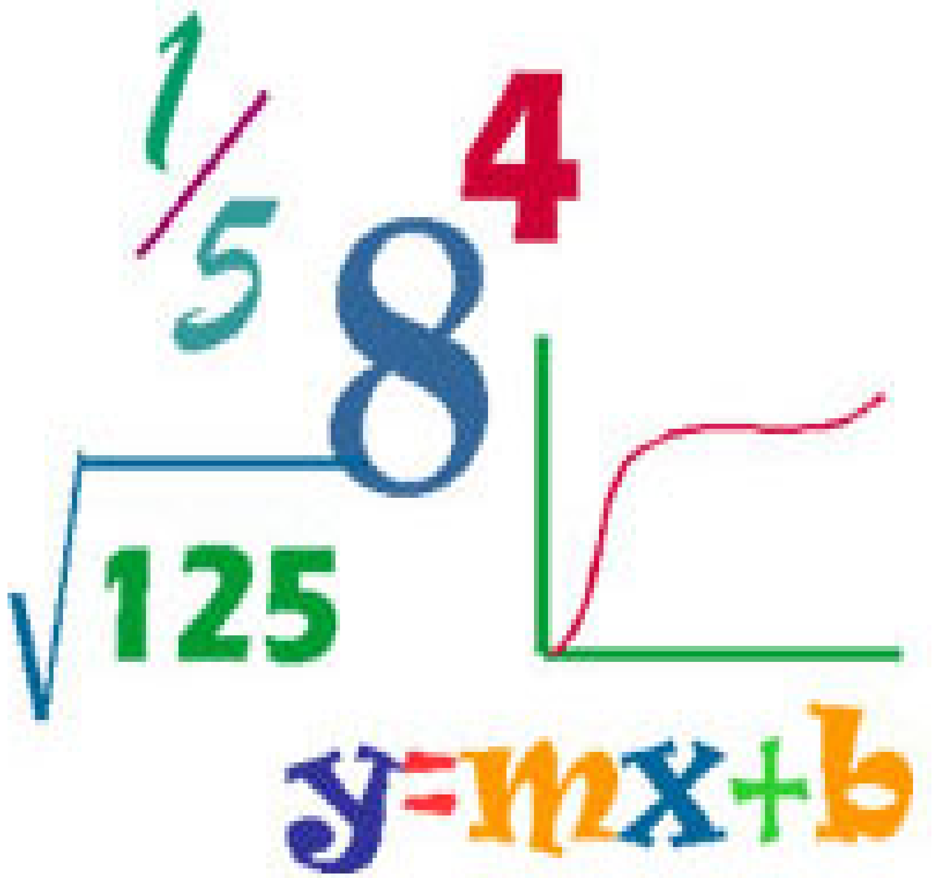
Linwood High

INTERMEDIATE 2 NOTES



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UNIT 1: CALCULATIONS INVOLVING PERCENTAGES

SIMPLE PERCENTAGES

Examples**:**

(1) **simple interest**

Invest £12000 for 8 months at 6% pa
(pa = per annum, per year)

$for\ 1\ year \quad £12000 \div 100 \times 6 = £720$

$for\ 8\ months \quad £720 \div 12 \times 8 = £480$

(2) **VAT**

Radio costs £60 excluding VAT at 20%.
Find the cost inclusive of .

$VAT = £60 \div 100 \times 20 = £12$

$cost = £60 + £12 = £72$

EXPRESSING AS A PERCENTAGE

$$\%\ change = \frac{change}{start} \times 100\%$$

Examples**:**

(1) **profit/loss**

A £15000 car is resold for £12000
Find the percentage loss.

$loss = £15000 - £12000 = £3000$

$\%\ loss = \frac{3000}{15000} \times 100\%$

$= 3000 \div 15000 \times 100\%$

$= 20\%$

(2) % inflation

Shopping costs £125 in 2005, £128 in 2006.
Calculate the rate of inflation.

$increase = £128 - £125 = £3$

$\%\ inflation = \frac{3}{125} \times 100\%$

$= 3 \div 125 \times 100\%$

$= 2 \cdot 4\%$

PERCENTAGE CHANGE

	original value		changed value
INCREASE: growth, appreciation, compound interest	100%	$\xrightarrow{+a\%}$	$(100+a)\%$
DECREASE: decay, depreciation	100%	$\xrightarrow{-a\%}$	$(100-a)\%$

For example,

8% increase: $100\% \xrightarrow{+8\%} 108\% = 1\cdot 08$ multiply quantity by $1\cdot 08$ for 8% increase

8% decrease: $100\% \xrightarrow{-8\%} 92\% = 0\cdot 92$ multiply quantity by $0\cdot 92$ for 8% decrease

Examples:

APPRECIATION AND DEPRECIATION

(1) A £240000 house appreciates in value by 5% in 2007, appreciates 10% in 2008 and depreciates by 15% in 2009. Calculate the value of the house at the end of 2009.

5% increase: $100\% + 5\% = 105\% = 1\cdot 05$

10% increase: $100\% + 10\% = 110\% = 1\cdot 10$

15% decrease: $100\% - 15\% = 085\% = 0\cdot 85$

$£240000 \times 1\cdot 05 \times 1\cdot 10 \times 0\cdot 85$

$= £235620$

or *evaluate year by year*

year 1

$5\% \times £240000 = £12000$

$£240000 + £12000 = £25200$

year 2

10% *of* $£252000 = £25200$

$£252000 + £25200 = £277200$

year 3

15% *of* $£277200 = £41580$

$£277200 - £41580 = £235620$

COMPOUND INTEREST

(2) Calculate the compound interest on £12000 invested at 5% pa for 3 years.

$£12000 \times (1\cdot 05)^3$ *ie.* $\times 1\cdot 05 \times 1\cdot 05 \times 1\cdot 05$ *or evaluate year by year*

$£12000 \times 1\cdot 157625$

$= £13891\cdot 50$

compound interest $= £13891\cdot 50 - £12000 = £1891\cdot 50$

UNIT 1: VOLUMES OF SOLIDS

SIGNIFICANT FIGURES

The number of significant figures indicates the accuracy of a **measurement**.

For example, 3400 centimetres = 34 metres = 0·034 kilometres
same measurement, same accuracy, each 2 significant figures.

significant figures: count the number of figures used, but
do **not** count **zeros** at the **end** of a number **without** a decimal point
do **not** count **zeros** at the **start** of a number **with** a decimal point.

These zeros simply give the place-value[1] of the figures and do not indicate accuracy.

rounding:

For example,

For example,	5713·4	has 5 significant figures
	5700	to 2 significant figures (this case, the nearest Hundred)
	0·057134	has 5 significant figures
	0·057	to 2 significant figures (this case, the nearest Thousandth)

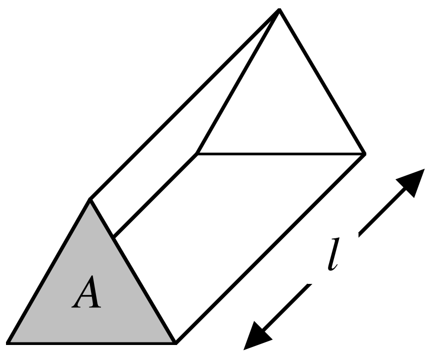
(note 0·057000 would be wrong)

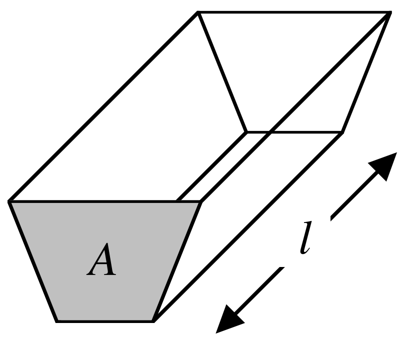
FORMULAE:

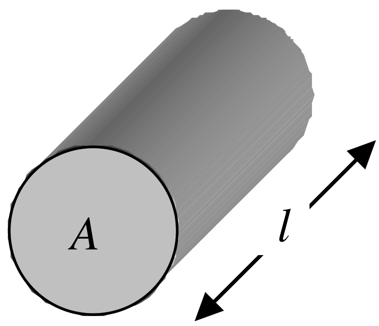
PRISM: a solid with the same cross-section throughout its length.
length l is at right-angles to the area A.

$$V = Al$$

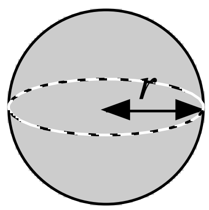
cylinder $V = \pi r^2 h$





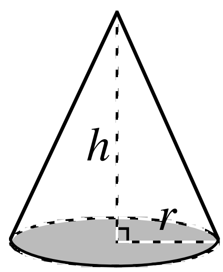


SPHERE



$$V = \frac{4}{3}\pi r^3$$

CONE

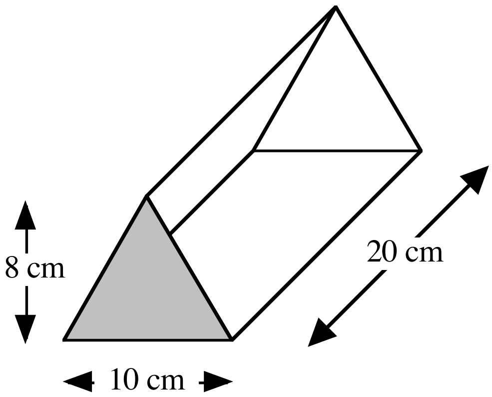


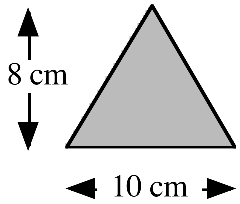
$$V = \frac{1}{3}\pi r^2 h$$

[1]place-value meaning **H**undreds, **T**ens , **U**nits, **t**enths, **h**undredths etc.

Examples:

(1) Calculate the volume.

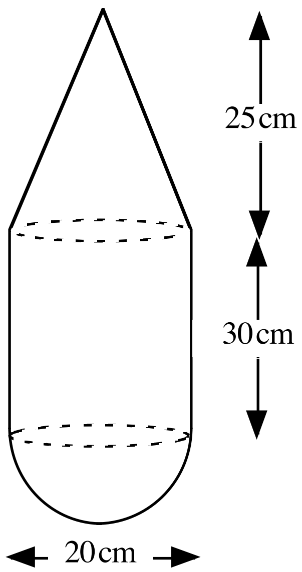




$$A = \tfrac{1}{2}bh$$
$$= 10 \times 8 \div 2$$
$$= 40\ cm^2$$

$$V = Al$$
$$= 40 \times 20$$
$$= 800\ cm^3$$

(2) Calculate the volume correct to **3 significant figures**.



$$radius = 20cm \div 2 = 10cm$$

$$V = \tfrac{1}{3}\pi r^2 h$$
$$= \tfrac{1}{3} \times \pi \times 10 \times 10 \times 25$$
$$= 2617 \cdot 993...\ cm^3$$

$$V = \pi r^2 h$$
$$= \pi \times 10 \times 10 \times 30$$
$$= 9424 \cdot 777...\ cm^3$$

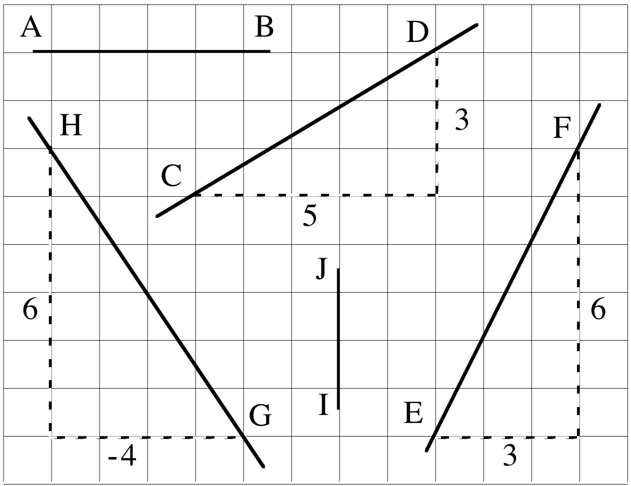
$$V = \tfrac{4}{3}\pi r^3 \div 2$$
$$= \tfrac{4}{3} \times \pi \times 10 \times 10 \times 10 \div 2$$
$$= 2094 \cdot 395...\ cm^3$$

$$total\ area = 2617 \cdot 993... + 9424 \cdot 777... + 2094 \cdot 395...$$
$$= 14137 \cdot 166...$$
$$\approx 14100\ cm^3$$

UNIT 1: LINEAR RELATIONSHIPS

GRADIENT The slope of a line is given by the ratio: $m = \dfrac{\textit{vertical change}}{\textit{horizontal change}}$

For example,



$m_{AB} = 0$ horizontal

$m_{CD} = \dfrac{3}{5}$ positive *m*

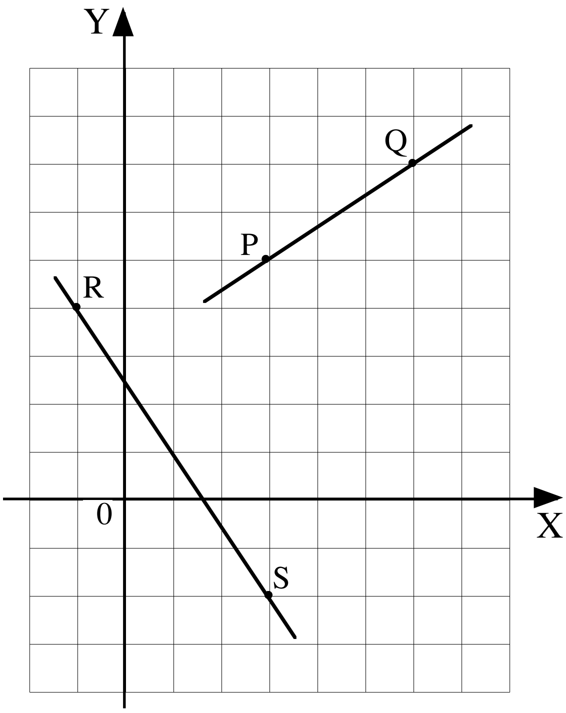
$m_{EF} = \dfrac{6}{3} = 2$

$m_{GH} = \dfrac{6}{-4} = -\dfrac{3}{2}$ negative *m*

m_{IJ} *is undefined* (*or infinite*) vertical

Using coordinates, **the gradient formula** is $m_{AB} = \dfrac{y_B - y_A}{x_B - x_A}$

For example,



$P\,(3,5)$, $Q\,(6,7)$

$$m_{PQ} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{7-5}{6-3} = \frac{2}{3}$$

note: same result for

$$\frac{5-7}{3-6} = \frac{-2}{-3} = \frac{2}{3}$$

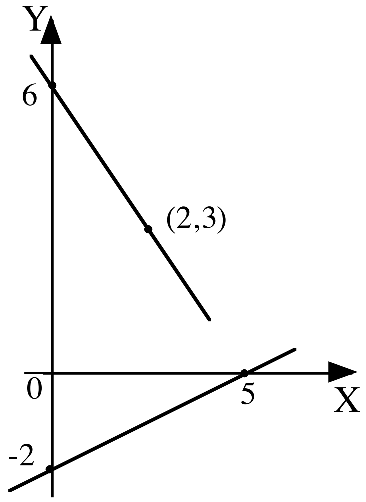
$R\,(-1,4)$, $S\,(3,-2)$

$$m_{RS} = \frac{y_S - y_R}{x_S - x_R} = \frac{-2-4}{3-(-1)} = \frac{-6}{4} = -\frac{3}{2}$$

EQUATION OF A STRAIGHT LINE

gradient m $$y = mx + C$$
y-intercept C units ie. meets the y-axis at $(0,C)$

For example,



$(2,3)$ $m = \frac{3-6}{2-0} = -\frac{3}{2}$ $y = mx + C$

$(0,6)$ $C = 6$ $y = -\frac{3}{2}x + 6$

$(5,0)$ $m = \frac{0-(-2)}{5-0} = \frac{2}{5}$ $y = mx + C$

$(0,-2)$ $C = -2$ $y = \frac{2}{5}x - 2$

Rearrange the equation to $y = mx + C$ for the gradient and y-intercept.

For example,

$3x + 2y - 12 = 0$

$2y = -3x + 12$ *isolate y- term*

$y = -\frac{3}{2}x + 6$ *obtain* $1y =$

$y = \ mx + C$ *compare to the general equation*

$m = -\frac{3}{2}$, $C = 6$, *line meets the y- axis at* $(0,6)$

UNIT 1: ALGEBRAIC OPERATIONS

REMOVING BRACKETS

Examples:

SINGLE BRACKETS

(1) $3x\left(2x - y + 7\right)$

$3x \times 2x = 6x^2$
$3x \times -y = -3xy$
$3x \times +7 = +21x$

$= 6x^2 - 3xy + 21x$

(2) $-2\left(3t + 5\right)$

$-2 \times 3t = -6t$
$-2 \times +5 = -10$

$= -6t - 10$

(3) $-3w\left(w^2 - 4\right)$

$-3w \times w^2 = -3w^3$
$-3w \times -4 = +12w$

$= -3w^3 + 12w$

Fully simplify:

(4) $2t\left(3 - t\right) + 5t^2$

$= 6t - 2t^2 + 5t^2$

$= 6t + 3t^2$

(5) $5 - 3\left(n - 2\right)$

$= 5 - 3n + 6$

$= 5 + 6 - 3n$

$= 11 - 3n$

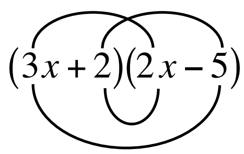
DOUBLE BRACKETS

(1) $\left(3x + 2\right)\left(2x - 5\right)$

$\left(3x + 2\right)\left(2x - 5\right)$
$= 3x\left(2x - 5\right) + 2\left(2x - 5\right)$
$= 6x^2 - 15x + 4x - 10$
$= 6x^2 - 11x - 10$

or

"FOIL"



$= 6x^2 - 15x + 4x - 10$
$= 6x^2 - 11x - 10$

(2) $\left(2t - 3\right)^2$

$= \left(2t - 3\right)\left(2t - 3\right)$
$= 2t\left(2t - 3\right) - 3\left(2t - 3\right)$
$= 4t^2 - 6t - 6t + 9$
$= 4t^2 - 12t + 9$

(3) $\left(w + 2\right)\left(w^2 - 3w + 5\right)$

$= w\left(w^2 - 3w + 5\right) + 2\left(w^2 - 3w + 5\right)$
$= w^3 - 3w^2 + 5w + 2w^2 - 6w + 10$
$= w^3 - 3w^2 + 2w^2 + 5w - 6w + 10$
$= w^3 - w^2 - w + 10$

FACTORSATION

COMMON FACTORS

Factors: divide into a number without a remainder. Factors of a number come in pairs. For example,

$12 = 1 \times 12 = 2 \times 6 = 3 \times 4$ $\quad$ *factors of* 12 *are* 1, 2, 3, 4, 6, 12

$18 = 1 \times 18 = 2 \times 9 = 3 \times 6$ $\quad$ *factors of* 18 *are* 1, 2, 3, 6, 9, 18

$4a = 1 \times 4a = 2 \times 2a = 4 \times a$ $\quad$ *factors of* $4a$ *are* $1, 2, 4, a, 2a, 4a$

$2a^2 = 1 \times 2a^2 = 2 \times a^2 = a \times 2a$ $\quad$ *factors of* $2a^2$ *are* $1, 2, a, 2a, a^2, 2a^2$

Highest Common Factor(HCF): the highest factors numbers share.
For example,
from the above lists of factors: $\quad HCF(12{,}18) = 6 \quad\quad HCF(4a{,}2a^2) = 2a$

Factorisation: HCFs are used to write expressions in **fully** factorised form.

Examples:

Factorise **fully**:

(1) $12x + 18y$

$6 \times 2x + 6 \times 3y$ *using* $HCF(12{,}18) = 6$

$= 6(2x + 3y)$

(2) $4a - 2a^2$

$2a \times 2 - 2a \times a$ *using* $HCF(4a{,}2a^2) = 2a$

$= 2a(2 - a)$

NOTE: the following answers are factorised but not **fully** factorised:

$2(6x + 9y)$ $\quad\quad$ $2(2a - a^2)$

$3(4x + 6y)$ $\quad\quad$ $a(4 - 2a)$

DIFFERENCE OF TWO SQUARES

Rule: $a^2 - b^2 = (a+b)(a-b)$

check: $(a+b)(a-b) = a(a-b) + b(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$

Examples:

Factorise **fully**:

(1) $4x^2 - 9$

$= (2x)^2 - 3^2$
$= (2x+3)(2x-3)$

(2) $t^2 - 1$

$= t^2 - 1^2$
$= (t+1)(t-1)$

(3) $n^4 - 1$

$= (n^2)^2 - 1^2$
$= (n^2+1)(n^2-1)$
$= (n^2+1)(n+1)(n-1)$

common factor first

(4) $8x^2 - 18$

$= 2(4x^2 - 9)$
$= 2(2x+3)(2x-3)$

(5) $t^3 - t$

$= t(t^2 - 1)$
$= t(t+1)(t-1)$

TRINOMIALS $ax^2 + bx + c$, $a = 1$ ie. $1x^2$
(Quadratic Expressions)

$ax^2 + bx + c = (x + ?)(x + ?)$ The missing numbers: are **a pair of factors of** c
sum to b

Examples:

Factorise **fully**:

(1) $x^2 + 5x + 6$

$1 \times 6 = 2 \times 3 = 6$
$2 + 3 = 5$
use $+2$ *and* $+3$

$= (x+2)(x+3)$

(2) $x^2 - 5x + 6$

$-1, -6$ *or* $-2, -3$
$-2 + (-3) = -5$
use -2 *and* -3

$= (x-2)(x-3)$

(3) $x^2 - 5x - 6$

$-1, 6$ or $1, -6$ or $-2, 3$ or $2, -3$
$1 + (-6) = -5$
use $+1$ *and* -6

$= (x+1)(x-6)$

TRINOMIALS $ax^2+bx+c\ ,a\neq 1$

Carry out a procedure which is a reversal of bracket breaking.

Examples:

(1) factorise $2t^2+7t+6$

$2\times 6=12$ *pairs of factors* $\underbrace{1,12\ \ or\ \ 2,6\ \ or\ \ 3,4}_{3+4=7}$

$2t^2\underline{\underline{+\mathbf{7}}}\,\mathbf{t}+6$

$=2t^2+\mathbf{4t}+\mathbf{3t}+6$ *replace* $+7t$ *by* $+4t+3t$ $(or\ +3t+4t)$

$=(2t^2+4t)+(3t+6)$ *bracket first and last pairs of terms*

$=2t\,(\mathbf{t}+\mathbf{2})+3(\mathbf{t}+\mathbf{2})$ *factorise each bracket using HCF*

$=(2t+3)(\mathbf{t}+\mathbf{2})$ *factorise: brackets are common factor*

(2) factorise $2t^2-7t+6$ **Watch!** Take care with **negative signs outside brackets**.

$2\times 6=12$ *pairs of factors* $\underbrace{1,12\ \ or\ \ 2,6\ \ or\ \ 3,4}_{3+4=7}$

$2t^2\underline{\underline{-\mathbf{7}}}\,\mathbf{t}+6$

$=2t^2-\mathbf{4t}-\mathbf{3t}+6$ *replace* $-7t$ *by* $-4t-3t$ $(or\ -3t-4t)$

$=(2t^2-4t)-(3t-6)$ *notice sign change in 2nd bracket,* $+6$ *to* -6

$=2t\,(\mathbf{t}-\mathbf{2})-3(\mathbf{t}-\mathbf{2})$

$=(2t-3)(\mathbf{t}-\mathbf{2})$

(3) factorise $2t^2-11t-6$

$2\times(-6)=-12$ *pairs of factors* $\underbrace{1,12\ \ or\ \ 2,6\ or\ \ 3,4}_{-12+1=-11}$

$2t^2\underline{\underline{-\mathbf{11}}}\,\mathbf{t}-6$ *one factor is negative*

$=2t^2-\mathbf{12t}+\mathbf{1t}-6$ *replace* $-11t$ *by* $-12t+1t$ (**not** $+1t-12t$)

$=(2t^2-12t)+(1t-6)$ *notice no sign change needed in 2nd bracket*

$=2t\,(\mathbf{t}-\mathbf{6})+1\,(\mathbf{t}-\mathbf{6})$ *notice 2nd bracket still requires common factor*

$=(2t+1)(\mathbf{t}-\mathbf{6})$

ALTERNATIVE METHOD:

Try out the possible combinations of the factors which could be in the brackets.

Examples: same quadratic expressions as the previous page.

(1) factorise $2t^2 + 7t + 6$

$2 \times 6 = 12$ *pairs of factors* $\underbrace{1,12 \;\; or \;\; 2,6 \;\; or \;\; 3,4}_{3+4=7}$

$2t^2 \underline{\underline{+\mathbf{7t}}} + 6$ *try combinations so that 3t and 4t are obtained*

factors of $2t^2$*: 2t , t*

factors of 6: 1 , 6 or 2 , 3

$1t$ $12t$

$2t^2 + 7t + 6$ *has no common factor. These have so can be ruled out*

$3t$ $4t$ ✓

$\underline{\underline{(2t+3)(t+2)}}$

(2) factorise $2t^2 - 7t + 6$

exactly as example (1) except -7t requires both negative, so -3 , -2 $\underline{\underline{(2t-3)(t-2)}}$

(3) factorise $2t^2 - 11t - 6$

$2 \times (-6) = -12$ *pairs of factors* $\underbrace{1,12 \;\; or \;\; 2,6 \;\; or \;\; 3,4}_{-12+1=-11}$ *one factor is negative*

$2t^2 \underline{\underline{-\mathbf{11t}}} - 6$ *try combinations so that* $-12t$ *and* $1t$ *are obtained*

$1t$ $-12t$ ✓

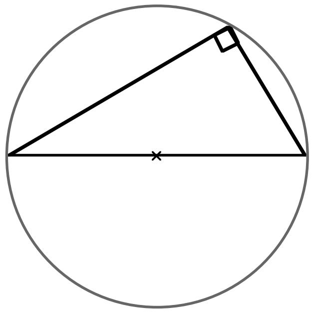
$2t^2 - 11t - 6$ *has no common factor. These have so can be ruled out*

$3t$ $-4t$

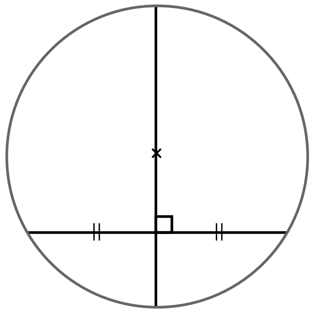
$\underline{\underline{(2t+1)(t-6)}}$

UNIT 1: PROPERTIES OF THE CIRCLE

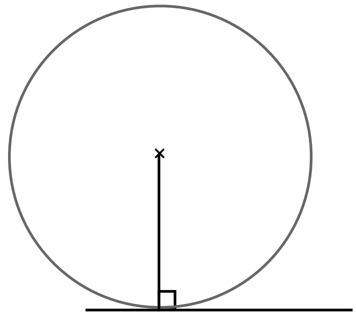
angle in a semicircle is a right-angle.



the perpendicular bisector of a chord is a diameter.



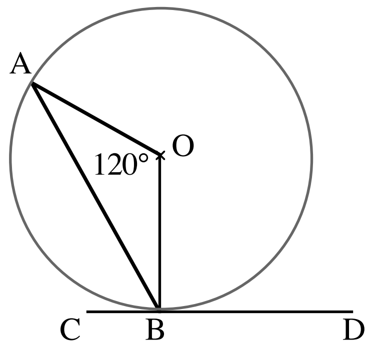
a tangent and the radius drawn to the point of contact form a right-angle.



ANGLES

Examples:

(1)



radius OA = OB so ΔAOB is isosceles and Δ angle sum 180° :

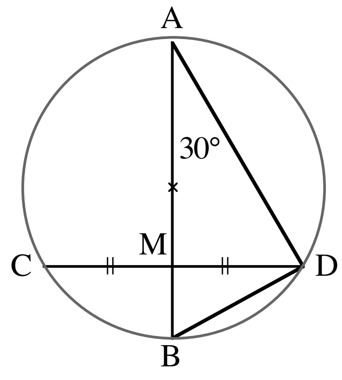
$\angle OBA = (180° - 120°) \div 2 = 30°$

tangent CD and radius OB : $\angle OBC = 90°$

Calculate the size of angle ABC.

$\angle ABC = 90° - 30° = 60°$

(2)



diameter AB bisects chord CD : $\angle AMD = 90°$

ΔAMD angle sum 180° :

$\angle ADM = 180° - 90° - 30° = 60°$

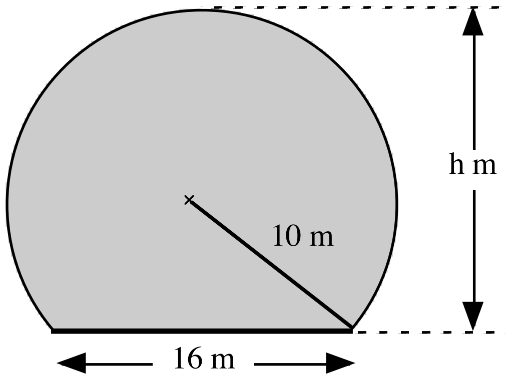
angle in a semicircle : $\angle ADB = 90°$

Calculate the size of angle BDC.

$\angle BDC = 90° - 60° = 30°$

PYTHAGORAS' THEOREM

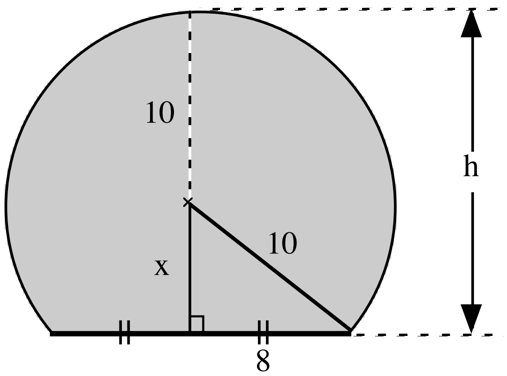
Example:



A circular road tunnel, radius 10 metres, is cut through a hill.

The road has a width 16 metres.

Find the height of the tunnel.



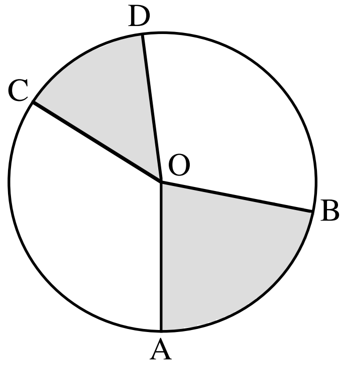
the diameter drawn is the perpendicular bisector of the chord:
Δ is right-angled so can apply Pyth. Thm.

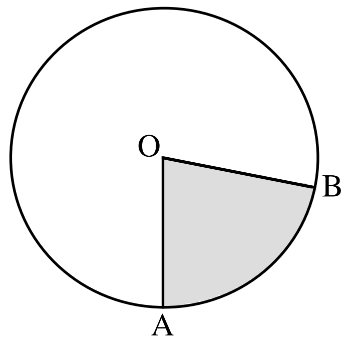
$$\begin{aligned} x^2 &= 10^2 - 8^2 \\ &= 100 - 64 \\ &= 36 \\ x &= \sqrt{36} \\ x &= 6 \end{aligned}$$

$$\begin{aligned} h &= x + 10 \\ &= 6 + 10 \\ h &= 16 \end{aligned}$$

height 16 *metres*

SECTORS



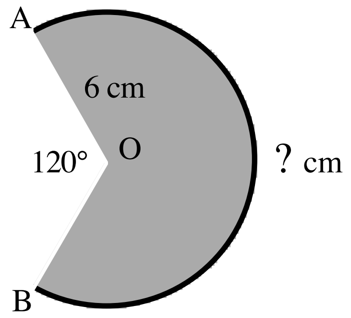


$$\frac{\angle AOB}{\angle COD} = \frac{arc\ AB}{arc\ CD} = \frac{area\ of\ sector\ AOB}{area\ of\ sector\ COD} \qquad \frac{\angle AOB}{360°} = \frac{arc\ AB}{\pi d} = \frac{area\ of\ sector\ AOB}{\pi r^2}$$

Choose the appropriate pair of ratios based on:
(i) the ratio which includes the quantity to be found
(ii) the ratio for which both quantities are known (or can be found).

Examples:

(1) Find the **exact** length of **major** arc AB.



$$\frac{\angle AOB}{360°} = \frac{arc\ AB}{\pi d}$$

$$\frac{240°}{360°} = \frac{arc\ AB}{\pi \times 12}$$

$$arc\ AB = \frac{240°}{360°} \times \pi \times 12$$

$$= 8\pi\ cm \quad (25 \cdot 132...)$$

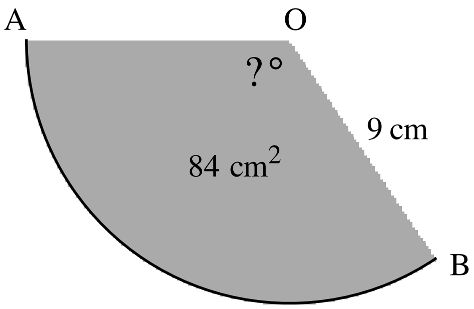
$$diameter\ d = 2 \times 6\ cm = 12\ cm$$

$$reflex\ \angle AOB = 360° - 120° = 240°$$

$$\frac{\angle AOB}{360°} = \frac{arc\ AB}{\pi d} = \frac{area\ of\ sector\ AOB}{\pi r^2}$$

✓ ? ✓ ✓ ✗ ✓

(2) Find the size of angle AOB.



$$\frac{\angle AOB}{360^\circ} = \frac{\text{area of sector } AOB}{\pi r^2}$$

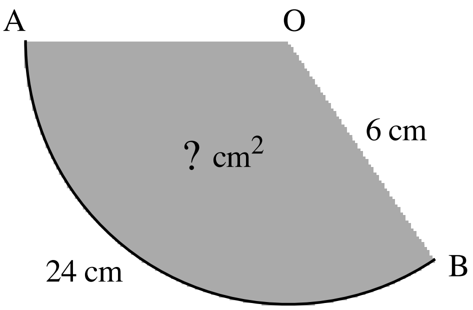
$$\frac{\angle AOB}{360^\circ} = \frac{84}{\pi \times 9 \times 9}$$

$$\angle AOB = \frac{84}{\pi \times 9 \times 9} \times 360^\circ$$

$$= 118 \cdot 835...$$

$$\angle AOB \approx 119^\circ$$

(3) Find the **exact** area of sector AOB.



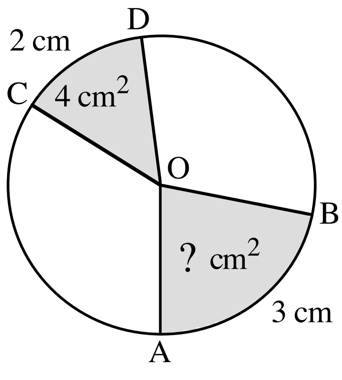
$$\frac{\text{arc } AB}{\pi d} = \frac{\text{area of sector } AOB}{\pi r^2}$$

$$\frac{24}{\pi \times 12} = \frac{\text{area of sector } AOB}{\pi \times 6 \times 6}$$

$$\text{area of sector } AOB = \frac{24}{\pi \times 12} \times \pi \times 6 \times 6$$

$$= 72\ cm^2$$

(4) Find the **exact** area of sector AOB.



$$\frac{\text{arc } AB}{\text{arc } CD} = \frac{\text{area of sector } AOB}{\text{area of sector } COD}$$

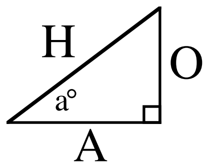
$$\frac{3}{2} = \frac{\text{area of sector } AOB}{4}$$

$$\text{area of sector } AOB = \frac{3}{2} \times 4$$

$$= 6\ cm^2$$

UNIT 2: TRIGONOMETRY

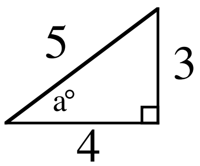
SOH-CAH-TOA The sides of a right-angled triangle are labelled:



Opposite: opposite the angle a°.
Adjacent: next to the angle a°.
Hypotenuse: opposite the right angle.

The ratios of sides $\frac{O}{H}, \frac{A}{H}$ and $\frac{O}{A}$ have values which depend on the size of angle a°.
These are called the sine, cosine and tangents of a°, written sin a° , cos a° and tan a°.

For example,



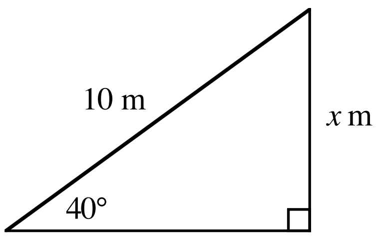
$S = \frac{O}{H}$ $\quad C = \frac{A}{H}$ $\quad T = \frac{O}{A}$

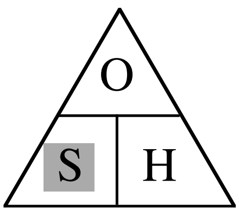
$\sin a^\circ = \frac{3}{5}$ $\quad \cos a^\circ = \frac{4}{5}$ $\quad \tan a^\circ = \frac{3}{4}$

FINDING AN UNKNOWN SIDE

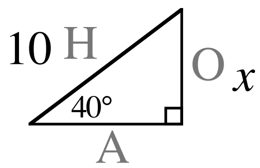
Examples:

(1) Find x.





$S = \frac{O}{H}$



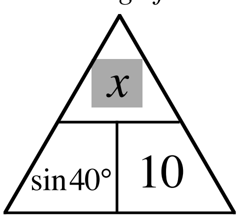
know H , find O

SOH-CAH-TOA

sine ratio uses O and H

$\sin 40^\circ = \frac{x}{10}$

rearrange for x



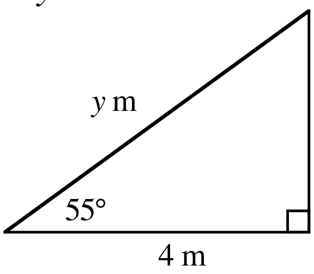
*ensure calculator set to **DEGREES***

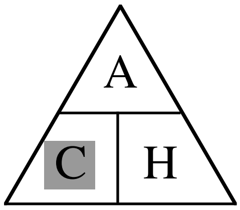
$x = 10 \times \sin 40^\circ$

$= 6 \cdot 427....$

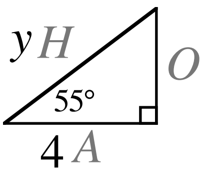
$x = 6 \cdot 4$

(2) Find y.





$$C = \frac{A}{H}$$



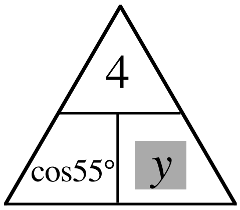
know A , find H

SOH-CAH-TOA

cosine ratio uses A and H

$$\cos 55^\circ = \frac{4}{y}$$

rearrange for y



$$y = \frac{4}{\cos 55^\circ}$$

4 ÷ cos55°, calculator set to DEGREES

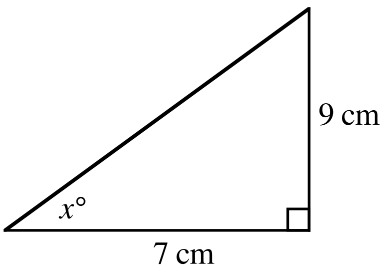
$$= 6 \cdot 973....$$

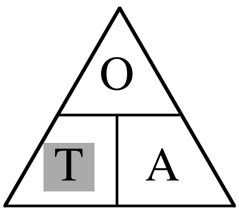
$$y = 7 \cdot 0$$

FINDING AN UNKNOWN ANGLE

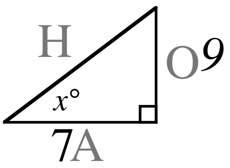
Example:

Find x.





$$T = \frac{O}{A}$$



know O , know A

SOH-CAH-TOA

tangent ratio uses O and A

$$\tan x^\circ = \frac{9}{7}$$

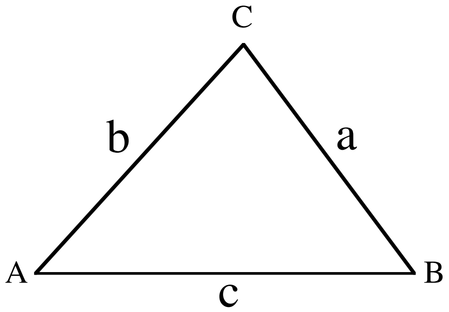
$$x = \tan^{-1}\left(\frac{9}{7}\right)$$

use brackets for (9 ÷ 7), calculator set to DEGREES

$$= 52 \cdot 125....$$

$$x = 52 \cdot 1$$

SINE RULE

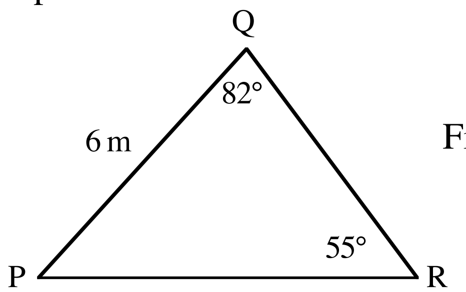


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

NOTE: requires at least one side and its opposite angle to be known.

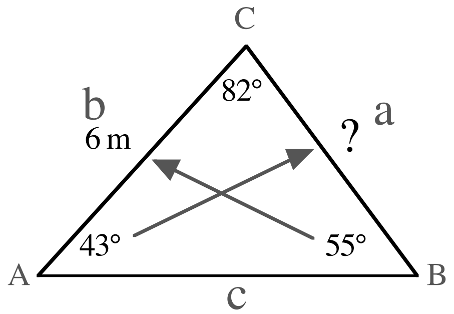
FINDING AN UNKNOWN SIDE

Example:



Find the length of side QR.

relabel triangle with a *as uknown side*
known angle/side pair labelled B and b



$$\frac{a \; ?}{\sin A \; \checkmark} = \frac{b \; \checkmark}{\sin B \; \checkmark} = \frac{c \; \times}{\sin C \; \checkmark}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 43°} = \frac{6}{\sin 55°}$$

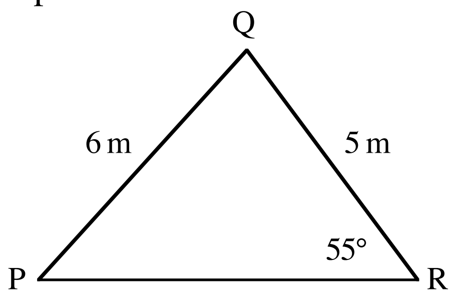
$$a = \frac{6}{\sin 55°} \times \sin 43°$$

$$= 4 \cdot 995.....$$

$$QR \approx 5 \cdot 0 \; m$$

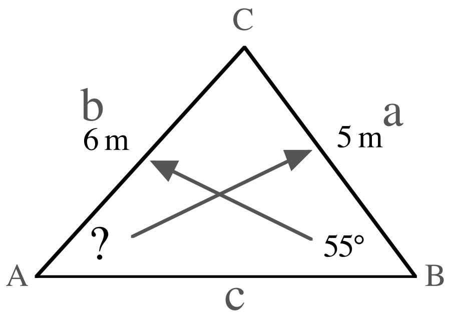
FINDING AN UNKNOWN ANGLE

Example:



Find the size of angle PQR.

cannot find angle PQR *directly but can find angle* QPR *first*
relabel triangle with A *as uknown angle* QPR
known angle/side pair labelled B and b
use the Sine Rule with the angles on the 'top'



$\frac{\sin A}{a}$ (? / ✓) $=$ $\frac{\sin B}{b}$ (✓ / ✓) $=$ $\frac{\sin C}{c}$ (✗ / ✗)

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{5} = \frac{\sin 55^\circ}{6}$$

$$\sin A = \frac{\sin 55^\circ}{6} \times 5$$

$$= 0 \cdot 682.....$$

$$A = \sin^{-1} 0 \cdot 682.....$$

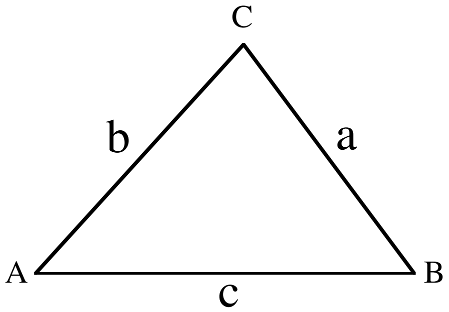
$$\angle QPR = 43 \cdot 049.....$$

$$\angle PQR = 180 - 55 - 43 \cdot 049.....$$

$$= 81 \cdot 950.....$$

$$\angle PQR \approx 82 \cdot 0^\circ$$

COSINE RULE



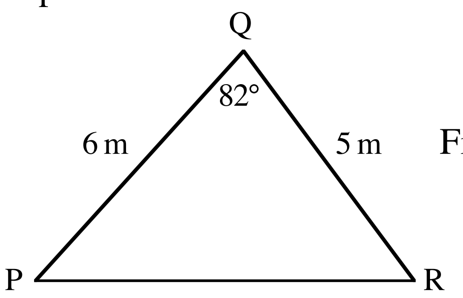
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

FINDING AN UNKNOWN SIDE $a^2 = b^2 + c^2 - 2bc\cos A$

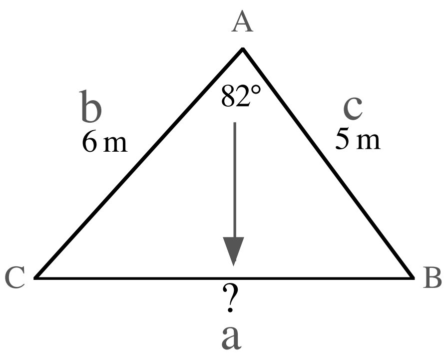
NOTE: requires knowing 2 sides and the angle between them.

Example:



Find the length of side PR.

relabel triangle with a *as uknown side*
known sides labelled b *and* c, *it doesn't matter which one is b or c*



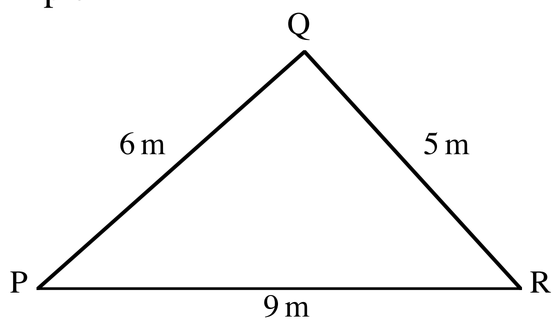
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos 82°$$
$$a^2 = 52 \cdot 649.....$$
$$a = \sqrt{52 \cdot 649.....}$$
$$= 7 \cdot 256.....$$
$$PR = 7 \cdot 3\ m$$

FINDING AN UNKNOWN ANGLE

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

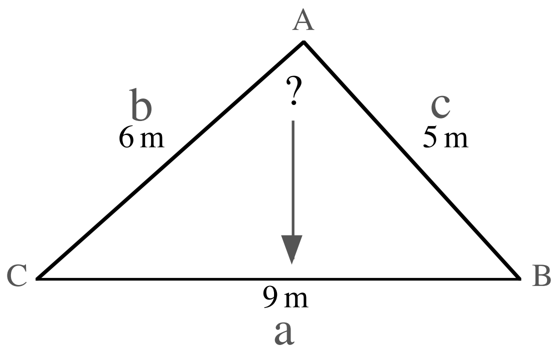
NOTE: requires knowing all 3 sides.

Example:



Find the size of angle PQR.

relabel the triangle with A *as the uknown angle and* a *as its opposite side*
other sides labelled b *and* c, *it doesn't matter which one is b or c*



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{6^2 + 5^2 - 9^2}{2 \times 6 \times 5}$$

$$= \frac{-20}{60}$$

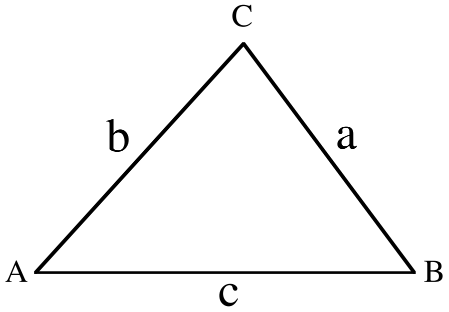
$$\cos A = -0 \cdot 333.....$$

$$A = \cos^{-1}(-0 \cdot 333.....)$$

$$= 109 \cdot 471.....$$

$$\angle PQR = 109 \cdot 5°$$

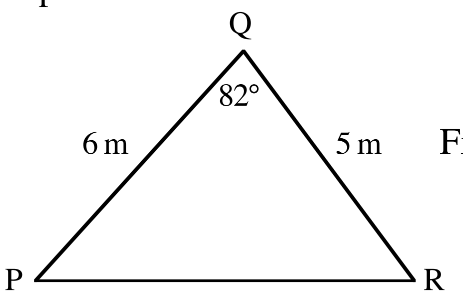
AREA FORMULA



$$Area\ \Delta ABC = \frac{1}{2}bc\sin A$$

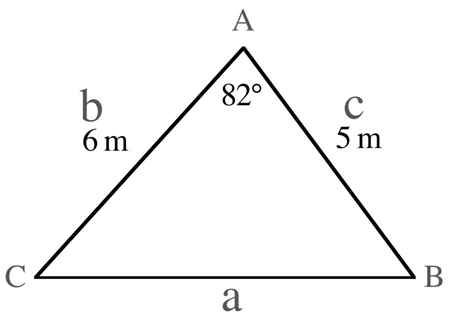
NOTE: requires knowing 2 sides and the angle between them.

Example:



Find the area of triangle PQR.

relabel triangle with A *as the known angle between 2 known sides*
the 2 known sides labelled b *and* c, *it doesn't matter which one is b or c*



$$\begin{aligned} Area\ \Delta ABC &= \frac{1}{2}bc\ \sin A \\ &= \frac{1}{2}\times 6\times 5\times \sin 82° \\ &= 14\cdot 854..... \\ Area &= 14\cdot 8\ m^2 \end{aligned}$$

UNIT 2: SIMULTANEOUS LINEAR EQUATIONS

EQUATION OF A LINE

The equation gives a rule connecting the x and y coordinates of any point on the line.

For example, $2x + y = 6$

sample points:

$(0, 6)$	$x = 0$, $y = 6$	$2 \times 0 + 6 = 6$
$(3, 0)$	$x = 3$, $y = 0$	$2 \times 3 + 0 = 6$
$(-2, 10)$	$x = -2$, $y = 10$	$2 \times (-2) + 10 = 6$
$(\frac{1}{2}, 5)$	$x = \frac{1}{2}$, $y = 5$	$2 \times \frac{1}{2} + 5 = 6$

Infinite points cannot be listed but can be shown as a graph.

SKETCHING STRAIGHT LINES

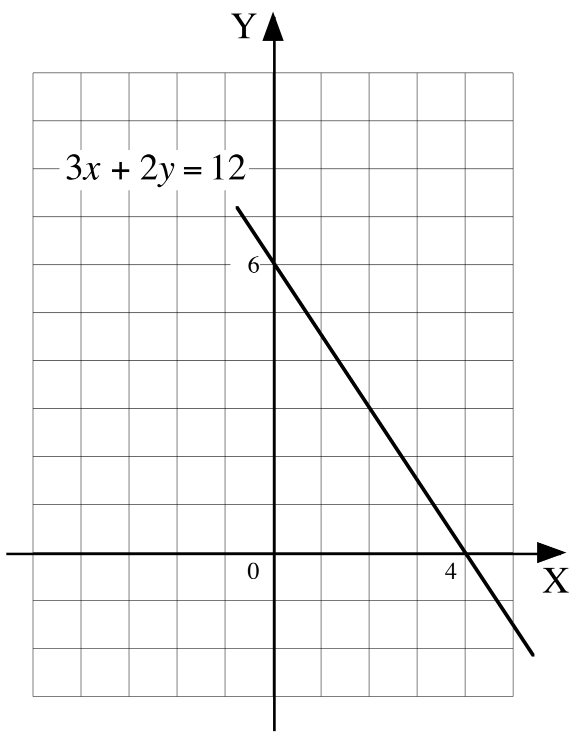
Show where the line meets the axes.

Example:

Sketch the graph with equation $3x + 2y = 12$.

$$3x + 2y = 12$$
$$3 \times 0 + 2y = 12 \quad \textit{substituted for } x = 0$$
$$2y = 12$$
$$y = 6 \quad \textit{plot } (0,6)$$

$$3x + 2y = 12$$
$$3x + 2 \times 0 = 12 \quad \textit{substituted for } y = 0$$
$$3x = 12$$
$$x = 4 \quad \textit{plot } (4,0)$$



SOLVE SIMULTANEOUS EQUATIONS: GRAPHICAL METHOD

Sketch the two lines and the point of intersection is the solution.

Example:

Solve **graphically** the system of equations: $y + 2x = 8$

$y - x = 2$

$y + 2x = 8 \quad (1)$

$y + 2 \times 0 = 8$ *substituted for* $x = 0$

$y = 8$ *plot* $(0,8)$

$y - x = 2 \quad (2)$

$y - 0 = 2$ *substituted for* $x = 0$

$y = 2$ *plot* $(0,2)$

$y + 2x = 8 \quad (1)$

$0 + 2x = 8$ *substituted for* $y = 0$

$2x = 8$

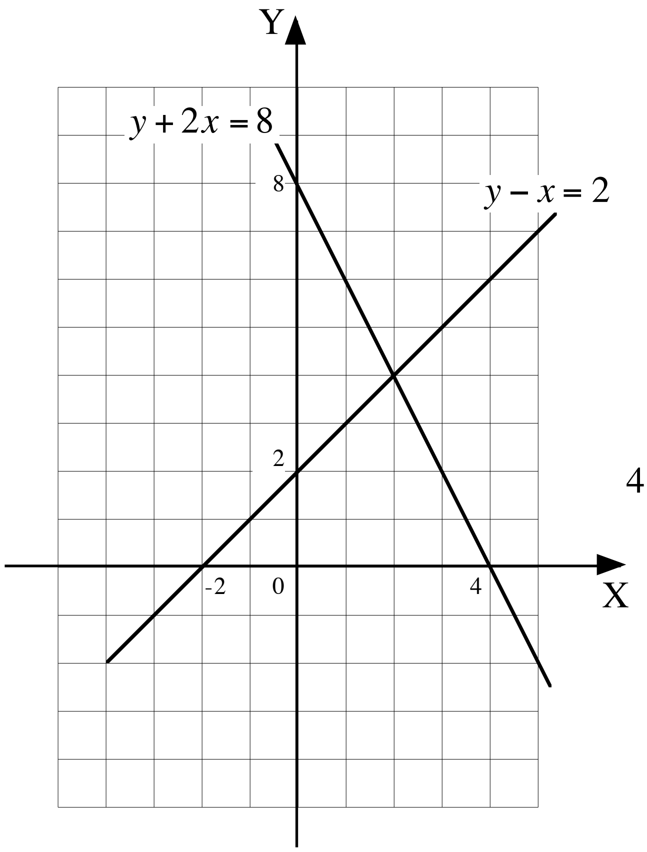
$x = 4$ *plot* $(4,0)$

$y - x = 2 \quad (2)$

$0 - x = 2$ *substituted for* $y = 0$

$-x = 2$

$x = -2$ *plot* $(-2,0)$



point of intersection $(2,4)$

CHECK:

$x = 2$ *and* $y = 4$

substituted in both equations

$y + 2x = 8 \quad (1)$

$4 + 2 \times 2 = 8$

$8 = 8$

$y - x = 2 \quad (2)$

$4 - 2 = 2$

$2 = 2$

SOLUTION:

$x = 2$ *and* $y = 4$

SOLVE SIMULTANEOUS EQUATIONS: SUBSTITUTION METHOD

Rearrange both equations to $y =$ and equate the two equations.
(or $x =$)

Example:
Solve **algebraically** the system of equations: $y + 2x = 8$
$y - x = 2$

$y + 2x = 8$ (1) *can choose to rearrange to* $y =$ *or* $x =$
$y = 8 - 2x$ *choosing* $y =$ *avoids fractions as* $x = 4 - \frac{1}{2}y$

$y - x = 2$ (2)
$y = x + 2$ *rearrange for* $y =$

$x + 2 = 8 - 2x$ *y terms equal*
$3x + 2 = 8$
$3x = 6$
$x = 2$

$y = x + 2$ (2) *can choose either equation (1) or (2)*
$= 2 + 2$ *substituted for* $x = 2$
$y = 4$

CHECK:
$y + 2x = 8$ (1) *using the other equation*
$4 + 2 \times 2 = 8$ *substituted for* $x = 2$ *and* $y = 4$
$8 = 8$

SOLUTION:
$x = 2$ *and* $y = 4$

SOLVE SIMULTANEOUS EQUATIONS: ELIMINATION METHOD

Can add or subtract multiples of the equations to eliminate either the x or y term.

Example:

Solve **algebraically** the system of equations: $4x + 3y = 5$
$5x - 2y = 12$

$4x + 3y = 5 \quad (1) \times 2$ — *can choose to eliminate x or y term*

$5x - 2y = 12 \quad (2) \times 3$ — *choosing y term,* $LCM(3y, 2y) = 6y$ (*least common multiple*)

$8x + 6y = 10 \quad (3)$ — *multiplied each term of (1) by 2 for* $+6y$

$15x - 6y = 36 \quad (4)$ — *multiplied each term of (2) by 3 for* $-6y$

$23x + 0 = 46 \quad (3) + (4)$ — *added "like" terms,*

$x = 2$ — $+6y$ *and* $-6y$ *added to 0 (ie eliminated)*

$4x + 3y = 5 \quad (1)$ — *can choose either equation (1) or (2)*

$4 \times 2 + 3y = 5$ — *substituted for* $x = 2$

$8 + 3y = 5$

$3y = -3$

$y = -1$

CHECK:

$5x - 2y = 12 \quad (2)$ — *using the other equation*

$5 \times 2 - 2 \times (-1) = 12$ — *substituted for* $x = 2$ *and* $y = -1$

$10 - (-2) = 12$

$12 = 12$

SOLUTION:

$x = 2$ *and* $y = -1$

UNIT 2: GRAPHS, CHARTS AND TABLES

Studying statistical information, it is useful to consider: (1) typical result: **average**
(2) distribution of results: **spread**

AVERAGES:

$$mean = \frac{total\ of\ all\ results}{number\ of\ results}$$

$$median = middle\ result\ of\ the\ ordered\ results$$

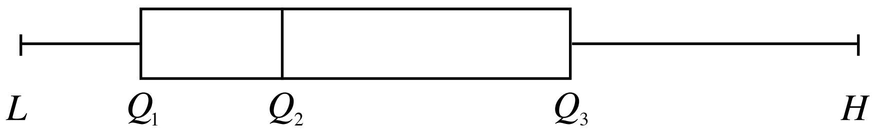
$$mode = most\ frequent\ result$$

SPREAD:

Ordered results are split into 4 equal groups so each contains 25% of the results.

The **5 figure summary** identifies: L , Q_1 , Q_2 , Q_3 , H
(lowest result , 1st , 2nd and 3rd quartiles , highest result)

A **Box Plot** is a statistical diagram that displays the 5 figure summary:



$$range,\ R = H - L$$

$$interquartile\ range,\ IQR = Q_3 - Q_1$$

$$semi-interquartile\ range,\ SIQR = \frac{Q_3 - Q_1}{2}$$

NOTE: If Q_1 , Q_2 or Q_3 fall between two results, the mean of the two results is taken.

For example,

12 ordered results: split into 4 equal groups of 3 results

10 11 13 ¦ Q_1 ¦ 17 18 20 ¦ Q_2 ¦ 20 23 25 ¦ Q_3 ¦ 26 27 29

$$Q_1 = \frac{13+17}{2} = 15 \quad , \quad Q_2 = \frac{20+20}{2} = 20 \quad , \quad Q_3 = \frac{25+26}{2} = 25 \cdot 5$$

Example:

Pulse rates: 66, 64, 71, 56, 60, 79, 77, 75, 69, 73, 75, 62, 66, 71, 66 beats per minute.

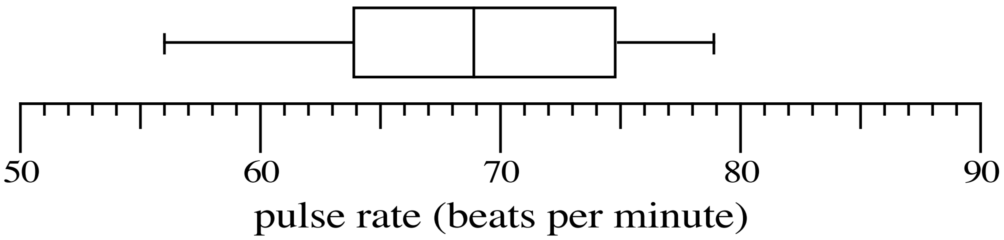
15 ordered results:

			Q_1				Q_2				Q_3			
56	60	62	**64**	66	66	66	**69**	71	71	73	**75**	75	77	79

5 Figure Summary:

$L = 56$, $Q_1 = 64$, $Q_2 = 69$, $Q_3 = 75$, $H = 79$

Box Plot:



Spread:

$$R = H - L = 79 - 56 = 23$$

$$IQR = Q_3 - Q_1 = 75 - 64 = 11$$

$$SIQR = \frac{Q_3 - Q_1}{2} = \frac{75 - 64}{2} = \frac{11}{2} = 5 \cdot 5$$

Averages: $(total = 66 + 64 + 71 + ... + 66 = 1030)$

$$MEAN = \frac{1030}{15} = 68 \cdot 666... = 68 \cdot 7$$

$$(Q_2)\,MEDIAN = 69$$

$$MODE = 66$$

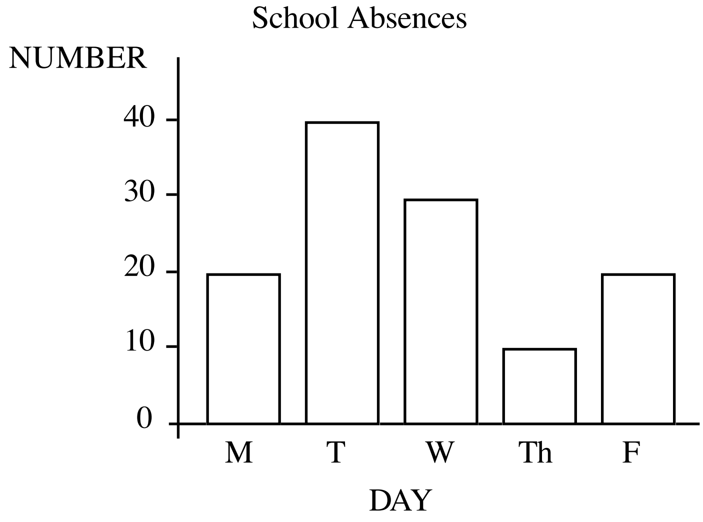
OTHER STATISTICAL DIAGRAMS

Examples:

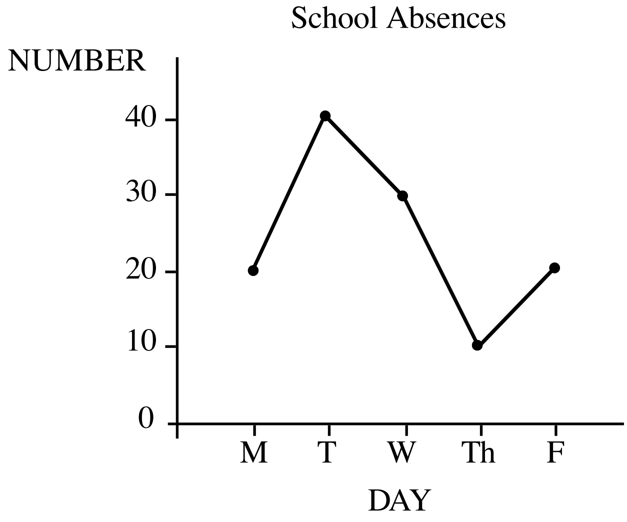
A school records the daily absences for a week.

DAY	Monday	Tuesday	Wednesday	Thursday	Friday
ABSENCES	20	40	30	10	20

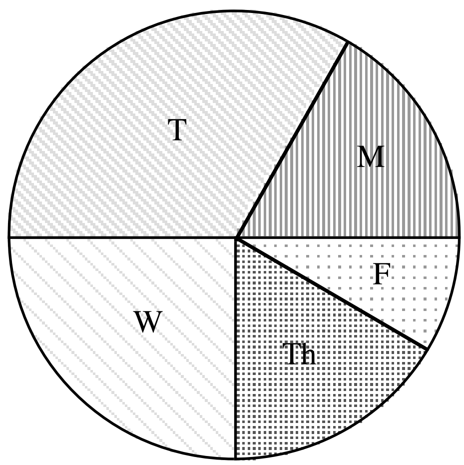
bar graph:



line graph:



pie chart:



$total\ absences = 120$

$M \quad \frac{20}{120} \times 360° = 60°$

$T \quad \frac{40}{120} \times 360° = 120°$

$W \quad \frac{30}{120} \times 360° = 90°$

$Th \quad \frac{10}{120} \times 360° = 30°$

$F \quad \frac{20}{120} \times 360° = 60°$

ordered stem-and-leaf:

Race Times (seconds):

10·4 , 10·2 , 9·9 , 12·1 , 11·7 , 10·9 , 9·9 , 11·4 , 10·6 , 11·5 , 10·1 , 9·8 , 10·2 , 11·3 , 11·0

unordered first

9	*9*	*9*	*8*			
10	*4*	*2*	*9*	*6*	*1*	*2*
11	*7*	*4*	*5*	*3*	*0*	
12	*1*					

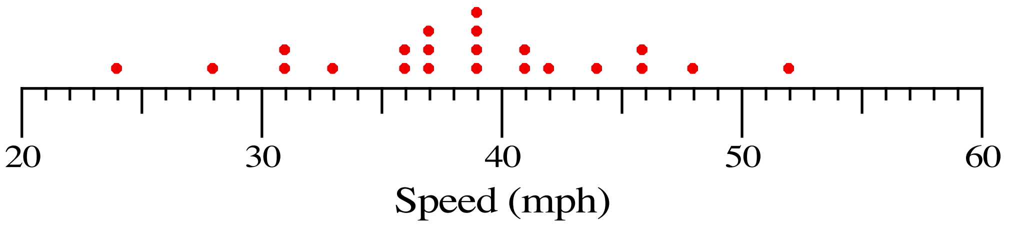
Race Time (seconds)

9	8	9	9			
10	1	2	2	4	6	9
11	0	3	4	5	7	
12	1					

n = 15 9 | 8 = 9.8

dot plot:

Car speeds (mph):



FREQUENCY DISTRIBUTION TABLES

Useful for dealing with a large number of results.

Example:

In a competition 50 people take part.

The table shows the distribution of points scored.

Scores (points)

result	frequency
10	4
11	5
12	9
13	12
14	10
15	7
16	3

mean:

result	frequency	result x frequency
10	4	40
11	5	55
12	9	108
13	12	156
14	10	140
15	7	105
16	3	48
TOTALS	50	652

$$MEAN = \frac{652}{50} = 13 \cdot 04$$

cummulative frequency:

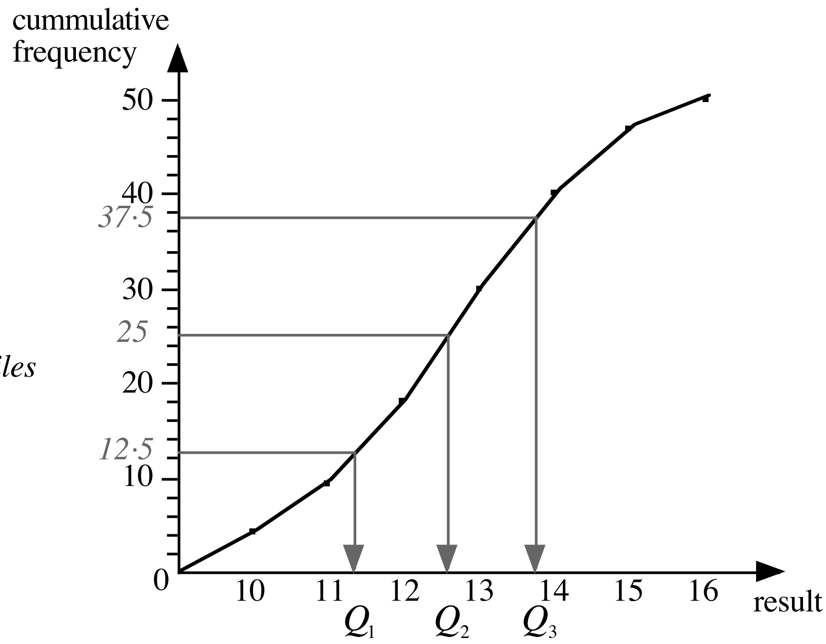
result	frequency	cummulative frequency
10	4	4
11	5	9
12	9	18
13	12	30
14	10	40
15	7	47
16	3	50

quarter the results: 50 ÷ 4 = 12 R 2

12 1 12 12 1 12
13th 25 / 26th 38th

$Q_1 = 12$, 13*th result included here
$Q_2 = 13$, 25*th* / 26*th result included here*
$Q_3 = 14$, 38*th result included here*

*divide the total frequency into 4 quarter intervals to **estimate** quartiles*
50 ÷ 4 = 12·5



UNIT 2: USE OF SIMPLE STATISTICS

STANDARD DEVIATION

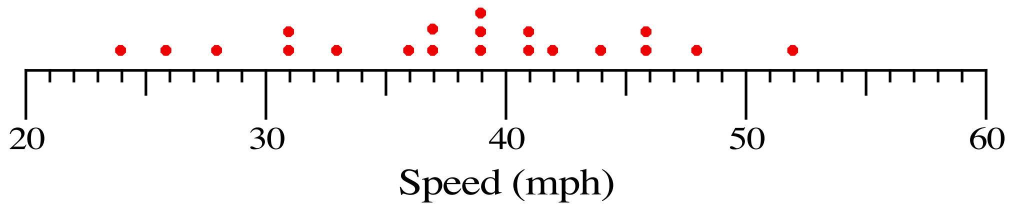
Is a measure of the spread (dispersion) of a set of data, giving a numerical value to how the data deviates from the mean.

Formulae:

mean $\bar{x} = \dfrac{\sum x}{n}$ standard deviation $s = \sqrt{\dfrac{\sum\left(x - \bar{x}\right)^2}{n-1}}$ or $s = \sqrt{\dfrac{\sum x^2 - \dfrac{\left(\sum x\right)^2}{n}}{n-1}}$

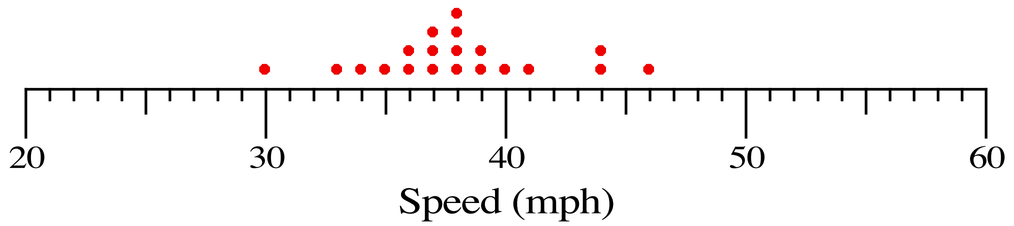
Examples,

(1) High Standard Deviation: results spread out



mean = 38 , standard deviation = 7·5

(2) Low Standard Deviation: results clustered around the mean



mean = 38 , standard deviation = 3·8

Calculations for Example (2):

x	$x-\bar{x}$	$\left(x-\bar{x}\right)^2$
30	-8	64
33	-5	25
34	-4	16
35	-3	9
36	-2	4
36	-2	4
37	-1	1
37	-1	1
37	-1	1
38	0	0
38	0	0
38	0	0
38	0	0
39	+1	1
39	+1	1
40	+2	4
41	+3	9
44	+6	36
44	+6	36
46	+8	64
totals 760	0	276

or

x	x^2
30	900
33	1089
34	1156
35	1225
36	1296
36	1296
37	1369
37	1369
37	1369
38	1444
38	1444
38	1444
38	1444
39	1521
39	1521
40	1600
41	1681
44	1936
44	1936
46	2116
totals 760	29156

$$\bar{x} = \frac{\sum x}{n} = \frac{760}{20} = 38$$

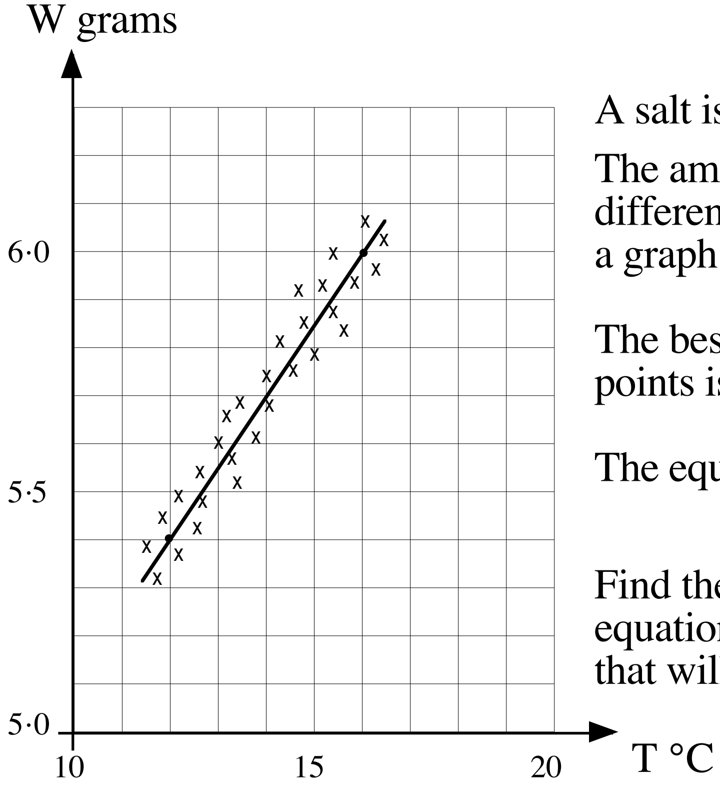
$$s = \sqrt{\frac{\sum\left(x-\bar{x}\right)^2}{n-1}} = \sqrt{\frac{276}{19}} = \sqrt{14\cdot 526....} = 3\cdot 811.... \approx 3\cdot 8$$

or

$$s = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}} = \sqrt{\frac{29156 - \frac{760^2}{20}}{19}} = \sqrt{\frac{276}{19}} = 3\cdot 811.... \approx 3\cdot 8$$

SCATTERGRAPHS AND LINE OF BEST FIT

Example:



A salt is dissolved in a litre of solvent.

The amount of salt that dissolves at different temperatures is recorded and a graph plotted.

The best-fitting straight line through the points is drawn.

The equation of the graph is of the form

$$W = mT + C.$$

Find the equation of the line and use the equation to calculate the mass of salt that will dissolve at 30 °C.

using two well-separated points on the line

$(16\ ,\ 6\cdot0)$

$(12\ ,\ 5\cdot4)$

$$m = \frac{6\cdot0 - 5\cdot4}{16-12} = \frac{0\cdot6}{4} = 0\cdot15$$

$$y = mx + C$$
$$W = 0\cdot15\,T + C$$

substituting for one point on the line $\underset{}{(\overset{T}{16}\ ,\ \overset{W}{6\cdot0})}$

$$6\cdot0 = 0\cdot15\times16 + C$$
$$6\cdot0 = 2\cdot4 + C$$
$$C = 3\cdot6$$
$$\underline{\underline{W = 0\cdot15\,T + 3\cdot6}}$$

$T = 30$

$$W = 0\cdot15\ \times30 + 3\cdot6$$
$$= 4\cdot5 + 3\cdot6$$
$$= 8\cdot1$$
$$\underline{\underline{8\cdot1\ \textit{grams}}}$$

PROBABILITY

The probability of an event A occuring is $P(A) = \dfrac{\text{number of outcomes involving A}}{\text{total number of outcomes possible}}$

Always $0 \le P \le 1$ and $P = 0$ impossible to occur , $P = 1$ certain to occur

The experimental results will differ from the theoretical probability.

Examples:

(1) A letter is chosen at random from the word ARITHMETIC.

4 vowels out of 10 letters, $P(vowel) = \dfrac{4}{10} = 0 \cdot 4$

(2) In an experiment a letter is chosen at random from the word ARITHMETIC and the results recorded.

letter	frequency	relative frequency
vowel	37	37÷100 = 0·37
consonant	63	63÷100 = 0·63
	total = 100	total = 1

Estimate of probability, $P(vowel) = 0 \cdot 37$

UNIT 3: MORE ALGEBRAIC OPERATIONS

ALGEBRAIC FRACTIONS

Examples:

SIMPLIFYING: fully factorise and 'cancel' common factors.

(1) $\dfrac{x^2-9}{x^2+2x-3}$

$= \dfrac{(x-3)(x+3)}{(x-1)(x+3)}$

$= \dfrac{x-3}{x-1}$

(2) $\dfrac{x-3}{2x^2-6x}$

$= \dfrac{1(x-3)}{2x(x-3)}$

$= \dfrac{1}{2x}$

(3) $\dfrac{3a^2b}{3a^2+3ab}$

$= \dfrac{3a\times ab}{3a(a+b)}$

$= \dfrac{ab}{a+b}$

ADD/SUBTRACT: a common denominator is required.

(4) $\dfrac{3}{2y} - \dfrac{4}{y^2}$

$= \dfrac{3y}{2y^2} - \dfrac{8}{2y^2}$

$= \dfrac{3y-8}{2y^2}$

(5) $\dfrac{3}{x-3} - \dfrac{3}{x+3}$

$= \dfrac{3(x+3)}{(x-3)(x+3)} - \dfrac{3(x-3)}{(x-3)(x+3)}$

$= \dfrac{3x+9-3x+9}{(x-3)(x+3)}$

$= \dfrac{18}{(x-3)(x+3)}$

MULTIPLY/DIVIDE:

(6) $\dfrac{3}{2(x+3)} \times \dfrac{(x+3)^2}{9}$

$= \dfrac{3(x+3)^2}{18(x+3)}$

$= \dfrac{3(x+3)\times(x+3)}{3(x+3)\times\ 6}$

$= \dfrac{x+3}{6}$

(7) $\dfrac{2}{y} \div \dfrac{4}{y^2}$

$= \dfrac{2}{y} \times \dfrac{y^2}{4}$

$= \dfrac{2y^2}{4y}$

$= \dfrac{y\times 2y}{2\times 2y}$

$= \dfrac{y}{2}$

TRANSPOSING FORMULAE (CHANGE OF SUBJECT)

Follow the rules for equations to isolate the **target term** and then the **target letter**.
(has target letter)

addition and subtraction

$x + a = b$	$x - a = b$
subtract a from each side	*add a to each side*
$x = b - a$	$x = b + a$

multiplication and division

$\dfrac{x}{a} = b$	$ax = b$
multiply each side by a	*divide each side by a*
$x = ab$	$x = \dfrac{b}{a}$

powers and roots

$x^2 = a$	$\sqrt{x} = a$
square root each side	*square each side*
$x = \sqrt{a}$	$x = a^2$

Examples:

Change the subject of the formula to r:

(1) $F = 3r^2 + p$

subtract p from each side

$F - p = 3r^2$

divide each side by 3

$\dfrac{F-p}{3} = r^2$

square root both sides

$\sqrt{\dfrac{F-p}{3}} = r$

subject of formula now r

$r = \sqrt{\dfrac{F-p}{3}}$

(2) $W = \dfrac{\sqrt{r} - n}{t}$

multiply both sides by t

$Wt = \sqrt{r} - n$

add n to both side

$Wt + n = \sqrt{r}$

square both sides

$(Wt + n)^2 = r$

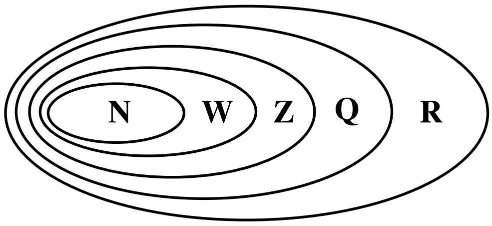
subject of formula now r

$r = (Wt + n)^2$

SURDS

NUMBER SETS:

Natural numbers N = {1, 2, 3...}
Whole numbers W = {0, 1, 2, 3...}
Integers Z = {... -3, -2, -1, 0, 1, 2, 3...}



Rational numbers, Q , can be written as a division of two integers.
Irrational numbers **cannot** be written as a division of two integers.

Real numbers, R , are all rational and irrational numbers.

SURDS ARE IRRATIONAL ROOTS.

For example, $\sqrt{2}$, $\sqrt{\frac{5}{9}}$, $\sqrt[3]{16}$ are surds.

whereas $\sqrt{25}$, $\sqrt{\frac{4}{9}}$, $\sqrt[3]{-8}$ are **not** surds as they are 5 , $\frac{2}{3}$ and -2 respectively.

SIMPLIFYING SURDS:

RULES: $\sqrt{mn} = \sqrt{m} \times \sqrt{n}$ $\qquad$ $\sqrt{\frac{m}{n}} = \frac{\sqrt{m}}{\sqrt{n}}$

Examples:

(1) Simplify $\sqrt{24} \times \sqrt{3}$

$\sqrt{24} \times \sqrt{3}$

$= \sqrt{72}$

$= \sqrt{36} \times \sqrt{2}$ $\quad$ 36 *is the largest square number which is a factor of* 72

$= 6 \times \sqrt{2}$

$= 6\sqrt{2}$

(2) Simplify $\sqrt{72} + \sqrt{48} - \sqrt{50}$

$\sqrt{72} + \sqrt{48} - \sqrt{50}$

$= \sqrt{36} \times \sqrt{2} + \sqrt{16} \times \sqrt{3} - \sqrt{25} \times \sqrt{2}$

$= 6\sqrt{2} + 4\sqrt{3} - 5\sqrt{2}$

$= 6\sqrt{2} - 5\sqrt{2} + 4\sqrt{3}$

$= \sqrt{2} + 4\sqrt{3}$

(3) Remove the brackets and fully simplify:

(a) $\left(\sqrt{3}-\sqrt{2}\right)^2$

$= \left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

$= \sqrt{3}\left(\sqrt{3}-\sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}-\sqrt{2}\right)$

$= \sqrt{9}-\sqrt{6} - \sqrt{6}+\sqrt{4}$

$= 3-\sqrt{6} - \sqrt{6}+2$

$= 5-2\sqrt{6}$

(b) $\left(3\sqrt{2}+2\right)\left(3\sqrt{2}-2\right)$

$= \left(3\sqrt{2}+2\right)\left(3\sqrt{2}-2\right)$

$= 3\sqrt{2}\left(3\sqrt{2}-2\right) + 2\left(3\sqrt{2}-2\right)$

$= 9\sqrt{4}-6\sqrt{2} + 6\sqrt{2} - 4$

$= 18-6\sqrt{2} + 6\sqrt{2} - 4$

$= 14$

RATIONALISING DENOMINATORS:

Removing surds from the denominator.

Examples:

Express with a rational denominator:

(1) $\dfrac{4}{\sqrt{6}}$

$\dfrac{4}{\sqrt{6}}$

$= \dfrac{4\times\sqrt{6}}{\sqrt{6}\times\sqrt{6}}$ *multiply the 'top' and 'bottom' by the surd on the denominator*

$= \dfrac{4\sqrt{6}}{6}$

$= \dfrac{2\sqrt{6}}{3}$

(2) $\dfrac{\sqrt{3}}{3\sqrt{2}}$

$\dfrac{\sqrt{3}}{3\sqrt{2}}$

$= \dfrac{\sqrt{3}\times\sqrt{2}}{3\sqrt{2}\times\sqrt{2}}$

$= \dfrac{\sqrt{6}}{3\times\sqrt{4}}$

$= \dfrac{\sqrt{6}}{6}$

INDICES

$$\text{base} \longrightarrow a^n \longleftarrow \text{index or exponent}$$

INDICES RULES: require the same base.

Examples:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$\frac{w^2 \times w^5}{w^3} = \frac{w^7}{w^3} = w^4$$

$$\left(a^m\right)^n = a^{mn}$$

$$\left(3^5\right)^2 = 3^{10}$$

$$(ab)^n = a^n b^n$$

$$\left(2a^3b\right)^2 = 2^2a^6b^2 = 4a^6b^2$$

$$\frac{1}{a^p} = a^{-p}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$a^0 = 1$$

$$\left(2\text{b}^3\right)^0 = 1$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

$$8^{\frac{4}{3}} = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$$

$$8^{-\frac{4}{3}} = \frac{1}{8^{\frac{4}{3}}} = \frac{1}{16}$$

UNIT 3: QUADRATIC FUNCTIONS

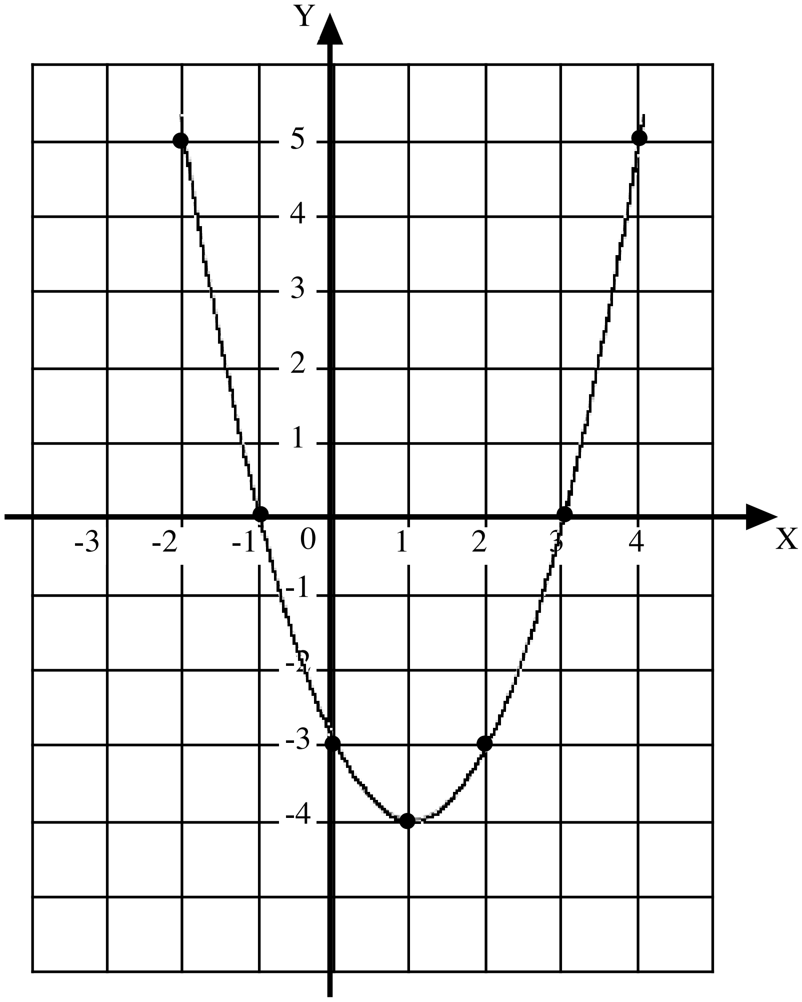
Form $y = ax^2 + bx + c, a \neq 0$, where a, b and c are constants.

The graph is a curve called a PARABOLA.

For example,

$y = x^2 - 2x - 3$

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-2x$	4	2	0	-2	-4	-6	-8
-3	-3	-3	-3	-3	-3	-3	-3
y	5	0	-3	-4	-3	0	5
points	(-2,5)	(-1,0)	(0,-3)	(1,-4)	(2,-3)	(3,0)	(4,5)



COMPLETED SQUARE:
Quadratic functions written in the form $y = \pm 1(x-a)^2 + b$, a and b are constants.

axis of symmetry $x = a$
turning point (a, b) , minimum for $+1$, maximum for -1

FACTORISED:
Quadratic functions written in the form $y = (x-a)(x-b)$, a and b are constants.

the zeros of the graph are a and b.
the axis of symmetry is $x = \dfrac{a+b}{2}$

For example,
$y = x^2 - 2x - 3$ can be written as $y = (x-1)^2 - 4$ or $y = (x+1)(x-3)$

meets the x-axis where $y = 0$
$(x+1)(x-3) = 0$
$x + 1 = 0$ *or* $x - 3 = 0$
$x = -1$ *or* $x = 3$
points $(-1, 0)$ *and* $(3, 0)$

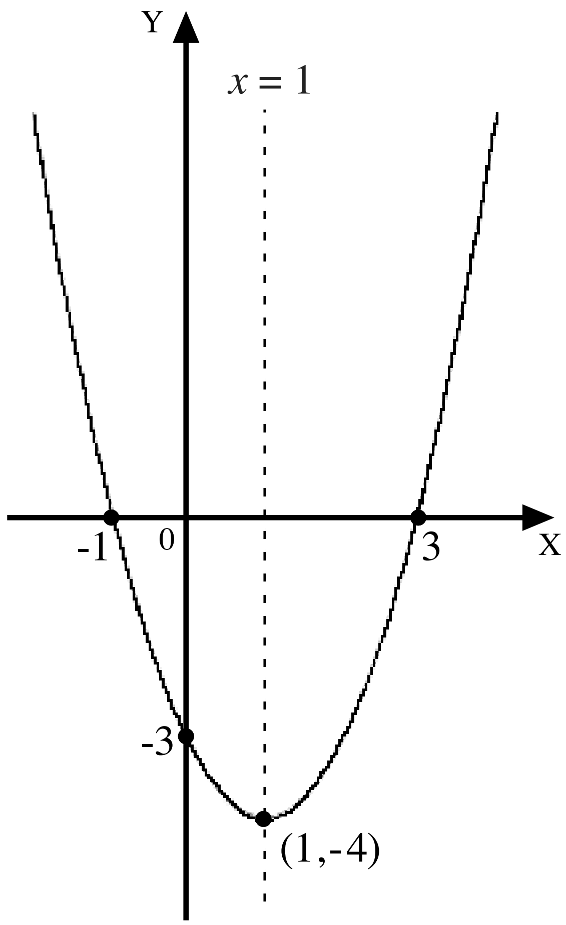
the **roots** *of the equation are* -1 *and* 3
the **zeros** *of the graph are* -1 *and* 3
axis of symmetry: $\dfrac{-1+3}{2}$, $x = 1$

meets the y-axis where $x = 0$
$y = (0-1)^2 - 4 = 1 - 4 = -3$
point $(0, -3)$

turning point: $y = +1(x-1)^2 - 4$

minimum turning point $(1, -4)$

axis of symmetry $x = 1$



QUADRATIC EQUATIONS

An equation of the form $ax^2 + bx + c = 0,\ a \neq 0$, where a, b and c are constants.

The value(s) of x that satisfy the equation are the **roots** of the equation.

FACTORISATION

If $b^2 - 4ac =$ a square number ie. $0, 1, 4, 9, 16.....$
then the quadratic expression can be factorised to solve the equation.

Examples:

Solve:

(1) $4n - 2n^2 = 0$

$$2n(2-n) = 0$$

$$2n = 0 \quad \text{or} \quad 2 - n = 0$$

$$\underline{\underline{n = 0 \quad \text{or} \quad n = 2}}$$

(2) $2t^2 + t - 6 = 0$

$$(2t-3)(t+2) = 0$$

$$2t - 3 = 0 \quad \text{or} \quad t + 2 = 0$$

$$2t = 3$$

$$\underline{\underline{t = \frac{3}{2} \quad \text{or} \quad t = -2}}$$

The equation may need to be rearranged:

(3) $(w+1)^2 = 2(w+5)$

$$w^2 + 2w + 1 = 2w + 10$$

$$w^2 - 9 = 0$$

$$(w+3)(w-3) = 0$$

$$w + 3 = 0 \quad \text{or} \quad w - 3 = 0$$

$$\underline{\underline{w = -3 \quad \text{or} \quad w = 3}}$$

(4) $x + 2 = \frac{15}{x}\ ,\ x \neq 0$

$$x(x+2) = 15$$

$$x^2 + 2x = 15$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

$$x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\underline{\underline{x = -5 \quad \text{or} \quad x = 3}}$$

QUADRATIC FORMULA

A quadratic equation $ax^2 + bx + c = 0$ can be solved using the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad , a \neq 0$$

Note: (1) **Use a calculator!**

(2) $b^2 - 4ac$ **will not be negative,** otherwise there is no solution.

Example:

Find the **roots** of the equation $3t^2 - 5t - 1 = 0$, correct to two decimal places.

$3t^2 - 5t - 1 = 0$

$at^2 + bt + c = 0$

$a = 3 \ , \ b = -5 \ , \ c = -1$

$b^2 - 4ac = (-5)^2 - 4 \times 3 \times (-1) = 37$

$-b = -(-5) = +5$

$2a = 2 \times 3 = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{5 \pm \sqrt{37}}{6}$$

$$= \frac{5 - \sqrt{37}}{6} \quad or \quad \frac{5 + \sqrt{37}}{6}$$

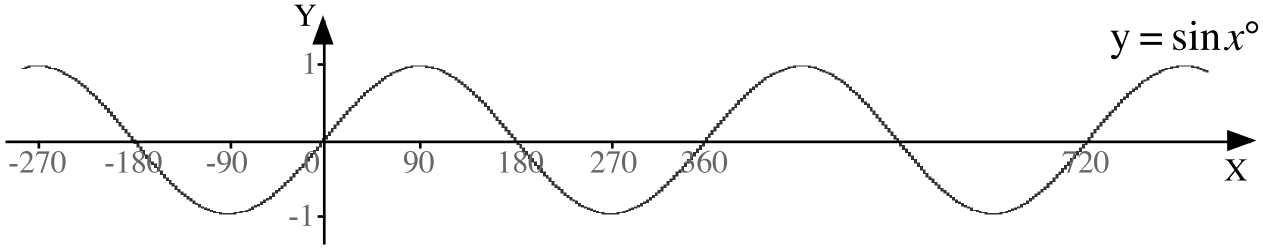
$$= \frac{-1 \cdot 0827....}{6} \quad or \quad \frac{11 \cdot 0827....}{6}$$

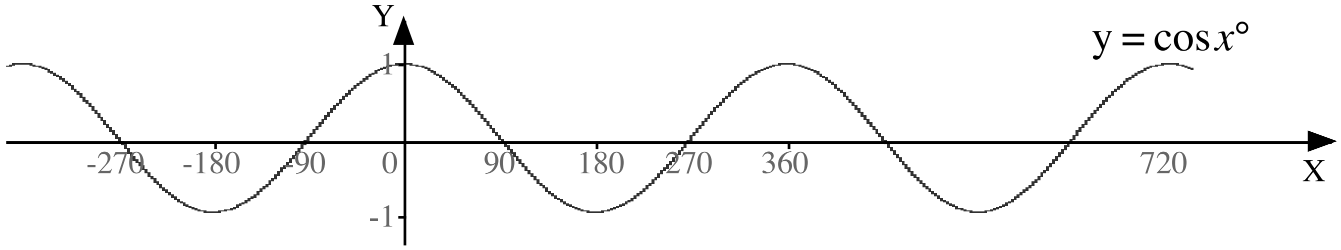
$$t = -0 \cdot 1804.... \quad or \quad 1 \cdot 8471....$$

roots are $-0 \cdot 18$ and $1 \cdot 85$

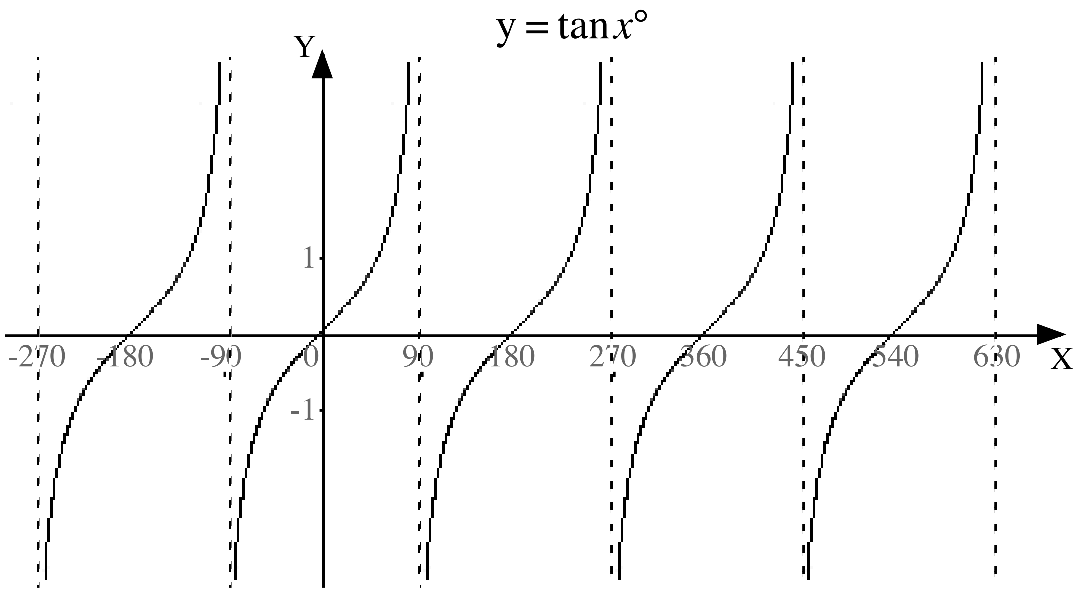
UNIT 3: FURTHER TRIGONOMETRY

GRAPHS





Each graph has a PERIOD of 360° (repeats every 360°).
The maximum value of each function is +1 , the minimum is -1.
The cosine graph is the sine graph shifted 90° to the left.



The tangent graph has a PERIOD of 180°.
The maximum value is positive infinity , the minimum is negative infinity.

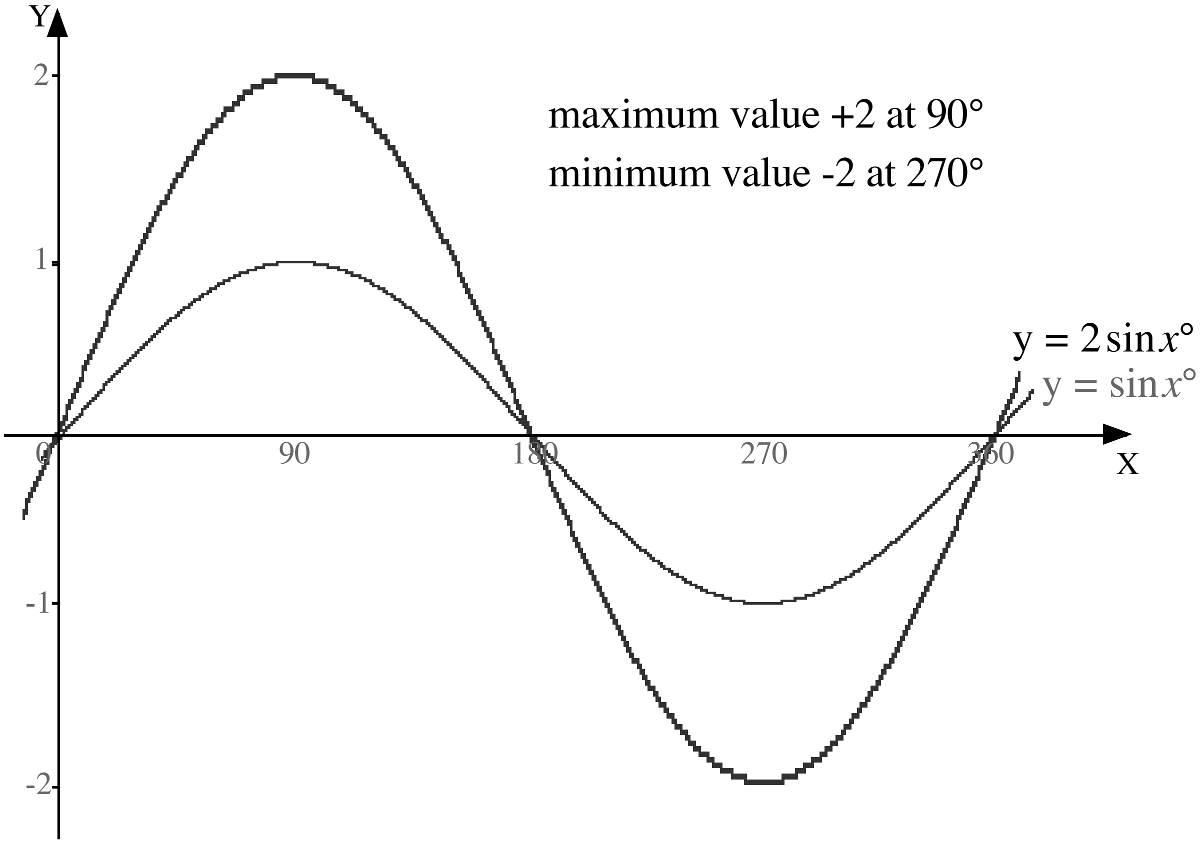
TRANSFORMATIONS Same rules for $y = \sin x°$ and $y = \cos x°$.

Y-STRETCH $y = \mathbf{n}\sin x°$ maximum value $+n$, minimum value $-n$.

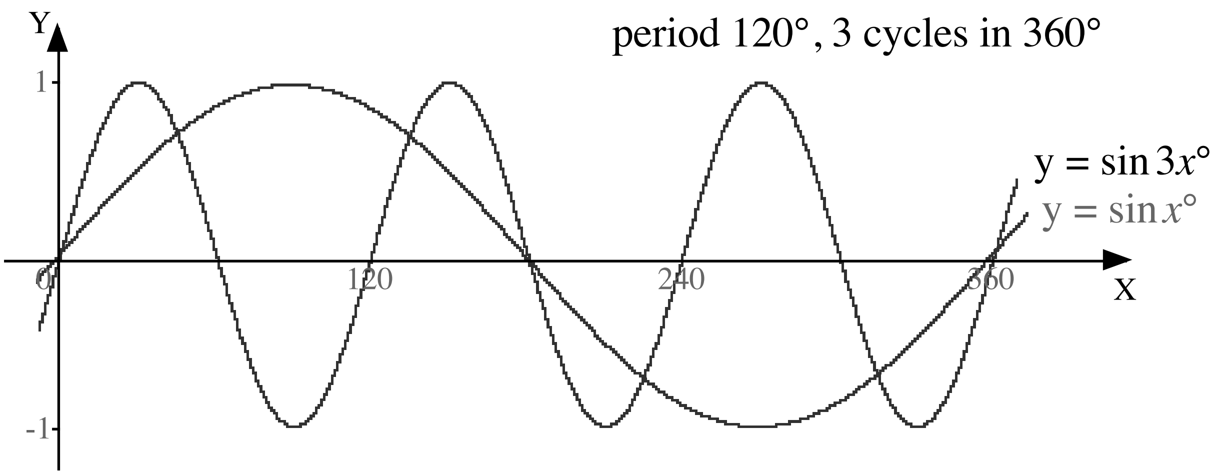
X-STRETCH $y = \sin \mathbf{n}x°$ has period $\frac{360°}{n}$. There are n cycles in 360°.

For example,

(1)

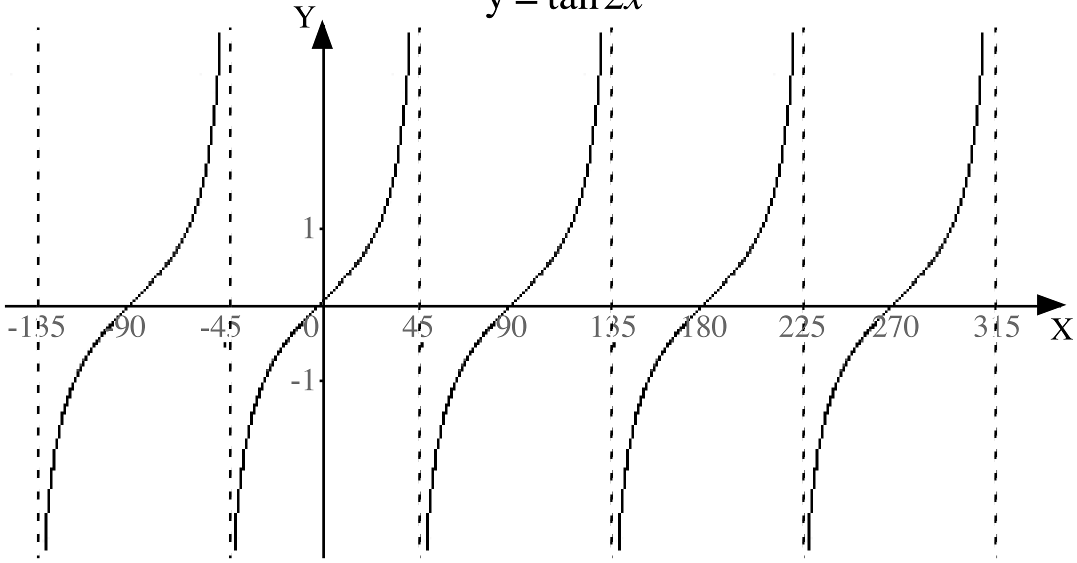


(2)



(3)

$y = \tan 2x°$

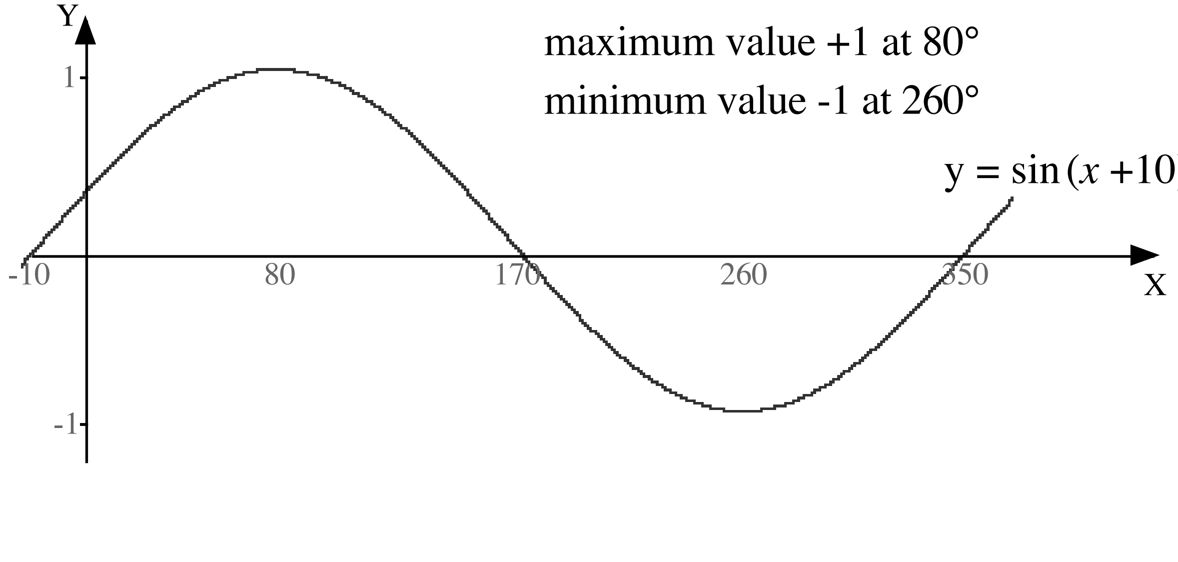


period 90°, 2 cycles in 180°

X-SHIFT $y = \sin(x + a)°$ graph shifted -a° horizontally.

For example,

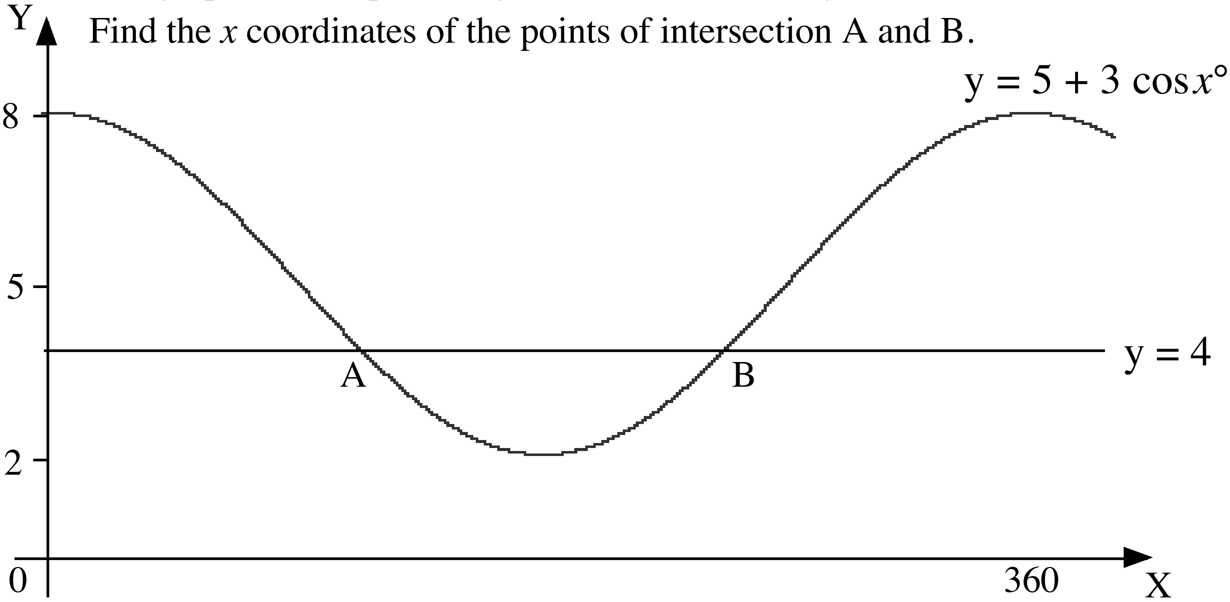
(4)



EQUATIONS

Example:

The graphs with equations $y = 5 + 3\cos x°$ and $y = 4$ are shown.
Find the x coordinates of the points of intersection A and B.



$$5 + 3\cos x° = 4$$
$$3\cos x° = -1$$
$$\cos x° = -\frac{1}{3}$$

$$\underline{\underline{x = 109\cdot5 \;\; or \;\; 250\cdot5}}$$

* **A**, **S**, **T**, **C** is where functions are **positive**:

✓S	A ×
cos -	cos +
180 - a = 109·5	a = cos $^{-1}$ $^{1}/_{3}$ = 70·528...
180 + a = 250·5	360 - a = 289·5
cos -	cos +
✓T	C ×

* A all functions are positive
S sine function only is positive
T cosine function only is positive
C tangent function only is positive

IDENTITIES

$$\sin^2 x° + \cos^2 x° = 1 \qquad\qquad \tan x° = \frac{\sin x°}{\cos x°}$$

Examples:

(1) If $\sin x° = \frac{1}{2}$, without finding x, find the **exact** values of $\cos x°$ and $\tan x°$.

$$\sin^2 x° + \cos^2 x° = 1$$

$$\left(\frac{1}{2}\right)^2 + \cos^2 x° = 1$$

$$\frac{1}{4} + \cos^2 x° = 1$$

$$\cos^2 x° = \frac{3}{4}$$

$$\cos x° = \frac{\sqrt{3}}{2}$$

$$\tan x° = \frac{\sin x°}{\cos x°}$$

$$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$\tan x° = \frac{1}{\sqrt{3}}$$

(2) Show that $\dfrac{1 - \cos^2 x°}{\sin x \cos x°} = \tan x$.

$$\frac{1 - \cos^2 x°}{\sin x \cos x°}$$

$$= \frac{\sin^2 x°}{\sin x \cos x°}$$

since $\sin^2 x° + \cos^2 x° = 1$

$1 - \cos^2 x° = \sin^2 x°$

$$= \frac{\sin x° \sin x°}{\sin x \cos x°}$$

$$= \frac{\sin x°}{\cos x°}$$

$$= \tan x$$