Surds - Lesson 5

Rationalising the Denominator 2

LI

- Rationalise a complicated denominator.

SC

- Conjugate surds.
- Expanding brackets.

Rationalising the denominator of a fraction means writing the fraction without a surd in the denominator (bottom)

Reminder

Multiplying numerator and denominator of a fraction by the same quantity does not change the fraction (because it's the same as multiplying the fraction by the number 1).

The conjugate of an expression with 2 terms is the expression obtained by changing the sign between the terms

Expression	Conjugate Expression
$x + y$	$x - y$
$-3p + 2$	$-3p - 2$

Surd Expression	Conjugate Surd Expression
$1 + \sqrt{5}$	$1 - \sqrt{5}$
$3 - \sqrt{7}$	$3 + \sqrt{7}$
$\sqrt{2} - 7$	$\sqrt{2} + 7$
$6 + 2\sqrt{3}$	$6 - 2\sqrt{3}$
$4\sqrt{11} - 9$	$4\sqrt{11} + 9$

A shorthand can be obtained for some surd calculations.

Multiplying brackets and simplifying the expression (or, applying a difference of 2 squares in reverse),

$$\left(a - \sqrt{b}\right)\left(a + \sqrt{b}\right)$$

gives,

$$a^2 - b$$

More generally,

$$\left(a - b\sqrt{c}\right)\left(a + b\sqrt{c}\right)$$

$$= a^2 - b^2 c$$

Example 1

$$(3 - \sqrt{7})(3 + \sqrt{7})$$

$$= 9 - 7$$

$$= \boxed{2}$$

Example 2

$$(5 - 2\sqrt{3})(5 + 2\sqrt{3})$$

$$= 5^2 - 2^2(3)$$

$$= 25 - 12$$

$$= \boxed{13}$$

Rationalising a Complicated Denominator

A 'complicated' denominator is one that has a term of the form :

$$a \pm b\sqrt{c}$$

How to rationalise a complicated denominator :

- Multiply top and bottom of fraction by the conjugate surd expression in the bottom.
- Simplify fully any surds on the top; simplify fully any fractions.

Example 3

Rationalise :

$$\frac{1}{2-\sqrt{5}} \quad \begin{matrix} \times\left(2+\sqrt{5}\right) \\ \times\left(2+\sqrt{5}\right) \end{matrix}$$

$$= \frac{2+\sqrt{5}}{\left(2-\sqrt{5}\right)\left(2+\sqrt{5}\right)}$$

$$= \frac{2+\sqrt{5}}{4-5}$$

$$= \frac{2+\sqrt{5}}{-1}$$

$$= \boxed{-2-\sqrt{5}}$$

Example 4

Rationalise :

$$\frac{1}{4+\sqrt{7}} \quad \begin{matrix} \times \left(4-\sqrt{7}\right) \\ \times \left(4-\sqrt{7}\right) \end{matrix}$$

$$= \frac{4-\sqrt{7}}{\left(4+\sqrt{7}\right)\left(4-\sqrt{7}\right)}$$

$$= \frac{4-\sqrt{7}}{16-7}$$

$$= \boxed{\frac{4-\sqrt{7}}{9}}$$

Example 5

Rationalise :

$$\frac{4}{6+\sqrt{3}} \quad \begin{matrix} \times \left(6-\sqrt{3}\right) \\ \times \left(6-\sqrt{3}\right) \end{matrix}$$

$$= \frac{4\left(6-\sqrt{3}\right)}{\left(6+\sqrt{3}\right)\left(6-\sqrt{3}\right)}$$

$$= \frac{4\left(6-\sqrt{3}\right)}{36-3}$$

$$= \boxed{\frac{4\left(6-\sqrt{3}\right)}{33}}$$

Example 6

Rationalise :

$$\frac{1 + \sqrt{5}}{2 - \sqrt{3}} \quad \begin{matrix} \times \left(2 + \sqrt{3}\right) \\ \times \left(2 + \sqrt{3}\right) \end{matrix}$$

$$= \frac{\left(1 + \sqrt{5}\right)\left(2 + \sqrt{3}\right)}{\left(2 - \sqrt{3}\right)\left(2 + \sqrt{3}\right)}$$

$$= \frac{2 + \sqrt{3} + 2\sqrt{5} + \sqrt{5}\sqrt{3}}{4 - 3}$$

$$= \boxed{2 + \sqrt{3} + 2\sqrt{5} + \sqrt{15}}$$

Questions

Rationalise the denominator.

a $\dfrac{1}{1-\sqrt{6}}$ **b** $\dfrac{1}{1+\sqrt{2}}$ **c** $\dfrac{1}{2-\sqrt{3}}$ **d** $\dfrac{3}{1-\sqrt{2}}$ **e** $\dfrac{7}{4+\sqrt{3}}$

f $\dfrac{8}{\sqrt{6}-\sqrt{2}}$ **g** $\dfrac{\sqrt{3}}{\sqrt{3}-6}$ **h** $\dfrac{1-\sqrt{2}}{1-\sqrt{3}}$ **i** $\dfrac{1+\sqrt{3}}{6+\sqrt{2}}$ **j** $\dfrac{5-\sqrt{2}}{\sqrt{3}-1}$

k $\dfrac{7}{2\sqrt{3}-1}$ **l** $\dfrac{1}{3\sqrt{7}+2}$ **m** $\dfrac{2\sqrt{5}}{2\sqrt{5}+1}$

Answers

a $\frac{-(1+\sqrt{6})}{5}$	**b** $-(1-\sqrt{2})$	**c** $2+\sqrt{3}$	**d** $-(3+3\sqrt{2})$
e $\frac{28-7\sqrt{3}}{13}$	**f** $2\sqrt{6}+2\sqrt{2}$	**g** $\frac{-(1+2\sqrt{3})}{11}$	**h** $\frac{\sqrt{2}+\sqrt{6}-\sqrt{3}-1}{2}$
i $\frac{6-\sqrt{2}+6\sqrt{3}-\sqrt{6}}{34}$	**j** $\frac{5\sqrt{3}+5-\sqrt{6}-\sqrt{2}}{2}$	**k** $\frac{7(2\sqrt{3}+1)}{11}$	**l** $\frac{3\sqrt{7}-2}{59}$
m $\frac{20-2\sqrt{5}}{19}$			