

Mathematics 2
Intermediate 2
Statistics
Supplement

4390

Autumn 1998

HIGHER STILL

Mathematics

Mathematics 2

Intermediate 2

Statistics

Supplement

Support Materials



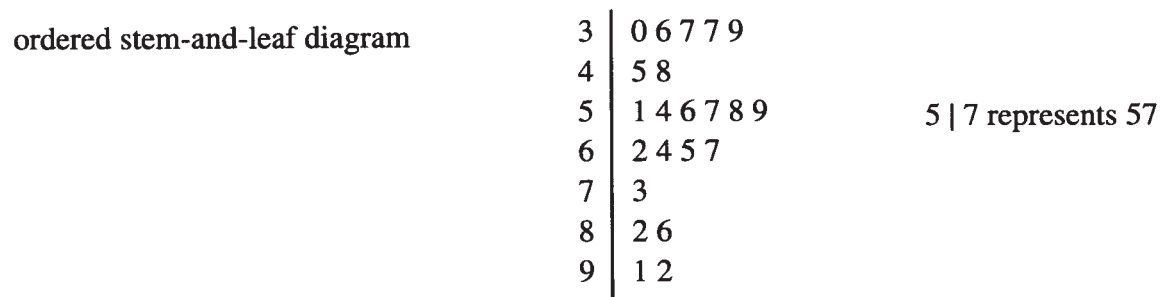
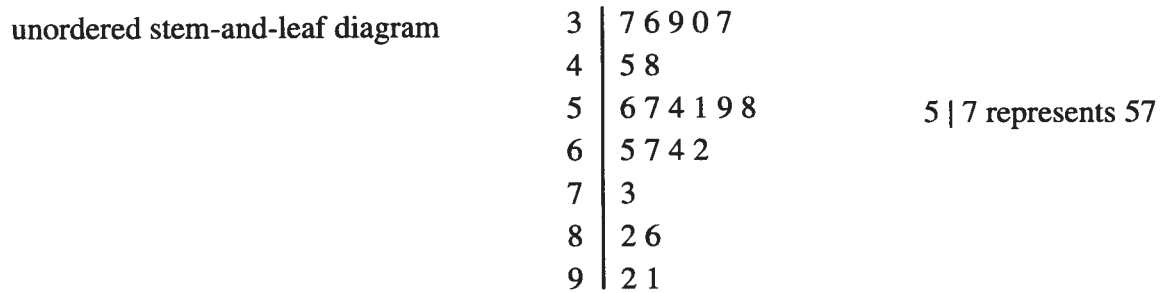
IMPORTANT

The attached pages should be used to replace the equivalent pages in the document Mathematics 2 (Int 2) Statistics which was distributed to centres in August 1998.

Stem-and-leaf diagrams

It is probable that students will not have met this type of graph previously. Stem-and-leaf diagrams can be introduced by means of the following example.

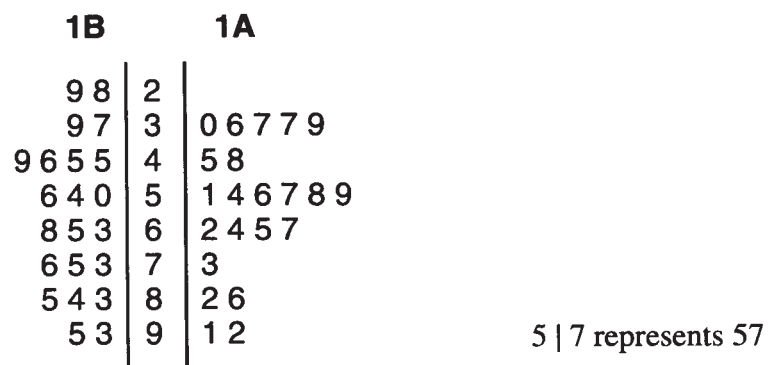
Class 1A's Test Results 56, 45, 37, 57, 82, 92, 36, 39, 54, 65, 67,
 30, 48, 51, 59, 73, 86, 91, 58, 64, 37, 62



A second class's scores can be brought in to illustrate a back to back stem-and-leaf diagram.

Class 1B's Test Results 65, 54, 73, 75, 29, 39, 63, 93, 45, 56, 76,
 50, 84, 45, 95, 37, 68, 49, 85, 46, 83, 28

Back to back stem-and-leaf diagrams can be used to compare two sets of data.



Exercise 6 may now be attempted.

Construction of Graphs

Constructing of graphs should not require a great deal of explanation.

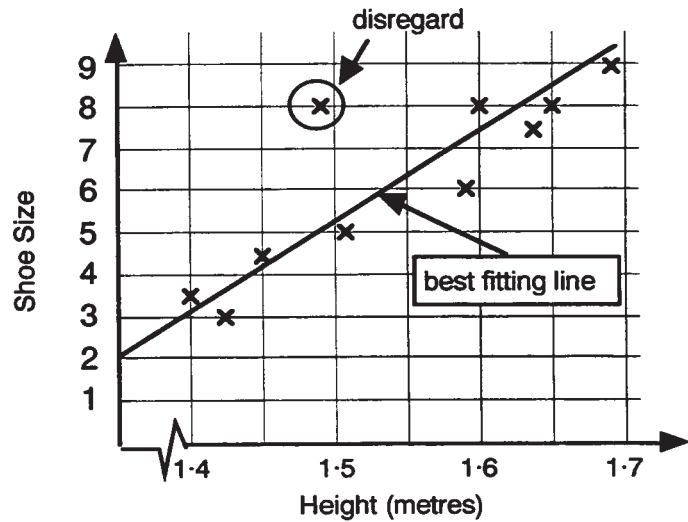
Construction of the line of best fit (or best fitting line) may be a new topic for many students. This can be introduced using the earlier example connecting shoe size to height to introduce the idea of correlation. (The equation of this line of regression will be dealt with in part B.)

Note that at both Intermediate 1 and 2 levels all examples should have 'strong' correlation either positive or negative.

A discussion should take place on how to find the 'best' fitting line.

This line can now be used to estimate the shoe size of someone who is 1.55 metres tall.

(Ans.= 6)



Exercise 7 can now be attempted.

Frequency tables

Students will probably have met frequency tables previously.

Frequency tables are used for organising data and tally marks are used to assist in the calculation of the frequency. Tally marks should be used to show 5 = |||| , not just five 'strokes'.

Example: A group of 50 year old men were asked, 'How many cars have you owned in your life' ?

The answers were:

2 5 4 3 4 2 3
 4 8 2 5 2 4 2
 3 4 6 3 5 6 4
 4 6 3 4 5 3 4

Construct a frequency table to show the results of the survey.

Solution:

Cars owned	Tally Marks	Frequency
2	 	5
3	 	6
4	 	9
5	 	4
6	 	3
7	0	0
8		1

Exercise 8 can now be attempted.

The Standard Deviation

Three ways of measuring the ‘measure of central tendency’ have been considered, i.e. the ‘middle’ – the **mean**, the **median** and the **mode**.

The **range** and **semi-interquartile range (SIQR)** provide a ‘measure of spread’, but both these measures of spread have drawbacks.

- The range will always include extreme measures and may not indicate situations where the majority of the measures are closely packed around the middle.
- SIQR completely ignores extreme measures and concentrates solely on the bulk of the numbers in the middle set of scores.

A third and far more accurate ‘measure of spread’ is the **standard deviation (s.d.)**. It takes into account where the bulk of the numbers lie, but it does not neglect the extremities.

At Intermediate 2 level, all examples which students will meet in assessment will be assumed to be samples. Therefore, the formula for the standard deviation is:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$\text{where } \bar{x} = \frac{\sum x}{n}$$

Note:

- point out clearly the \bar{x} notation
- define $\sum x$ as the ‘sum of all the x ’s’.

The definition for the **s.d.** could simply be given, but the teacher/lecturer may wish to go through the ‘logical steps’ to show where it arises from using this example:

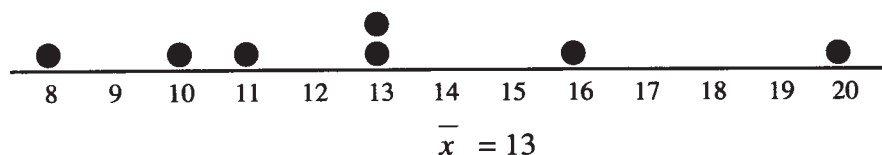
Example. Find the **standard deviation** of the following sample of ages:

8, 10, 11, 13, 13, 16, 20

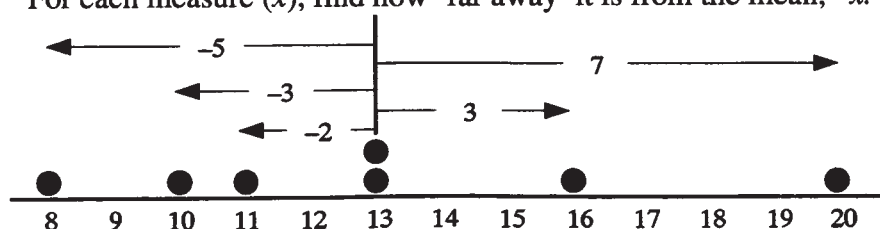
Step 1 Calculate the **mean**

$$\Rightarrow \bar{x} = \frac{\sum x}{n} = \frac{8 + 10 + 11 + 13 + 13 + 16 + 20}{7} = 13$$

Step 2 Look at the spread of the numbers using a **dot plot**.



Step 3 For each measure (x), find how ‘far away’ it is from the mean, \bar{x} .



Step 4 Introduce the table to simplify this.

(differences)

Score (x)	$(x - \bar{x})$	$(x - \bar{x})^2$
8	$8 - 13 = -5$	$(-5)^2 = 25$
10	$10 - 13 = -3$	$(-3)^2 = 9$
11	$11 - 13 = -2$	$(-2)^2 = 4$
13	$13 - 13 = 0$	$(0)^2 = 0$
13	$13 - 13 = 0$	$(0)^2 = 0$
16	$16 - 13 = 3$	$(3)^2 = 9$
20	$20 - 13 = 7$	$(7)^2 = 49$
		$\Sigma(x - \bar{x})^2 = 96$

Explain the reason for the 'squaring' is to eliminate the negative signs.
i.e. The sum of the squares of the differences (deviations from the mean) is **always** positive.

Step 5 The 'average' of the square of these differences is now found by dividing by 6.

$$\Rightarrow \frac{\Sigma(x - \bar{x})^2}{n-1} = \frac{96}{6} = 16$$

Step 6 These differences were squared to eliminate the negative signs.

Therefore to find the **standard deviation** the square root of the answer must be taken.

$$\Rightarrow \text{standard deviation} = \sqrt{\frac{\Sigma(x - \bar{x})^2}{(n-1)}} = \sqrt{16} = 4$$

This is a more accurate 'measure of spread' since it concentrates on where the large bulk of measures lie but it also takes into account the extremes.

Exercise 5 could now be attempted.

After exercise 5, students can be shown how to use the statistical functions on a scientific calculator and can then check their answers quickly to the questions in exercise 5.

The following notes explain how to do so using a **Casio fx - 82SX** but the steps are similar on most calculators.

CASIO fx - 82SX

Step 1 Select the STATISTICS mode by pressing **MODE** **.**

Step 2 Press **SHIFT** **SAC** to clear all memories.

Step 3 Enter the data as follows (use the example on the previous page)

8 **M+** 10 **M+** 11 **M+** 13 **M+**
13 **M+** 16 **M+** 20 **M+**

Step 4 To display the summary of your data, use the **SHIFT** button as follows:

SHIFT 6 gives **n** (the number of measures) = 7

SHIFT 5 gives $\sum x$ (the sum of the x's) = 91

SHIFT 7 gives \bar{x} (the mean) = 13

SHIFT 9 gives **s.d.** (the standard deviation) = 4

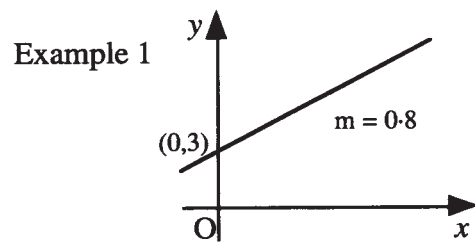
Lines of Regression

Exercise 6 is used as a revision covering gradients and equations of lines. Revise that (every) line can be written in the form

$$y = mx + c \quad (\text{rather than } y = ax + b)$$

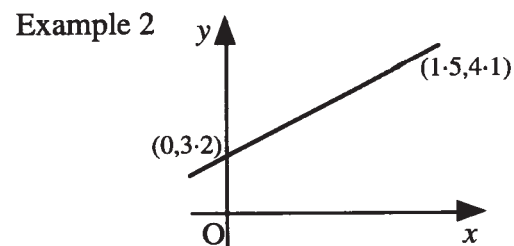
including how to find the gradient. (either by using the gradient formula or simply by counting across and up)

Three examples:



$$y = mx + c$$

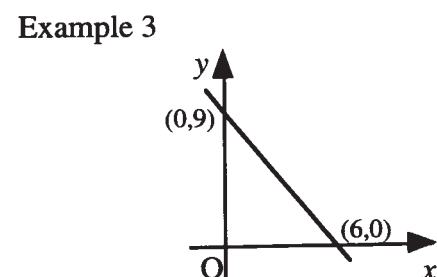
$$y = 0.8x + 3$$



$$m = \frac{\text{vertical}}{\text{horizontal}} = \frac{4.1 - 3.2}{1.5 - 0} = 0.6$$

$$y = mx + c$$

$$y = 0.6x + 3.2$$



$$m = \frac{\text{vertical}}{\text{horizontal}} = \frac{0 - 9}{6 - 0} = -1.5$$

$$y = mx + c$$

$$y = -1.5x + 9$$

Exercise 6 could now be attempted.

Line of best fit

Students can now be shown how to draw a **line of best fit** (by eye!) through a series of points when there is an obvious connection (or **correlation**) between the two variables.

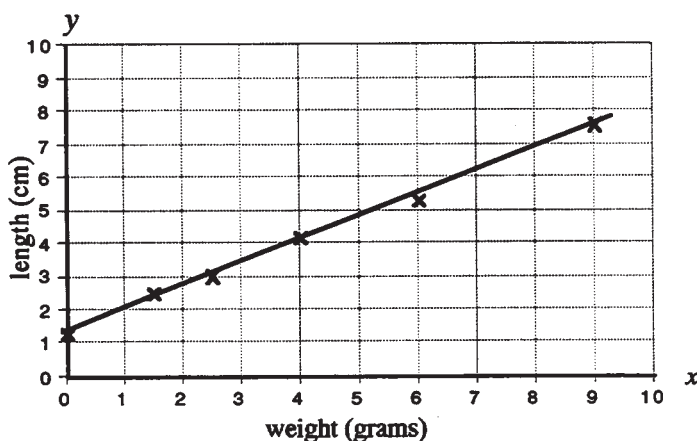
Question 1 (a) and (b) from Exercise 7 can be used as an introduction or the following example can be used.

Weights are placed on the end of a spring and the extension measured. The table shows the results.

Weights (grams) x	0	1.5	2.5	4	6	9
Length of Spring (cm) y	1.3	2.5	3.0	4.2	5.3	7.6

- Plot the points using a scale 1 cm = 1 unit on both axes.
- Draw the best fitting line (the line of regression)
- Use it to estimate the length of the spring when a 5 gram weight is attached.

Solution:



$$\begin{aligned}\text{gradient} &= \frac{7.6 - 1.3}{9 - 0} \\ &= 0.7 \text{ (approx.)}\end{aligned}$$

$$\text{y-intercept} = 1.3 \text{ (approx.)}$$

$$\text{equation} \Rightarrow y = 0.7x + 1.3$$

$$\begin{aligned}\text{when } x &= 5 \\ \Rightarrow y &= 0.7 \times 5 + 1.3 \\ \Rightarrow y &= 4.8 \text{ cm}\end{aligned}$$

Exercise 7 may now be attempted.

'Simple' Probability

The following points should be included in a discussion on 'simple' probability.

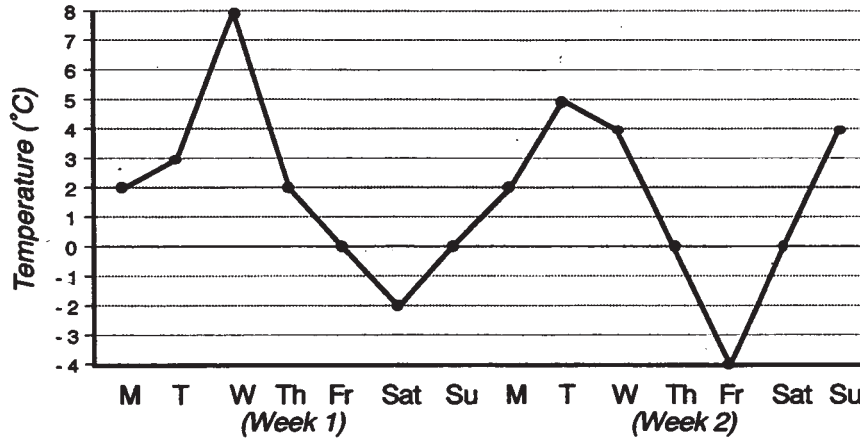
- A 'cert.' is 1 or 100%
- A 'no chance' is 0
- $P = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$
- Answers can be expressed as fractions, decimals or percentages.

Students should be encouraged to leave their answers in simplest form.

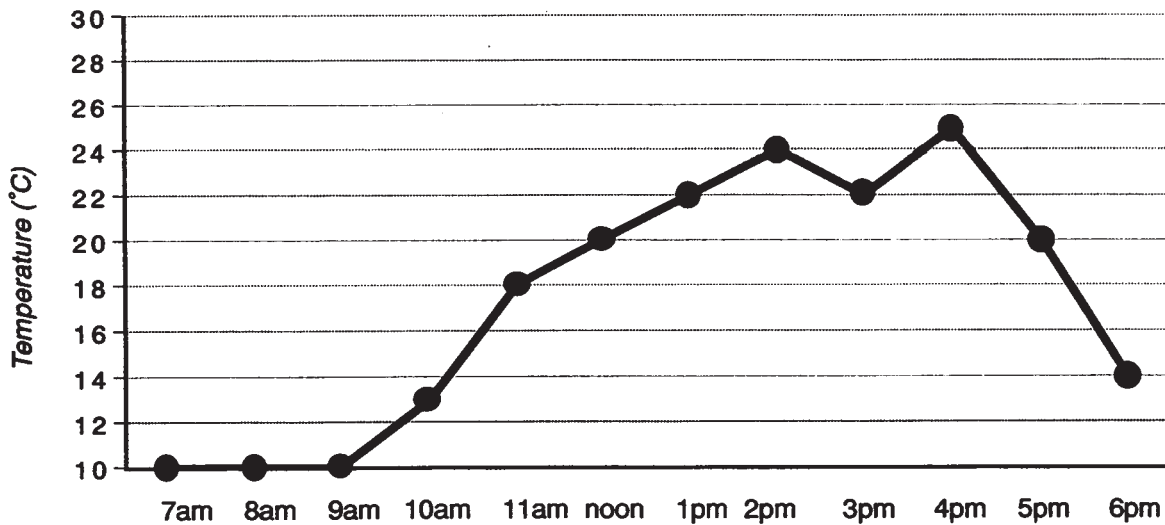
The Line Graph

Exercise 2

1. The temperature on top of a town hall was taken every morning at 1100. This line graph was made up from the recordings.

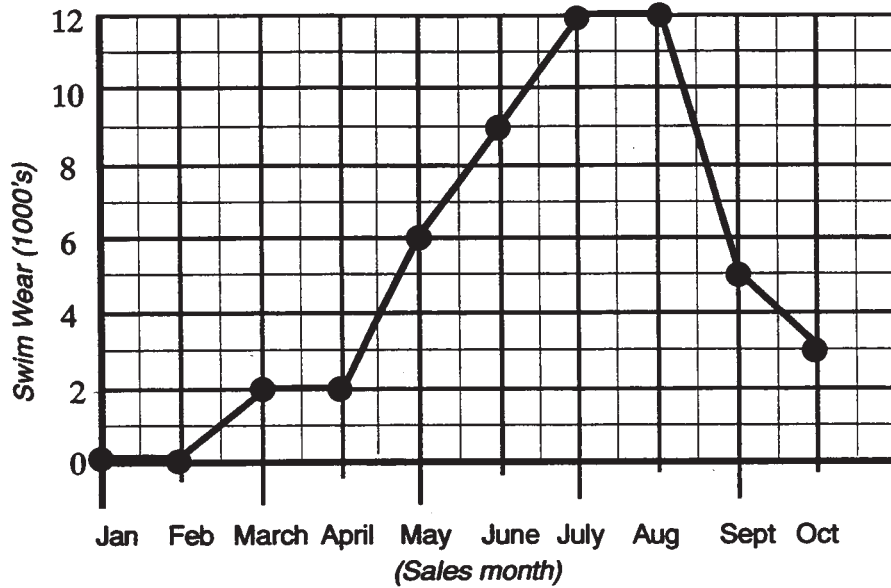


- What was the temperature on:
 - the Wednesday of week 1
 - the Friday of week 2?
- What was the fall in temperature from Tuesday, week 2 – until Friday, week 2?
- Between which two days was there the biggest rise in temperature?
- What was the trend of the graph from Wednesday to Saturday of the first week?



2. Someone was also recording the temperature of the rooms inside the town hall every hour from when she arrived at 7 a.m. until 6 p.m.
- When do you think that the heating was switched on?
 - What was the highest temperature?
 - When was this?
 - Suggest a reason for the sudden dip from 2 p.m. till 3 p.m.
 - Between which times did the temperature rise the most slowly?
 - What was the longest time that the temperature kept rising ?
 - When was the heating officially switched off?

3. The line graph shows the number of swimming trunks, bikinis etc. bought in a superstore during the months of January to October in a normal year.

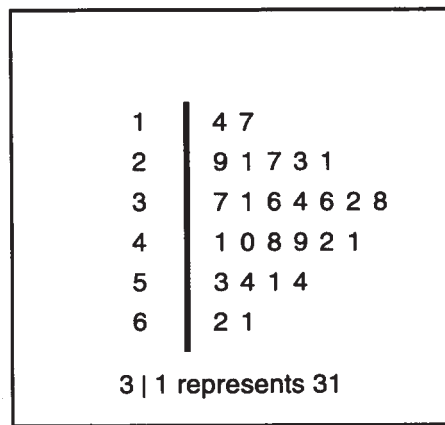


- Why are sales so low during January?
- When do the sales remain the same? (Three answers required.)
- During which month do the sales rise the most?
- When is the most dramatic fall in sales and by how much?
- When are sales at a peak? Why is this?
- Although sales are not great in September – why do you think that swim wear is still being bought?

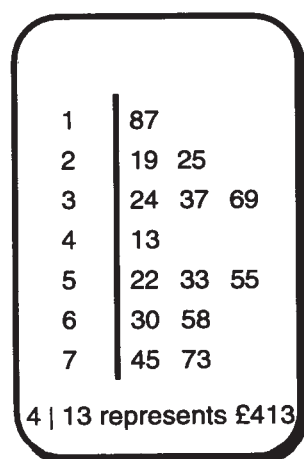
The Stem-and-Leaf Diagram

Exercise 6

1. The stem-and-leaf diagram shows the number of cases of dog food sold at a supermarket each week over a 6 month period.
Write out the weekly sales, in order, and find how many weeks the sales were more than the target of 35.



2.



This 'two digit' stem-and-leaf diagram shows the daily takings over a fortnight, (in pence) for Grant's Sweet Shop.

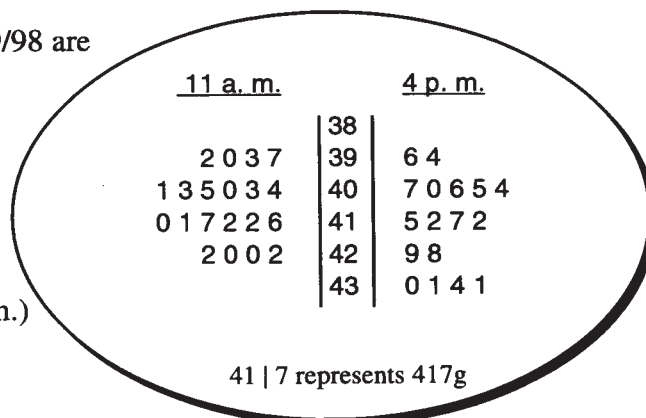
Write out (in order) the fourteen daily takings.

3. A machine weighs out coffee and puts it into packets. The machine is checked over at 11 a.m. and again at 4 p.m. to see if its output is satisfactory.

The results of the inspection on 15/9/98 are shown below in a back-to-back stem-and-leaf chart.

The weights are in grams.

A sample of 20 packets was weighed at both times.



- (a) At which time (11 a.m. or 4 p.m.) did the machine produce:
(i) 411 grams?
(ii) 434 grams?
- (b) Of the 20 packets sampled at 11 a.m., how many weighed between 391 and 406 grams?

The Construction of Graphs

Exercise 7

1. The Park Sports Centre began its winter programme with the following number of under 16s enrolling on the opening night.

	<i>Swimming</i>	<i>Fencing</i>	<i>Football</i>	<i>Aerobics</i>	<i>Keep Fit</i>
Girls	120	108	9	62	19
Boys	85	32	105	22	5

Draw a compound BAR CHART to illustrate this information.

2. The following table shows the weekly wages of young, part-time workers in Orkam Superstore.

Wage in £'s	10–19	20–29	30–39	40–49	50–59	60–69	70–79
No. Workers	3	7	23	29	38	14	1

Draw a BAR CHART of the distribution.

3. The height of a sun flower was measured each week for 8 weeks. Here are the results:

Week Number	1	2	3	4	5	6	7	8
Height (cm)	5	8	20	30	45	59	65	89

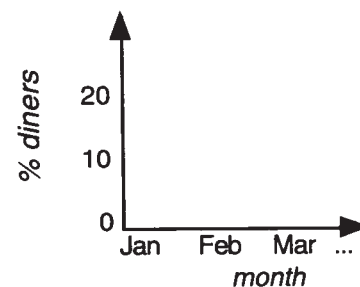
Show this information in a LINE GRAPH.

4. A sample of school children was looked at in 1997 for 6 months, regarding their eating habits in school at lunchtime. A comparison was made between those who had school lunches and those who brought their own food.

Month	Jan	Feb	March	April	May	June
% school lunch	75	70	60	60	30	5
% own food	10	10	30	35	40	65

The results are shown in the table:

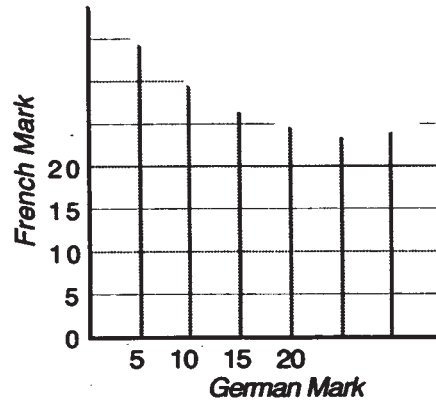
- (a) Use the scale shown to draw LINE GRAPHS of the data on the same diagram.
- (b) Give at least **three** comments on the results of the survey.



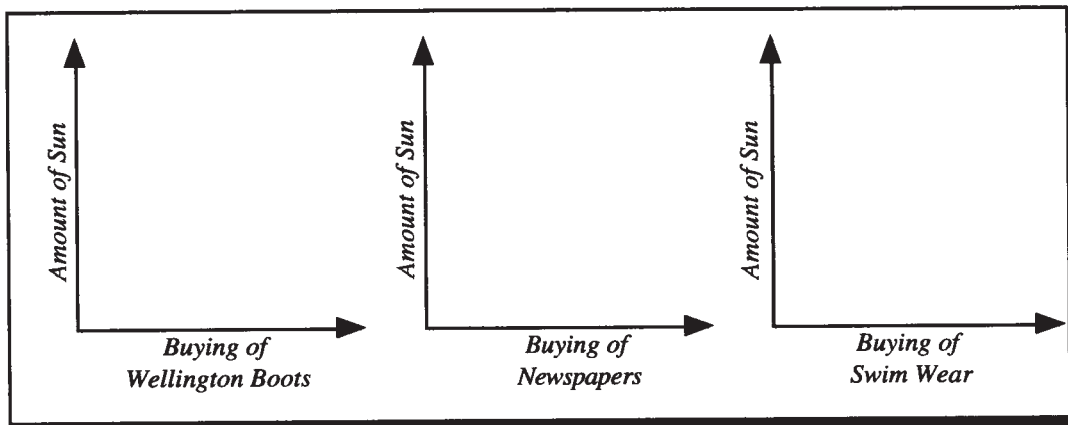
5. Here is a table of exam marks (out of 50) from a modern languages department.

<i>Pupils Name</i>	Ali	Bo	Ed	Dan	Flo	Hal	Nan	Pen	Rab	Sid
<i>French Mark</i>	5	10	10	20	15	30	30	40	40	45
<i>German Mark</i>	5	5	10	15	20	25	30	35	35	45

- Draw a diagram similar to the one shown, using the same scale for each axis.
- Plot the information from the table to make a SCATTER DIAGRAM.
- Describe some connections between the French marks and the German marks.
- Draw a line of best fit through points.
- Use the graph to estimate what the German mark would be if French was 35.
- One person seems to go against the trend. Who is it? . . . and what makes you think that?



6. Copy these axes and draw SCATTER DIAGRAMS which would show the connections between the items shown. (about 10 to 15 crosses would do).



7. Brad and his pals record the number of take-away meals they deliver each evening, and the time it takes them.

Time (mins)	25	22	20	20	15	13	9	15	17	14
No. Meals	40	33	30	32	22	20	13	20	21	19

The average time is 17 minutes and the average number of take-away meals is 25.

- Draw up a set of axes on squared paper, with meals on the horizontal axis and time on the vertical axis. Using suitable scales draw a SCATTER DIAGRAM.
- Draw a line of best fit through the point (25,17). (meals = 25, time = 17)
- Estimate the time it would take Brad & Co. to deliver 28 meals.

8. Construct a simple stem-and-leaf diagram to display the following golf scores, shot by the U.S.A. team in The Ryder Cup, Spain in 1997.

66	79	80	78	69	66	68	80	77
78	93	78	67	62	74	64	81	75
60	66	78	77	84	76	72	69	90
63	62	84	75	75	71	65	64	73

9. A factory has two machines for packing flour into 2 kg packets. 40 packets from each machine were weighed, in grams, and the results were as follows.

MACHINE 1

2020	2014	2019	2005	2020	2023	2015	2006	2013	2012
2021	2011	2019	2021	2013	2000	2020	2026	2018	2007
2038	2018	2016	2017	2010	2019	2013	2012	2020	2021
2002	2015	2013	2017	2019	2020	2016	2025	2014	2028

MACHINE 2

2005	2016	2019	2017	2029	2018	2020	2023	2014	2022
2020	2018	2020	2000	2020	2033	2010	2013	2030	2005
2013	2019	2021	2016	2012	2017	1999	2021	2014	2009
1998	2002	2004	2005	2007	2011	2016	2020	2001	2003

Make a back-to-back stem-and-leaf diagram for the figures. Compare the two sets.

10. A teacher wanted to compare the marks of her two first year classes in a test. She had the feeling that one class was a good bit better than the other. Here are the results of the test:

CLASS 1X												
5	7	14	17	19	24	24	25	26	26	26	27	27
27	27	28	29	29	31	31	33	36	38	39	43	
CLASS 1Y												
8	10	11	13	15	17	18	18	18	19	20	22	23
23	24	25	25	26	26	48	49	50	50	50	50	

As the mean (average) mark of each class is almost identical the teacher has to find another method to compare the marks.

- (a) Make a back-to-back stem-and-leaf diagram as shown and complete it.
 (b) If the teacher made the 'pass' mark '25 out of 50'.

Comment on the results with regard to numbers who 'passed' and how many got high marks.

class 1X		class 1Y
	0	
	1	
	2	
	3	
	4	
	5	

Standard Deviation

mean $\bar{x} = \frac{\sum x}{n}$

standard deviation $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$ where n = the number of scores

Exercise 5

1. Follow through this example carefully to find the standard deviation of this sample of scores.

5, 7, 8, 10, 10, 13, 17

- (a) Find the mean \bar{x} of the set of seven numbers.
 (b) Copy and complete the table and find the total of the $(x - \bar{x})^2$ column. (i.e. find $\sum (x - \bar{x})^2$)
 (c) Now calculate the standard deviation, s , of the seven numbers.

Score x	$x - \bar{x}$	$(x - \bar{x})^2$
5	$5 - 10 = -5$	$(-5)^2 = 25$
7	$7 - 10 = -3$	$(-3)^2 = 9$
8		

2. At Halloween, a group of children each counted out the number of apples they received.

13, 20, 16, 24, 19, 16

- (a) Calculate the mean number of apples.
 (b) Draw up a table and find the standard deviation.
3. During one week in January 1997 a man recorded the wind speed, in knots, at noon each day in his back garden.

11, 2, 0, 2, 18, 16, 7

- (a) Find the mean and range.
 (b) Rearrange the above recordings and find the median and semi-interquartile range.
 (c) Calculate the standard deviation.
4. Three groups of Primary 6 children sat the same test.

Blue Group: 12, 13, 14, 14, 14, 14, 15, 16
Red Group: 11, 11, 12, 14, 18, 18
Yellow Group: 8, 8, 9, 9, 14, 19, 19, 20, 20.

- (a) Check that the mean for each of the three groups is exactly the same.
 (b) Use a separate table each time to calculate the standard deviation for each group.
 (c) Comment on the differences in the standard deviations of the groups.

5. A golfer takes a note of his golf scores for each game he plays throughout the golf season.

94, 112, 88, 92, 100, 87, 90, 91, 88, 96, 94, 102, 83, 85, 86

- (a) Calculate the golfer's mean score.
(b) Draw up a table and hence calculate the standard deviation.

6. Bob grows brussel sprouts in his garden. He is hoping to win a prize in the next Campsie Show. He has been advised that if he uses Make-it-grow fertiliser then his sprouts will be bigger than normal.

He weighs each sprout carefully. The weights are in grams.

49, 50, 55, 47, 62, 35, 28, 59, 53, 54, 43, 58, 47, 69, 75, 34

- (a) Calculate the mean weight of the sprouts.
(b) Draw up a table and hence calculate the standard deviation.

Exercise 5

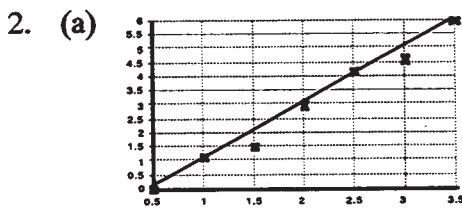
- (a) mean = 10 (b) total = 96 (c) standard deviation = 4
- (a) mean = 18 (b) standard deviation = 3.85
- (a) mean = 8, range = 18 (b) median = 7, SIQR = 7
(c) standard deviation = 7.19
- (a) mean = 14 (b) blue s.d. = 1.20, red s.d. = 3.29, yellow s.d. = 5.52
(c) comments.
- (a) mean = 92.5 (b) standard deviation = 7.61
- (a) mean = 51.125, standard deviation = 12.48

Exercise 6

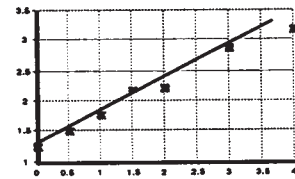
- (a) $y = 3x + 1$ (b) $y = \frac{1}{2}x + 4$ (c) $y = 0.3x + 2.1$ (d) $y = -1.3x + 10$
- (a) $m = 1.5$ ii) $y = 1.5x + 3$ (b) $m = 2$ ii) $y = 2x - 1$
(c) $m = 0.5$ ii) $y = 0.5x + 4$ (d) $m = -1.5$ ii) $y = -1.5x + 8$
- (a) $y = 1.2x + 2.1$ (b) $y = 1.5x + 1.9$ (c) $y = -2.6x + 6.6$

Exercise 7

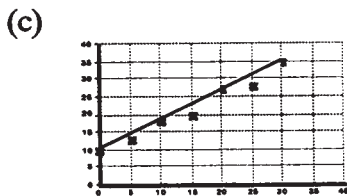
- (a) $y = 0.75x + 0.5$ (b) $y = 1.5x + 1$



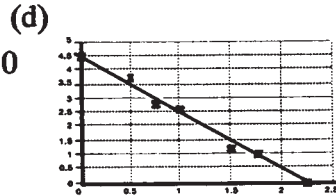
(b) $y = 2x - 1$



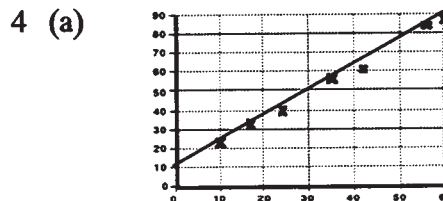
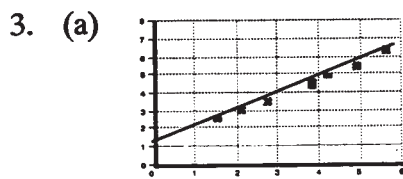
$y = 0.5x + 1.25$



(d) $y = 0.8x + 10$

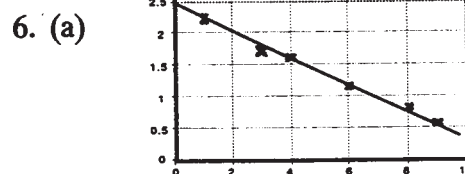
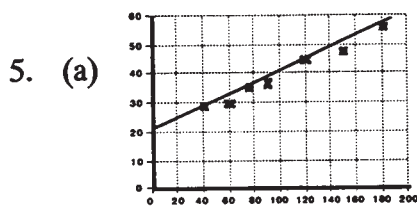


$y = -2x + 4.5$



- (b) $y = 0.9x + 1.1$ (approx)
(c) 2.8

- (c) $y = 1.3x + 10$
(d) 74



- (c) $y = 0.2x + 20$ (d) 53

- (b) $y = -0.2x + 2.4$ (c) 1.4 (d) 12 days

Exercise 8

1. (a) $3/18 = 1/6$ (b) $5/6$
2. (a) $1/6$ (b) $1/2$ (c) $1/3$ (d) $1/2$
3. (a) $8/24$ (b) $4/24$ (c) $2/24$ (d) $6/24$ (e) $4/24$
4. (a) $1/49$ (b) $24/49$ (c) $1/7$ (d) $1/7$
5. 30% (or 3 out of 10)
6. (a) $1/13$ (b) $1/52$ (c) $3/13$ (d) $1/2$ (e) $1/4$
7. (a) $1/5$ (b) $2/5$
8. (a) $7/20$ (b) $1/20$ (c) $8/20 = 2/5$ (d) $3/5$
9. (a) 120 (b) $1/6$ (c) $5/12$ (d) $3/4$
10. (a) $1/8$ (b) $5/16$ (c) $11/16$
11. (a) $7/25$ (b) $2/5$ (c) $17/25$ (d) $17/25$ (e) 0
12. (a) $27/124$ (b) $39/124$ (c) $9/124$
13. (a) (1,1)(1,2)(1,3)(1,4)(1,5)(1,6) (b) 36
 (2,1)(2,2)(2,3)(2,4)(2,5)(2,6)
 (3,1)(3,2)(3,3)(3,4)(3,5)(3,6) (c) i) $1/36$ ii) $1/6$
 (4,1)(4,2)(4,3)(4,4)(4,5)(4,6)
 (5,1)(5,2)(5,3)(5,4)(5,5)(5,6) (iii) $1/6$ iv) $5/18$
 (6,1)(6,2)(6,3)(6,4)(6,5)(6,6)
14. (a) 5560 (b) i) 0.014 ii) 0.0719 iii) 0.719
 (c) There were 10 times as many using Radia as Rainbo
 This showed up when the probability was 10 times that of Rainbo.
15. (a) 0.5 (b) 0.12 (c) 0.07 (d) 0 (e) 0.2
16. (a) 150
 (b) i) 0.007 ii) 0.1 iii) 0.03 iv) 0 v) 0.073

Checkup Exercise 2

1. Range Mean Mode Median
 (a) 13 8 14 8
 (b) £13 £9 none £10
 (c) 4 cm 3 cm 3 cm 3 cm
2. (a) 5 (b) 53 (c) 53 (d) 53.2
3. Median = £6.26, Mean = £6.86. Yes. He was correct!
4. (a) 7.3 (b) 9 out of 10 !
 (c) No, the teacher included herself, knocking the mean out
 (d) Median
5. (a) Mean 4.9, Mode 6, Median 5
 (b) Mode will tell him which size he sells most.
6. (a) i) 6 ii) 4 iii) 5 (b) Mean gives better mark.