2x = 8$

$y - x = 2$

$y + 2x = 8$ (1)

$y + 2 \times 0 = 8$ *substituted for* $x = 0$

$y = 8$ *plot* $(0,8)$

$y + 2x = 8$ (1)

$0 + 2x = 8$ *substituted for* $y = 0$

$2x = 8$

$x = 4$ *plot* $(4,0)$

$y - x = 2$ (2)

$y - 0 = 2$ *substituted for* $x = 0$

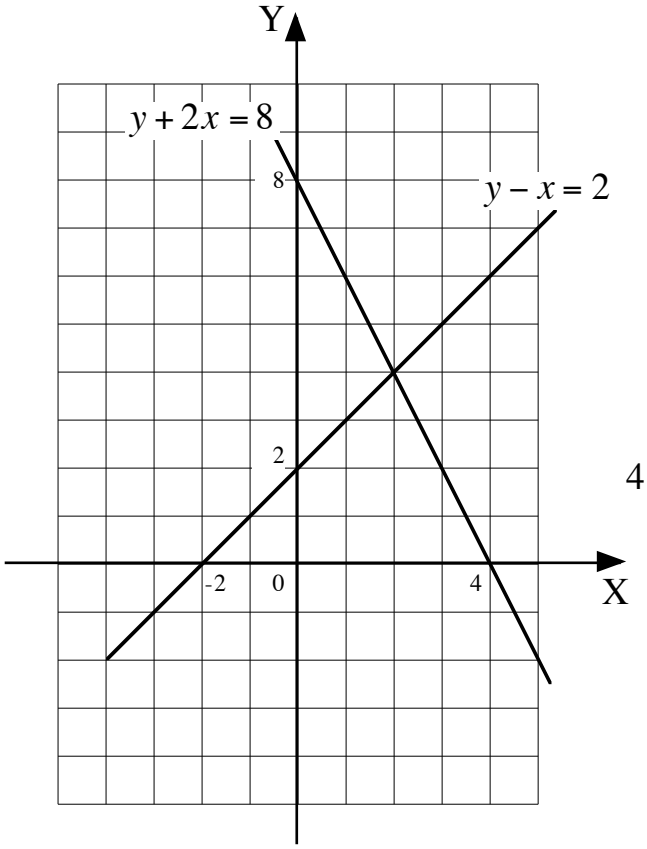
$y = 2$ *plot* $(0,2)$

$y - x = 2$ (2)

$0 - x = 2$ *substituted for* $y = 0$

$-x = 2$

$x = -2$ *plot* $(-2,0)$



point of intersection $(2,4)$

CHECK:

$x = 2$ *and* $y = 4$

substituted in both equations

$y + 2x = 8$ (1) $\qquad$ $y - x = 2$ (2)

$4 + 2 \times 2 = 8$ $\qquad$ $4 - 2 = 2$

$8 = 8$ $\qquad$ $2 = 2$

SOLUTION:

$x = 2$ *and* $y = 4$

SOLVE SIMULTANEOUS EQUATIONS: SUBSTITUTION METHOD

Rearrange both equations to $y =$ and equate the two equations.
(or $x =$)

Example:
Solve **algebraically** the system of equations: $y + 2x = 8$
$y - x = 2$

$y + 2x = 8$ (1) *can choose to rearrange to $y =$ or $x =$*

$y = 8 - 2x$ *choosing $y =$ avoids fractions as* $x = 4 - \frac{1}{2}y$

$y - x = 2$ (2)

$y = x + 2$ *rearrange for $y =$*

$x + 2 = 8 - 2x$ *y terms equal*

$3x + 2 = 8$

$3x = 6$

$x = 2$

$y = x + 2$ (2) *can choose either equation (1) or (2)*

$= 2 + 2$ *substituted for $x = 2$*

$y = 4$

CHECK:

$y + 2x = 8$ (1) *using the other equation*

$4 + 2 \times 2 = 8$ *substituted for $x = 2$ and $y = 4$*

$8 = 8$

SOLUTION:

$x = 2$ *and* $y = 4$

SOLVE SIMULTANEOUS EQUATIONS: ELIMINATION METHOD

Can add or subtract multiples of the equations to eliminate either the x or y term.

Example:

Solve **algebraically** the system of equations: $4x + 3y = 5$

$5x - 2y = 12$

$4x + 3y = 5$	(1) × 2	*can choose to eliminate x or y term*
$5x - 2y = 12$	(2) × 3	*choosing y term*, $LCM\,(3y, 2y) = 6y$ (*least common multiple*)
$8x + 6y = 10$	(3)	*multiplied each term of* (1) *by* 2 *for* $+6y$
$15x - 6y = 36$	(4)	*multiplied each term of* (2) *by* 3 *for* $-6y$
$23x + 0 = 46$	(3) + (4)	*added "like" terms,*
$x = 2$		$+6y$ *and* $-6y$ *added to* 0 (*ie eliminated*)
$4x + 3y = 5$	(1)	*can choose either equation* (1) *or* (2)
$4 \times 2 + 3y = 5$		*substituted for* $x = 2$
$8 + 3y = 5$		
$3y = -3$		
$y = -1$		

CHECK:

$5x - 2y = 12$	(2)	*using the other equation*
$5 \times 2 - 2 \times (-1) = 12$		*substituted for* $x = 2$ *and* $y = -1$
$10 - (-2) = 12$		
$12 = 12$		

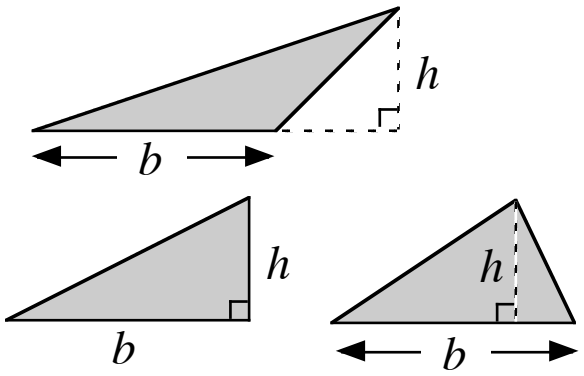
SOLUTION:

$x = 2$ *and* $y = -1$

CHAPTER 11: AREAS AND VOLUMES

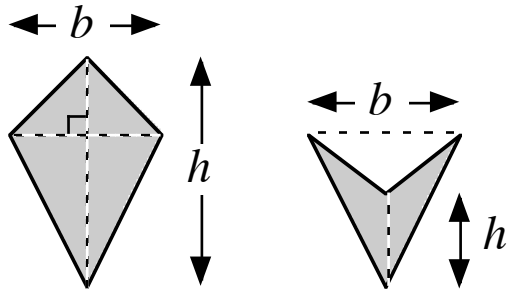
FORMULAE:

triangles $A = \frac{1}{2}bh$

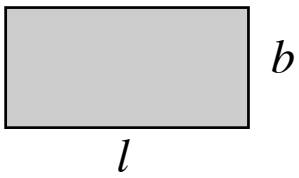


kites (incudes the rhombus)

$A = \frac{1}{2}$ *the product of the diagonals*

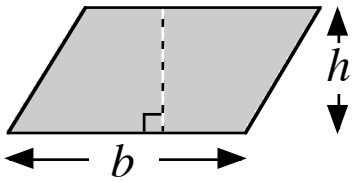


rectangle



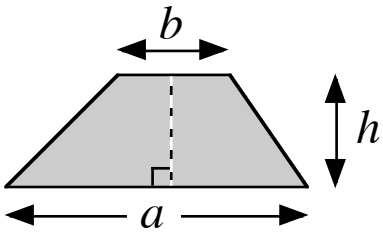
$A = lb$

parallelogram

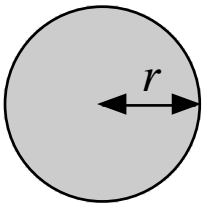


$A = bh$

trapezium



$A = \frac{1}{2}h(a+b)$



circle

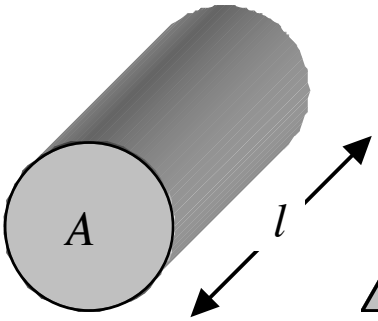
$A = \pi r^2$

$C = \pi d$

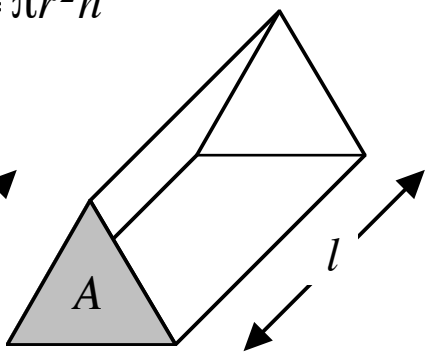
***Prism**: a solid with the same cross-section throughout its length.*

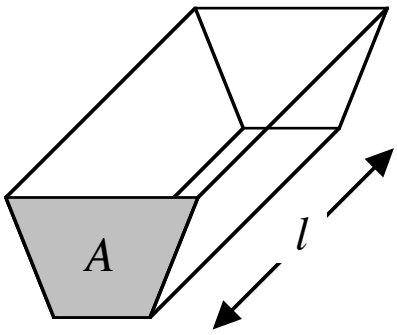
*Length **l** is at right-angles to the area **A**.*

cylinder $V = \pi r^2 h$



$V = Al$

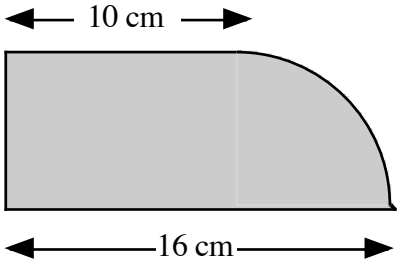




AREA AND PERIMETER

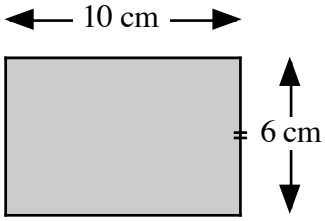
Example:

Calculate: (1) the area
(2) the perimeter



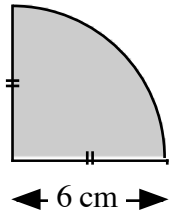
Split the shape into known shapes.

(1)



$A = lb$
$= 10 \times 6$
$= 60\ cm^2$

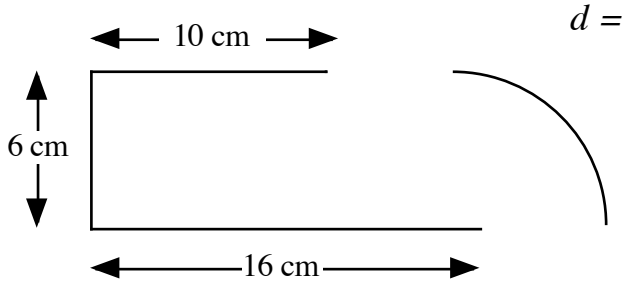
$A = \frac{1}{4}\pi r^2$
$= \pi \times 6 \times 6 \div 4$
$= 28 \cdot 274...\ cm^2$



$r = 16\ cm - 10\ cm = 6\ cm$

$total\ area = 60 + 28 \cdot 274... = 88 \cdot 274... = 88 \cdot 3\ cm^2$

(2)



$d = 6\ cm \times 2 = 12\ cm$

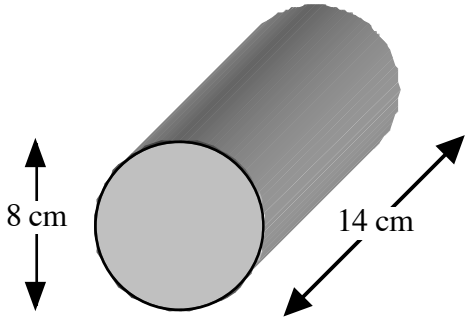
$\frac{1}{4}C = \frac{1}{4}\pi d$
$= \pi \times 12 \div 4$
$= 9 \cdot 424...\ cm$

$perimeter = 10 + 6 + 16 + 9 \cdot 424... = 41 \cdot 424... = 41 \cdot 4\ cm$

VOLUMES

Examples:

(1) Calculate the volume

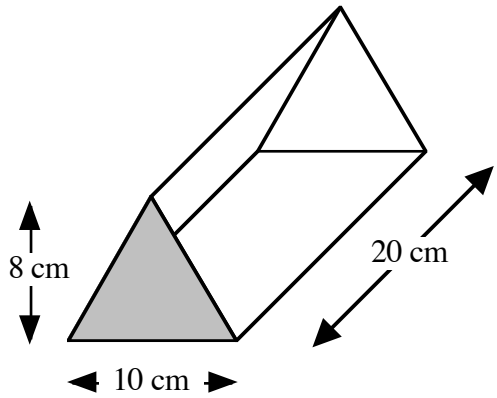


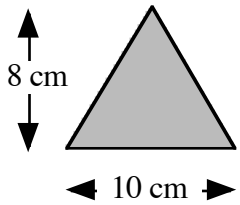
$$radius = 8\ cm \div 2 = 4\ cm$$

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 4 \times 4 \times 14 \\ &= 703 \cdot 716... \\ &= 703 \cdot 7\ cm^3 \end{aligned}$$

(*exact answer* 224π)

(2) Calculate the volume

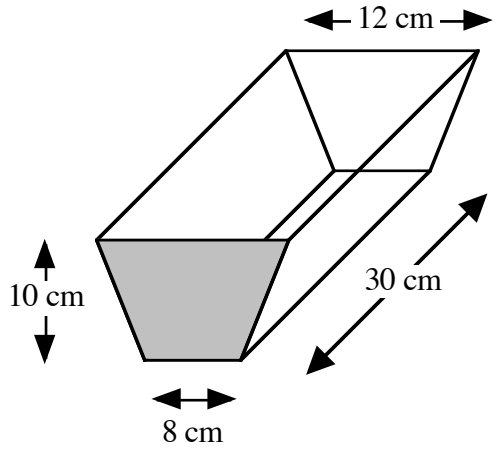


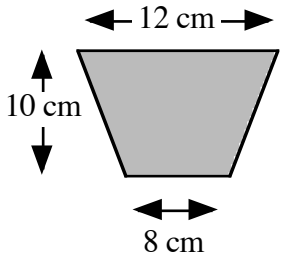


$$\begin{aligned} A &= \tfrac{1}{2}bh \\ &= 10 \times 8 \div 2 \\ &= 40\ cm^2 \end{aligned}$$

$$\begin{aligned} V &= Al \\ &= 40 \times 20 \\ &= 800\ cm^3 \end{aligned}$$

(3) Calculate the volume

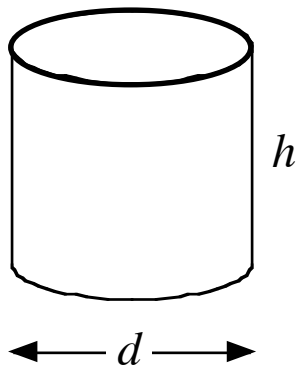


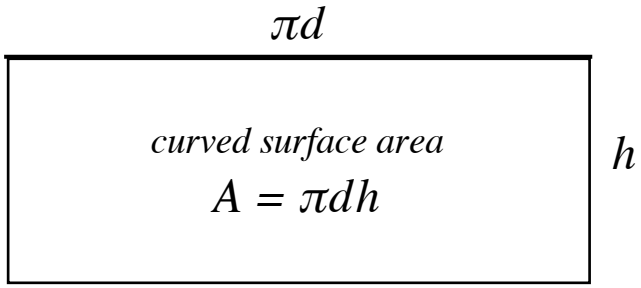


$$\begin{aligned} A &= \tfrac{1}{2}h(a+b) \\ &= 10 \times (8+12) \div 2 \\ &= 10 \times 20 \div 2 \\ &= 100\ cm^2 \end{aligned}$$

$$\begin{aligned} V &= Al \\ &= 100 \times 30 \\ &= 3000\ cm^3 \end{aligned}$$

SURFACE AREA
CYLINDER:

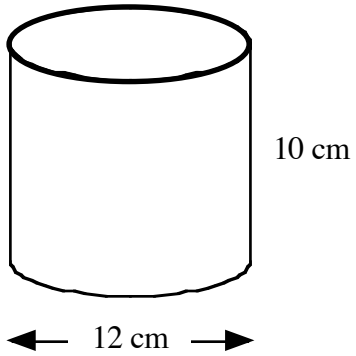




total surface area

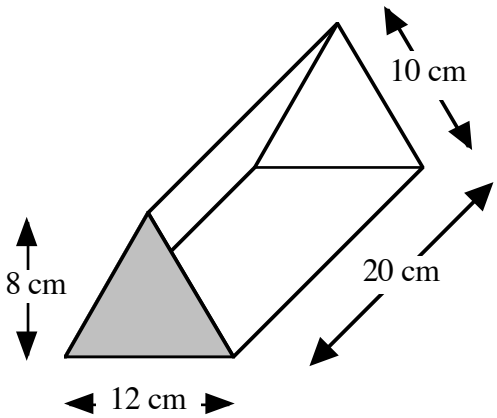
$$A = \pi dh + 2\pi r^2$$

Examples:
(1) Calculate the total surface area.



$$\begin{aligned} A &= \pi dh + 2\pi r^2 \\ &= \pi \times 12 \times 10 + 2 \times \pi \times 6 \times 6 \\ &= 376 \cdot 991... + 226 \cdot 194... \quad \text{or} \quad 120\pi + 72\pi \\ &= 603 \cdot 185... \qquad\qquad\qquad\qquad = 192\pi\ cm^2 \\ &= 603\ cm^2 \qquad\qquad\qquad\qquad (\textit{exact answer}) \end{aligned}$$

triangular prism
(2) Calculate the total surface area.



triangles	rectangles	
$A = \frac{1}{2}bh$	$A = lb$	$A = lb$
$= 12 \times 8 \div 2$	$= 10 \times 20$	$= 12 \times 20$
$= 48\ cm^2$	$= 200\ cm^2$	$= 240\ cm^2$

$$\textit{total area} = 2 \times 48 + 2 \times 200 + 240 = 736\ cm^2$$

CHAPTER 12: FACTORSATION

COMMON FACTORS

Factors: divide into a number without a remainder. Factors of a number come in pairs. For example,

$12 = 1 \times 12 = 2 \times 6 = 3 \times 4$ $\quad$ *factors of* 12 *are* 1, 2, 3, 4, 6, 12

$18 = 1 \times 18 = 2 \times 9 = 3 \times 6$ $\quad$ *factors of* 18 *are* 1, 2, 3, 6, 9, 18

$4a = 1 \times 4a = 2 \times 2a = 4 \times a$ $\quad$ *factors of* $4a$ *are* $1, 2, 4, a, 2a, 4a$

$2a^2 = 1 \times 2a^2 = 2 \times a^2 = a \times 2a$ $\quad$ *factors of* $2a^2$ *are* $1, 2, a, 2a, a^2, 2a^2$

Highest Common Factor(HCF): the highest factors numbers share.
For example,
from the above lists of factors: $HCF(12,18) = 6$ $\quad$ $HCF(4a, 2a^2) = 2a$

Factorisation: HCFs are used to write expressions in **fully** factorised form.

Examples:

Factorise **fully**:

(1) $12x + 18y$

$6 \times 2x + 6 \times 3y$ *using* $HCF(12,18) = 6$

$= 6(2x + 3y)$

(2) $4a - 2a^2$

$2a \times 2 - 2a \times a$ *using* $HCF(4a, 2a^2) = 2a$

$= 2a(2 - a)$

NOTE: the following answers are factorised but not **fully** factorised:

$2(6x + 9y)$ $\quad$ $2(2a - a^2)$

$3(4x + 6y)$ $\quad$ $a(4 - 2a)$

DIFFERENCE OF TWO SQUARES

Rule: $a^2 - b^2 = (a+b)(a-b)$

check: $(a+b)(a-b) = a(a-b) + b(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$

Examples:

Factorise **fully**:

(1) $4x^2 - 9$

$= (2x)^2 - 3^2$

$= (2x+3)(2x-3)$

(2) $t^2 - 1$

$= t^2 - 1^2$

$= (t+1)(t-1)$

(3) $n^4 - 1$

$= (n^2)^2 - 1^2$

$= (n^2+1)(n^2-1)$

$= (n^2+1)(n+1)(n-1)$

common factor first

(4) $8x^2 - 18$

$= 2(4x^2 - 9)$

$= 2(2x+3)(2x-3)$

(5) $t^3 - t$

$= t(t^2 - 1)$

$= t(t+1)(t-1)$

TRINOMIALS $ax^2 + bx + c$, $a = 1$ ie. $1x^2$

(Quadratic Expressions)

$ax^2 + bx + c = (x + ?)(x + ?)$ The missing numbers: are **a pair of factors of** c **sum to** b

Examples:

Factorise **fully**:

(1) $x^2 + 5x + 6$

$1 \times 6 = 2 \times 3 = 6$

$2 + 3 = 5$

use $+2$ *and* $+3$

$= (x+2)(x+3)$

(2) $x^2 - 5x + 6$

$-1, -6$ *or* $-2, -3$

$-2 + (-3) = -5$

use -2 *and* -3

$= (x-2)(x-3)$

(3) $x^2 - 5x - 6$

$-1, 6$ or $1, -6$ or $-2, 3$ or $2, -3$

$1 + (-6) = -5$

use $+1$ *and* -6

$= (x+1)(x-6)$

TRINOMIALS ax^2+bx+c , $a \neq 1$

Carry out a procedure which is a reversal of bracket breaking.

Examples:

(1) factorise $2t^2 + 7t + 6$

$$2 \times 6 = 12 \quad \textit{pairs of factors} \quad \underbrace{1,12 \;\; or \;\; 2,6 \;\; or \;\; 3,4}_{3+4=7}$$

$$2t^2 \underline{\underline{+7}}\mathbf{t} + 6$$

$= 2t^2 + \mathbf{4t} + \mathbf{3t} + 6$ *replace* $+7t$ *by* $+4t+3t$ (*or* $+3t+4t$)

$= (2t^2 + 4t) + (3t + 6)$ *bracket first and last pairs of terms*

$= 2t(\mathbf{t+2}) + 3(\mathbf{t+2})$ *factorise each bracket using HCF*

$= (2t+3)(\mathbf{t+2})$ *factorise: brackets are common factor*

(2) factorise $2t^2 - 7t + 6$ **Watch!** Take care with **negative signs outside brackets**.

$$2 \times 6 = 12 \quad \textit{pairs of factors} \quad \underbrace{1,12 \;\; or \;\; 2,6 \;\; or \;\; 3,4}_{3+4=7}$$

$$2t^2 \underline{\underline{-7}}\mathbf{t} + 6$$

$= 2t^2 - \mathbf{4t} - \mathbf{3t} + 6$ *replace* $-7t$ *by* $-4t-3t$ (*or* $-3t-4t$)

$= (2t^2 - 4t) - (3t - 6)$ *notice sign change in 2nd bracket,* $+6$ *to* -6

$= 2t(\mathbf{t-2}) - 3(\mathbf{t-2})$

$= (2t-3)(\mathbf{t-2})$

(3) factorise $2t^2 - 11t - 6$

$$2 \times (-6) = -12 \quad \textit{pairs of factors} \quad \underbrace{1,12 \;\; or \;\; 2,6 \;\; or \;\; 3,4}_{-12+1=-11}$$

$2t^2 \underline{\underline{-11}}\mathbf{t} - 6$ *one factor is negative*

$= 2t^2 - \mathbf{12t} + \mathbf{1t} - 6$ *replace* $-11t$ *by* $-12t+1t$ (**not** $+1t-12t$)

$= (2t^2 - 12t) + (1t - 6)$ *notice no sign change needed in 2nd bracket*

$= 2t(\mathbf{t-6}) + 1(\mathbf{t-6})$ *notice 2nd bracket still requires common factor*

$= (2t+1)(\mathbf{t-6})$

ALTERNATIVE METHOD:

Try out the possible combinations of the factors which could be in the brackets.

Examples: same quadratic expressions as the previous page.

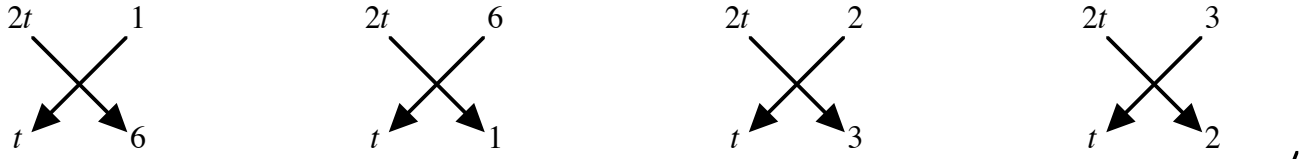
(1) factorise $2t^2 + 7t + 6$

$2 \times 6 = 12$ *pairs of factors* $\underbrace{1,12 \text{ or } 2,6 \text{ or } 3,4}_{3+4=7}$

$2t^2 \underline{\underline{+\mathbf{7t}}} + 6$ *try combinations so that 3t and 4t are obtained*

factors of $2t^2$: $2t$, t

factors of 6: 1 , 6 *or* 2 , 3



$1t$ $12t$

$2t^2 + 7t + 6$ *has no common factor. These have so can be ruled out*

$3t$ $4t$ ✓

$\underline{\underline{(2t+3)(t+2)}}$

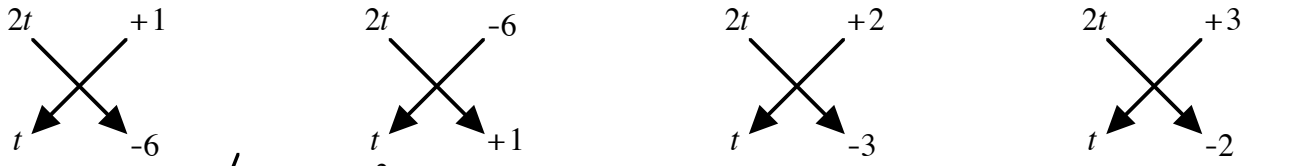
(2) factorise $2t^2 - 7t + 6$

exactly as example (1) except -7t requires both negative, so -3 , -2 $\underline{\underline{(2t-3)(t-2)}}$

(3) factorise $2t^2 - 11t - 6$

$2 \times (-6) = -12$ *pairs of factors* $\underbrace{1,12 \text{ or } 2,6 \text{ or } 3,4}_{-12+1=-11}$ *one factor is negative*

$2t^2 \underline{\underline{-\mathbf{11t}}} - 6$ *try combinations so that* $-12t$ *and* $1t$ *are obtained*



$1t$ $-12t$ ✓

$2t^2 - 11t - 6$ *has no common factor. These have so can be ruled out*

$3t$ $-4t$

$\underline{\underline{(2t+1)(t-6)}}$

CHAPTER 13: PERSONAL FINANCE

PERCENTAGE CHANGE

	original value		changed value
INCREASE: growth, appreciation, compound interest	100%	$\xrightarrow{+a\%}$	$(100+a)\%$
DECREASE: decay, depreciation	100%	$\xrightarrow{-a\%}$	$(100-a)\%$

For example,

8% increase: $100\% \xrightarrow{+8\%} 108\% = 1\cdot08$ multiply quantity by $1\cdot08$ for 8% increase

8% decrease: $100\% \xrightarrow{-8\%} 92\% = 0\cdot92$ multiply quantity by $0\cdot92$ for 8% decrease

Examples:

APPRECIATION AND DEPRECIATION

(1) A £240000 house appreciates in value by 5% in 2007, appreciates 10% in 2008 and depreciates by 15% in 2009. Calculate the value of the house at the end of 2009.

5% increase: $100\% + 5\% = 105\% = 1\cdot05$

10% increase: $100\% + 10\% = 110\% = 1\cdot10$

15% decrease: $100\% - 15\% = 085\% = 0\cdot85$

$£240000 \times 1\cdot05 \times 1\cdot10 \times 0\cdot85$

$= £235620$

or *evaluate year by year*

year 1

$5\% \times £240000 = £12000$

$£240000 + £12000 = £25200$

year 2

10% *of* $£252000 = £25200$

$£252000 + £25200 = £277200$

year 3

15% *of* $£277200 = £41580$

$£277200 - £41580 = £235620$

COMPOUND INTEREST

(2) Calculate the compound interest on £12000 invested at 5% pa for 3 years.

$£12000 \times (1\cdot05)^3$ *ie.* $\times 1\cdot05 \times 1\cdot05 \times 1\cdot05$ *or evaluate year by year*

$£12000 \times 1\cdot157625$

$= £13891\cdot50$

compound interest $= £13891\cdot50 - £12000 = £1891\cdot50$

INCOME TAX

Tax is paid on **taxable income**, gross annual income less **allowances**.

Example:

Tom earns £28 800 a year and after allowances of £6 300 he will pay tax at 20%.
Calculate his income after tax.

20% tax subtracted

$$100\% \xrightarrow{-20\%} 080\% = 0 \cdot 80$$

taxable income = £28800 – £6300 = £22500

£22500 × 0·80 = £18000

REVERSING PERCENTAGE CHANGE

Divide by the factor which produced the increase.

Examples:

(1) Including VAT of 20%, a radio costs £96. Find the original cost exclusive of VAT.

20% VAT added

$$100\% \xrightarrow{+20\%} 120\% = 1 \cdot 20$$

£x × 1·20 = £96

£x = £96 ÷ 1·20

= £80

(2) A camera costs £120 after a discount of 25% is applied. Find the original cost.

25% discount subtracted

$$100\% \xrightarrow{-25\%} 075\% = 0 \cdot 75$$

£x × 0·75 = £120

£x = £120 ÷ 0·75

= £160

CHAPTER 14: FORMULAE

TRANSPOSING FORMULAE (CHANGE OF SUBJECT)

Follow the rules for equations to isolate the **target term** and then the **target letter**.
(has target letter)

addition and subtraction

$x + a = b$

subtract a from each side

$x = b - a$

$x - a = b$

add a to each side

$x = b + a$

multiplication and division

$\frac{x}{a} = b$

multiply each side by a

$x = ab$

$ax = b$

divide each side by a

$x = \frac{b}{a}$

powers and roots

$x^2 = a$

square root each side

$x = \sqrt{a}$

$\sqrt{x} = a$

square each side

$x = a^2$

Examples:

Change the subject of the formula to r:

(1) $F = 3r^2 + p$

subtract p from each side

$F - p = 3r^2$

divide each side by 3

$\frac{F - p}{3} = r^2$

square root both sides

$\sqrt{\frac{F - p}{3}} = r$

subject of formula now r

$r = \sqrt{\frac{F - p}{3}}$

(2) $W = \frac{\sqrt{r} - n}{t}$

multiply both sides by t

$Wt = \sqrt{r} - n$

add n to both side

$Wt + n = \sqrt{r}$

square both sides

$(Wt + n)^2 = r$

subject of formula now r

$r = (Wt + n)^2$

If the **target term is negative** , rearrange to the other side so it is positive
If the **target term is in brackets** , break the brackets
If there are **two target terms** , gather the target terms and factorise for the target letter.

(3) $g = a(v - ar)$

break the brackets

$g = av - a^2r$

add a^2r *to each side*

$g + a^2r = av$

subtract g from each side

$a^2r = av - g$

divide both sides by a^2

$r = \dfrac{av - g}{a^2}$

(4) $h = \dfrac{\pi r + t}{r}$

multiply both sides by r

$hr = \pi r + t$

subtract πr *from each side*

$hr - \pi r = t$

factorise for r

$r(h - \pi) = t$

divide both sides by $(h - \pi)$

$r = \dfrac{t}{(h - \pi)}$ *brackets not needed*

EFFECT OF CHANGE

Examples:

(1) The volume of a cylinder is given by

$$V = \pi r^2 h$$

Describe the effect on volume of:

(i) doubling h

$V = \pi r^2 2h$

$= 2\pi r^2 h$

2 times bigger

(ii) trebling r

$V = \pi(3r)^2 h$

$= \pi 9r^2 h$

$= 9\pi r^2 h$

9 times bigger

(2) The length of a prism is given by

$$l = \frac{V}{A}$$

Describe is the effect on length of:

(i) doubling V

$l = \dfrac{V}{A}$

$= \dfrac{2V}{A}$

$= 2\dfrac{V}{A}$

2 times bigger

(ii) trebling A

$l = \dfrac{V}{A}$

$= \dfrac{V}{3A}$

$= \frac{1}{3}\dfrac{V}{A}$

$\frac{1}{3}$ *of size*

CONSTRUCTING FORMULAE

An arithmetic process can be generalised using letters to represent quantities.

Examples:

(1)

CAR HIRE CHARGES		
minimum charge:		£20 per day
mileage charge:	first 200 miles	no charge
	over 200 miles	each additional mile 20p

(a) Find the cost of hiring a car for 6 days and travelling a distance 500 miles.

(b) A car is hired for d days and the mileage is n miles where n > 200.
Write a formula for the cost £C of hiring the car.

(a) *The calculations are set out carefully for each stage.*

minimum charge (£) : $\mathbf{6} \times 20 = 120$ *note* $20p$ *is* £$0 \cdot 20$

mileage charge (£) : $\mathbf{500} - 200 = 300$ $300 \times 0 \cdot 20 = 60$

total cost: £C = £120 + £60 = £180

(b) *The appropriate letters can replace the numbers.*

minimum charge (£) : $\mathbf{d} \times 20 = 20d$

mileage charge (£) : $\mathbf{n} - 200$ $(n - 200) \times 0 \cdot 20 = 0 \cdot 2(n - 200)$

total cost (£) , $C = 20d + 0 \cdot 2\,(n - 200)$

(2) Odd numbers starting with 3 are added. Generalise for adding n odd numbers.

4 terms: $3 + 5 + 7 + 9 = 4 \times 6$

5 terms: $3 + 5 + 7 + 9 + 11 = 5 \times 7$

6 terms: $3 + 5 + 7 + 9 + 11 + 13 = 6 \times 8$

4 *terms*: 4×6
5 *terms*: 5×7
6 *terms*: 6×8
n terms: $n \times (n + 2)$
$= n(n + 2)$

The first number matches the number of terms.
The second number is always 2 more than this.

CHAPTER 15: PROBABILITY

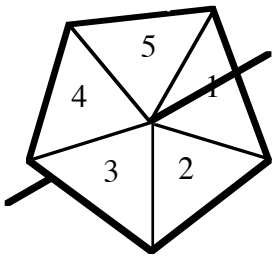
The probability of an event A occuring is $\quad P(A) = \dfrac{\text{number of outcomes involving A}}{\text{total number of outcomes possible}}$

Always $0 \le P \le 1$ and $P = 0$ impossible to occur , $P = 1$ certain to occur

$$P(\text{not A}) = 1 - P(A)$$

$$\text{number of expected outcomes involving event A} = \text{number of trials} \times P(A)$$

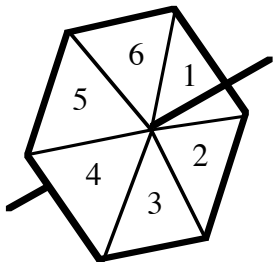
For example,
In a game, scores from two spinners are added together. The possible totals are shown.



SPINNER 1

SPINNER 2

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11



3 ways out of 30 to score a total of 9, $P(9) = \dfrac{3}{30} = \dfrac{1}{10}$

27 ways to score a total which is **not** 9,

$$P(not\ 9) = \frac{27}{30} = \frac{9}{10} \qquad \text{or} \qquad P(not\ 9) = 1 - P(9) = 1 - \frac{1}{10} = \frac{9}{10}$$

cannot score 12 , $\qquad P(12) = 0$

must score less than 12 $\qquad P(< 12) = 1$

spin the pair of spinners 60 times,

$$\begin{aligned} number\ of\ 9s\ expected &= number\ of\ trials \times P(9) \\ &= 60 \times \frac{1}{10} \\ &= 6 \end{aligned}$$