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Unit 2 : Properties of Functions - Lesson 4

Modulus Graphs, Critical Points and Extrema

LI

- Sketch the graph of the Modulus of a function.
- Find Critical Points of a function.

SC

- Differentiation.
- Graph sketching.

Jan 10-12:11

Sketching Modulus Graphs

The modulus (aka absolute value) of a number p is :

$$|p| = \begin{cases} p & (\text{if } p \geq 0) \\ -p & (\text{if } p < 0) \end{cases}$$

The modulus of a function f is the modulus of all y - values :

$$|f| = \begin{cases} f & (\text{if } f \geq 0) \\ -f & (\text{if } f < 0) \end{cases}$$

To sketch the graph of the modulus of a function f, reflect in the x - axis any parts of the graph that are below the x - axis

Critical Points

A critical point of a function is a point where either :

- $f'(x) = 0$ (stationary point).
- $f'(x)$ does not exist.

$f'(x)$ will not exist when the graph has a 'sharp corner' or at an endpoint

Dec 12-10:17

Extrema

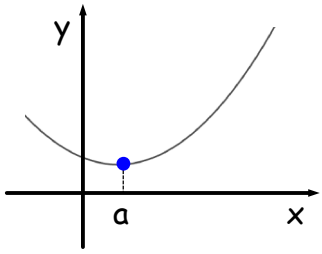
Extrema for a function refer to maximum and minimum values (i.e. y - values)

3 Types of Extrema

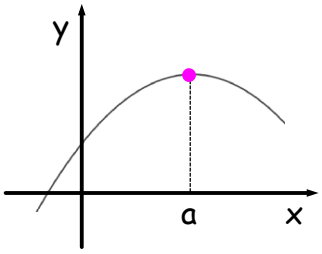
- Local.
- Endpoint.
- Global.

A function has a local minimum (local maximum) at $x = a$ if there is an interval containing $x = a$ for which $f(a) \leq (\geq) f(x)$ for all x - values in that interval

Local Minimum

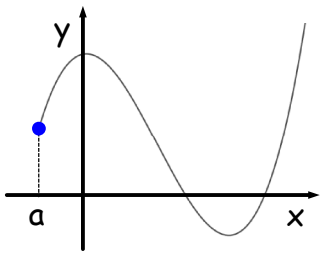


Local Maximum

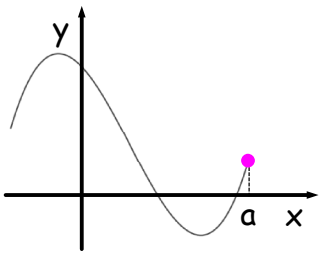


If $x = a$ is an endpoint in dom f, f has an endpoint minimum (endpoint maximum) at $x = a$ if there is a point $x = p$ in dom f such that $f(a) \leq (\geq) f(x)$ for all x - values in the interval $[a, p)$ or $(p, a]$

Endpoint Minimum



Endpoint Maximum



A function has a global minimum (global maximum) at $x = a$ if $f(a) \leq (\geq) f(x)$ for all x - values in the domain of f

All local extrema occur at critical points;
but not all critical points are local extrema

Every global extremum is either a local extremum or endpoint extremum

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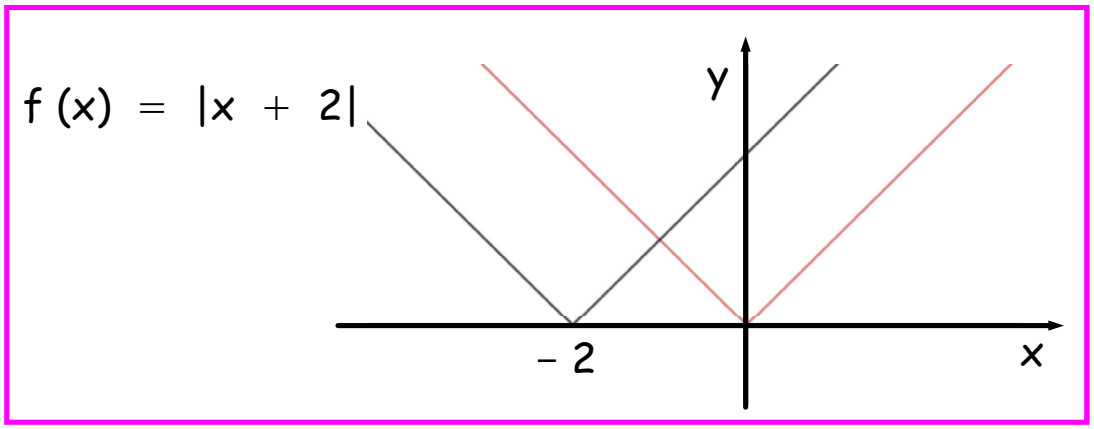
Example 1

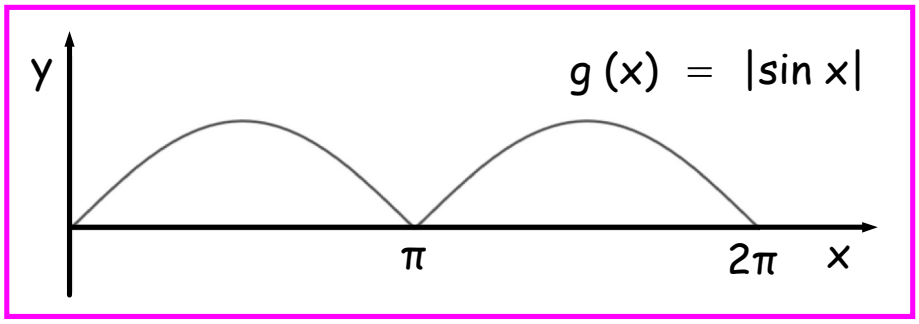
Sketch the graphs of :

(a) $f(x) = |x + 2|$.
(b) $g(x) = |\sin x|$ $(0 \leq x \leq 2\pi)$.

(a) The graph of f is the graph of $y = |x|$ shifted 2 units to the left.



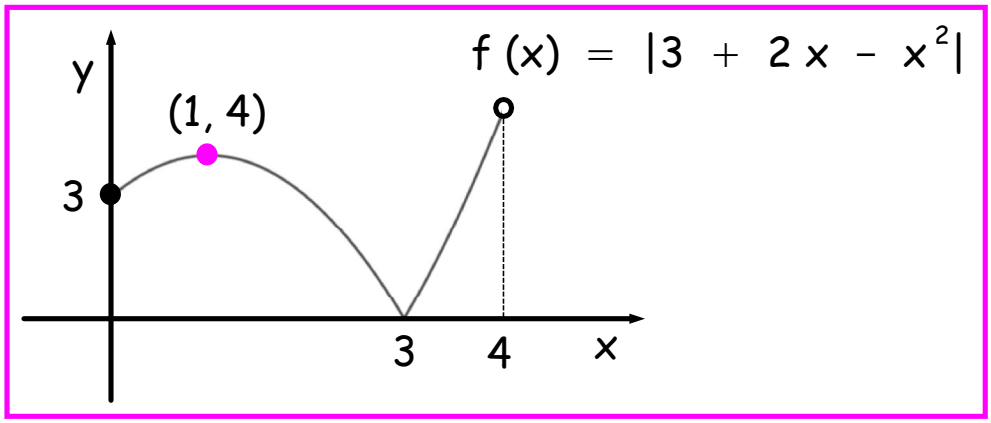
(b)



Example 2

Sketch the graph of $f(x) = |3 + 2x - x^2|$ for $x \in [0, 4)$ and identify its extrema.

Consider $Q(x) = 3 + 2x - x^2 = (3 - x)(1 + x)$. This allows us to sketch the graph of Q and hence $f\ (= |Q|)$. Q has a stationary point at $(1, 4)$. The graph of f is :



Critical points occur at $(1, 4)$ (it's a stationary point), $(0, 3)$ and $(3, 0)$ ($f'(0)$ and $f'(3)$ don't exist). $(4, 5)$ is not a critical point of f as 4 is not in the domain of f.

$(1, 4)$:	local maximum
$(0, 3)$:	(local) endpoint minimum
$(3, 0)$:	global minimum

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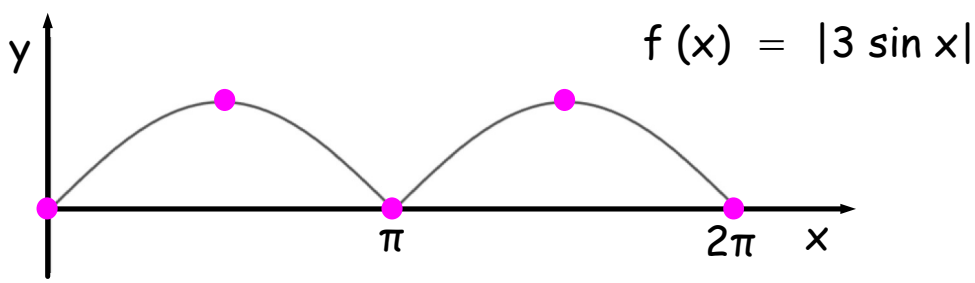
Example 3

Classify the extrema of $f(x) = |3 \sin x|$ for $x \in [0, 2\pi]$.



$(0, 0), (\pi, 0), (2\pi, 0)$: global minima

$(\pi/2, 3), (3\pi/2, 3)$: global maxima

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AH Maths - MiA (2nd Edn.)

- pg. 66 Ex. 5.2 All Q.
- pg. 71 Ex. 5.6 Q 1 a, b, e, 2 a, 3 a, b, c.

Feb 10-15:47

Ex. 5.2

For each of these functions sketch the graphs of $f(x)$ and $|f(x)|$.

1 $f(x) = x - 1$

2 $f(x) = 2x - 1$

3 $f(x) = \cos x°, 0 \leq x \leq 360$

4 $f(x) = x^2 - 1$

5 $f(x) = 1 - x^2$

6 $f(x) = x^2$

7 $f(x) = x^3$

8 $f(x) = \ln x, x > 0$

9 $f(x) = \tan x°, 0 \leq x \leq 360$

Ex. 5.6

1 Consider the nature of the end-points in these functions.

a $f(x) = x^2$ domain $[-1, 2]$ that is $-1 \leqslant x \leqslant 2$

b $f(x) = 4 - x^2$ domain $[-1, 1]$

e $f(x) = |2x|$ domain $[-2, 3)$ that is $-2 \leqslant x < 3$

2 Identify the critical points in these functions.

a $f(x) = x^2 + 2x + 1$ domain $[-2, 3]$

3 i Find the critical points.

ii Identify end-points and local extrema.

iii Hence identify the global maxima and minima where they exist.

a $f(x) = x^2 - 4x - 5$ domain $[-2, 4]$

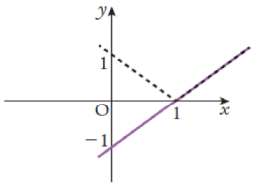
b $f(x) = |\cos x|$ domain $\left[\frac{\pi}{4}, \frac{2\pi}{3}\right]$

c $f(x) = |\ln x|$ domain $(0, e]$

Answers to AH Maths (MiA), pg. 66, Ex. 5.2

In each example the solid line is $f(x)$ and the broken line $|f(x)|$

1



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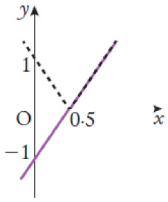

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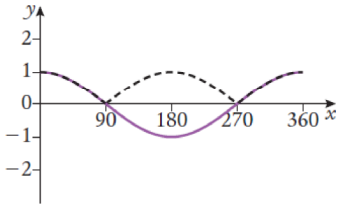

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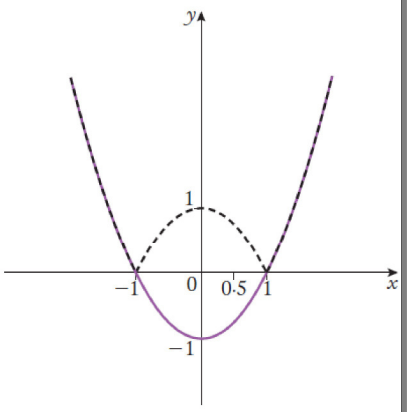

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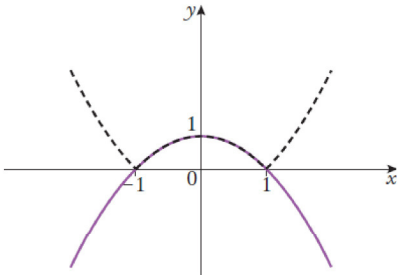

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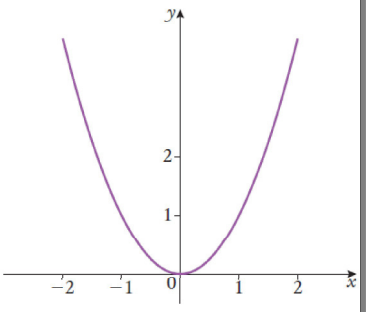

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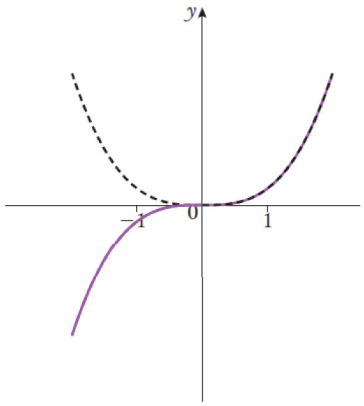

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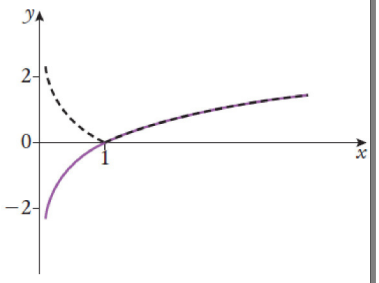

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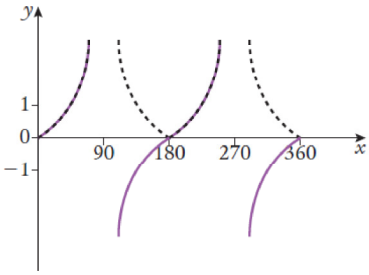

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Answers to AH Maths (MiA), pg. 71, Ex. 5.6

1 a $(-1, 1)$ end point max; $(2, 4)$ end point max.

b $(-1, 3)$ end point min; $(1, 3)$ end point min.

e $(-2, 4)$ end point max; $(3, 6)$ 3 not in domain.

2 a $(-1, 0)$ local min; $(3, 16)$ end point max; $(-2, 1)$ end point max.

3 a i ii $(-2, 7)$ end pt max; $(2, -9)$ local min; $(4, -5)$ end pt max.

iii $(2, -9)$ global min; $(-2, 7)$ global max.

b i ii $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ end pt max; $\left(\frac{\pi}{2}, 0\right)$ local min; $\left(\frac{2\pi}{3}, \frac{1}{2}\right)$ end pt max.

i ii $\left(\frac{\pi}{2}, 0\right)$ global min; $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ global max.

c i ii $(1, 0)$ local min; $(e, 1)$ local max

iii $(1, 0)$ global min; no global max.