Calderglen High School

Mathematics Department

Higher Mathematics
Home Exercise Programme

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Home Exercise 1

The Laws of Indices		
Rule 1 : $x^p \times x^q = x^{(p+q)}$	Rule 2 : $x^p \div x^q = x^{(p-q)}$	Rule 3 : $(x^p)^q = x^{pq}$
Rule 4 : $(xy)^p = x^p \times x^q$	Rule 5 : $x^0 = 1$	Rule 6 : $x^{-p} = \dfrac{1}{x^p}$
Rule 7 : $\sqrt[n]{x^m} = (\sqrt[n]{x})^m = x^{\frac{m}{n}}$		

1. Simplify each of the following

   (a) $3x^2 \times 4x^7$ (b) $5x^{-4} \times 2x^2$

   (c) $12x^2 \div 4x^5$ (d) $6x^2 \div 3x^{-4}$

   (e) $(3x^{-4})^2$ (f) $(x^2y^3)^4$

   (g) Express $2x^{-3}$ with a positive index

   NC

   (Standard Grade : The Laws of Indices) **(7)**

2. Simplify $x^2(2x^3 - 1)$

   NC

   (Standard Grade : The Laws of Indices) **(2)**

3. Express each of the following as a surd in its simplest form

   (a) $\sqrt{44}$ (b) $\sqrt{189}$

   NC

   (Standard Grade : Simplifying Surds) **(2)**

4. Solve each of the following equations. **Show all relevant working. An answer only will receive no marks.**

   (a) $2^x = 128$ (b) $9^x = 243$

   NC

   (Standard Grade : Solving Exponential Equations) **(5)**

Time : 25 minutes **Total : 16**

End of Home Exercise 1

Home Exercise 2

The Laws of Indices		
Rule 1 : $x^p \times x^q = x^{(p+q)}$	Rule 2 : $x^p \div x^q = x^{(p-q)}$	Rule 3 : $(x^p)^q = x^{pq}$
Rule 4 : $(xy)^p = x^p \times x^q$	Rule 5 : $x^0 = 1$	Rule 6 : $x^{-p} = \dfrac{1}{x^p}$
Rule 7 : $\sqrt[n]{x^m} = (\sqrt[n]{x})^m = x^{\frac{m}{n}}$		

1. Simplify each of the following

(a) $6x^{\frac{1}{2}} \times 2x^{\frac{1}{4}}$

(b) $5x^{-\frac{3}{2}} \times 2x^{\frac{1}{4}}$

(c) $14x^{\frac{3}{2}} \div 2x^{\frac{3}{4}}$

(d) $2x^{-\frac{1}{2}} \div 4x^{-2}$

(e) $(16x^{-2})^{\frac{1}{2}}$

(f) $(x^6y^9)^{-\frac{1}{3}}$

(g) Express $\frac{1}{3}x^{-\frac{4}{5}}$ with a positive index

NC

(Standard Grade : The Laws of Indices) **(7)**

2. Simplify $3x^{-\frac{2}{3}}(x^{\frac{1}{3}} - 5)$

NC

(Standard Grade : The Laws of Indices) **(2)**

3. Express each of the following as a surd in its simplest form

(a) $4\sqrt{200}$

(b) $\sqrt{8} \times \sqrt{12}$

(c) $3\sqrt{5} \times 5\sqrt{3}$

(d) $(\sqrt{5} - \sqrt{2})(\sqrt{5} - \sqrt{2})$

NC

(Standard Grade : Simplifying Surds) **(7)**

4. Solve each of the following equations. **Show all relevant working. An answer only will receive no marks.**

(a) $5^x = \dfrac{1}{125}$

(b) $4^x = \dfrac{1}{32}$

NC

(Standard Grade : Solving Exponential Equations) **(7)**

Time : 35 minutes **Total : 23**

End of Home Exercise 2

Home Exercise 3

1. Differentiate each of the following each of the following

(a) $f(x) = 4\sqrt[3]{x^2}$

(b) $f(x) = \dfrac{5}{x^3}$

(c) $f(x) = 5x^2 + \dfrac{3}{\sqrt[4]{x}}$

(d) $f(x) = \dfrac{4 - 5x + 3x^3}{x^2}$

(Unit 1 : Outcome 3 : Differentiation : Using the rules) **NC**

(2,2,3,6)

2. (a) D(-6,1), E(4,5) and F(8,-3) are the vertices of triangle DEF.
Find the equation of the median from D.

(b) K(6,6), L(-3,0) and M(0,-3) are the vertices of triangle KLM.
Find the equation of the altitude from K.

(c) P is the point (-3,1) and Q is (5,5).
Find the equation of the perpendicular bisector of PQ.

(Unit 1 : Outcome 1 : Straight Lines : Altitudes, medians and perpendicular bisectors of a triangle) **NC**

(3,3,4)

Time : 35 minutes **Total : 23**

End of Home Exercise 3

Home Exercise 4

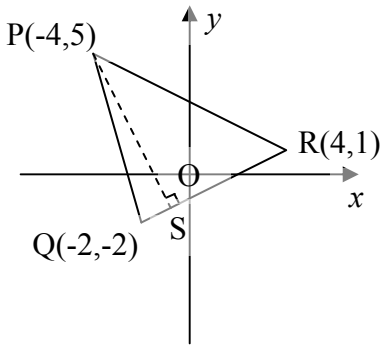
1. Find the equation of the perpendicular bisector of the line joining A(2,-1) and B(8,3)

(Unit 1 : Outcome 1 : Straight Lines : Altitudes, medians and perpendicular bisectors of a triangle) **NC**

(4)

2. P(-4,5), Q(-2,-2) and R(4,1) are the vertices of triangle PQR as shown in the diagram.

Find the equation of PS, the altitude from P

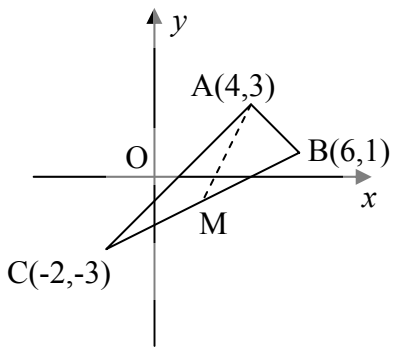


(Unit 1 : Outcome 1 : Straight Lines : Altitudes, medians and perpendicular bisectors of a triangle) **NC**

(3)

3. A triangle ABC has vertices A(4,3), B(6,1) and C(-2,-3) as shown in the diagram.

Find the equation of AM, the median from A.



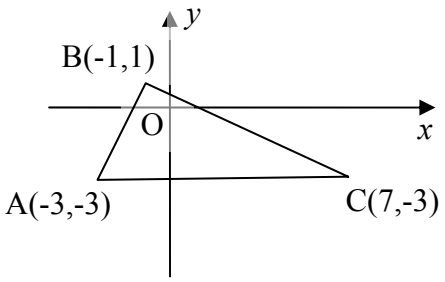
(Unit 1 : Outcome 1 : Straight Lines : Altitudes, medians and perpendicular bisectors of a triangle) **NC**

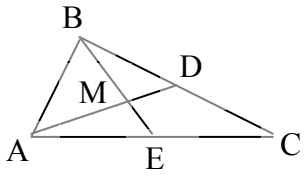
(3)

4. …..over

Home Exercise 4

4.





(a) Show that triangle ABC is right angled at B.

(b) The medians AD and BE intersect at M

(i) Find the equations of AD and BE.

(ii) Hence find the coordinates of M.

(Unit 1 : Outcome 1 : Straight Lines : Altitudes, medians and perpendicular bisectors of a triangle) **NC**

(3,5,3)

Time : 35 minutes	**Total : 21**

End of Home Exercise 4

Home Exercise 5

1. Differentiate each of the following with respect to x

(a) $f(x)=\dfrac{2x^2+4x+3}{2x^2}$ (b) $f(x)=\dfrac{3x^2-4x}{\sqrt[3]{x^2}}$

(Unit 1 : Outcome 3 : Differentiation : Using the rules) **NC**

(5,4)

2. (a) Show that there is only one tangent to the curve $y=4x^2-16x+5$ with a gradient of 8

(b) Find the point of contact on the curve and the equation of the tangent.

(Unit 1 : Outcome 3 : Differentiation : Finding the equation of a tangent to a curve) **NC**

(3,2)

3. $f(x)=2x-1$ $g(x)=3-2x$ $h(x)=\dfrac{1}{4}(5-x)$

(a) Find a formula for *k(x)* where *k(x)=f* [*g(x)*]

(b) Find a formula for *h*[*k(x)*]

(c) What is the connection between the functions *h* and *k*.

(Unit 1 : Outcome 2 : Functions : Composition of functions and inverse functions) **NC**

(2,2,1)

4. The functions *f* and *g* are defined on a suitable domain by

$f(x)=x^2-1$ $g(x)=x^2+2$

(a) Find an expression for *f* [*g(x)*]

(b) Factorise *f* [*g(x)*]

(Unit 1 : Outcome 2 : Functions : Composition of functions) **NC**

(2,2)

5. The functions *f* and *g*, defined on a suitable domains, are given by

$f(x)=\dfrac{1}{x^2-4}$ $g(x)=2x+1$

(a) Find an expression for *h(x)* = *g*[*f(x)*]. Give your answer as a single fraction.

(b) State a suitable domain for *h*.

(Unit 1 : Outcome 2 : Functions : Composition of functions) **NC**

(3,1)

Time : 40 minutes **Total : 27**

End of Home Exercise 5

Home Exercise 6

1. Find the derivative of each of the following.

   (a) $f(x) = \dfrac{2x^3 - 3x^2 + 4}{x^3}$ (b) $f(x) = \left(2x^3 + \dfrac{1}{x}\right)^2$

   (Unit 1 : Outcome 3 : Differentiation : Using the rules) **NC**

   (5,6)

2. Find the equation of the tangent to the curve $y = x + \dfrac{1}{x}$ at the point where $x = \dfrac{1}{2}$.

   (Unit 1 : Outcome 3 : Differentiation : Finding the equation of a tangent to a curve) **NC**

   (5)

3. Show that the tangents to the curve $y = x^3$ at the points where $x = -1$ and $x = 1$ are parallel.

   (Unit 1 : Outcome 1 : Straight Line : Parallel lines)

   (Unit 1 : Outcome 3 : Differentiation : The equation of a tangent to a curve) **NC**

   (3)

4. For the function defined by $f(x) = x^3 - 6x^2 + 5$ find the intervals on which it is increasing.

   (Unit 1 : Outcome 3 : Differentiation : Increasing/decreasing functions) **NC**

   (5)

5. Sketch the graph of the curve with equation $y = 12x - x^3$.

   (Unit 1 : Outcome 3 : Differentiation : Curve sketching) **C**

   (9)

Time : 50 minutes **Total : 33**

End of Home Exercise 6

Home Exercise 7

1. Find the derivative of each of the following

(a) $f(x) = \dfrac{(4-x)(5-2x)}{x^2}$ (b) $f(x) = \dfrac{x+1}{2x^{\frac{1}{3}}}$

(Unit 1 : Outcome 3 : Differentiation : Using the rules) **NC**

(5,4)

2. A fishing club uses a loch which has been stocked with 2000 fish.

Due to fish being caught by members and dying of natural causes, the fish population is reduced by 18% per month.

(a) If members are allowed to fish, without the loch being re-stocked, during which month will the number of fish in the loch fall below 750?

(b) The club committee decide that the number of fish in the loch should not be allowed to fall below 1200.

In order to ensure that this does not happen, it is decided that 180 fish should be added at the end of each month.

Will this maintain a level of at least 1200 fish at all times?
Give a reason for your answer.

(Unit 1 : Outcome 4 : Recurrence relations : Limit of a sequence) **C**

(3,5)

3. A sequence is defined by the recurrence relation

$$U_n = 0.9U_{n-1} + 2 \qquad U_1 = 3$$

(a) Calculate the value of U_2.

(b) What is the smallest value of n for which $U_n > 10$?

(c) Find the limit of this sequence as $n \to \infty$.

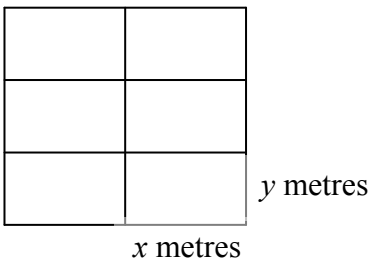
(Unit 1 : Outcome 4 : Recurrence relations : The limit of a sequence) **C**

(1,1,2)

4. …..over

4. A zookeeper wants to fence off six individual pens, as shown in the diagram opposite.

Each pen is a rectangle measuring x metres by y metres.



(a) (i) Express the total length of fencing in terms of x and y.

(ii) Given that the total length of fencing is 360 metres, show that the total area, A m^2, of the six pens is given by

$$A(x) = 240x - \frac{16}{3}x^2$$

(b) Find the value of x and y which give the maximum area and write down this maximum area.

(Unit 1 : Outcome 3 : Differentiation : Optimisation) **C**

(4,6)

Time : 45 minutes **Total : 31**

End of Home Exercise 7

Home Exercise 8

1. Trees are sprayed weekly with the pesticide, 'Killpest', whose manufacturers claim it will destroy 65% of all pests. Between weekly sprayings, it is estimated that 500 new pests invade the trees.

   A new pesticide,'Pestkill' comes on to the market. The manufacturers claim that it will destroy 85% of existing pests but it is estimated that 650 new pests per week will invade the trees.

   Which pesticide will be more effective in the long term?

   (Unit 1 : Outcome 4 : Recurrence relations : Limit of a sequence) **C**

   (7)

2. Express $f(x)=(2x-1)(2x+5)$ in the form $a(x+b)^2+c$

   (Unit 1 : Outcome 2 : Functions : Completing the square) **NC**

   (3)

3. A curve has equation $y = x^4 - 4x^3 + 3$

   (a) Find algebraically the coordinates of the stationary points. **C**

   (b) Determine the nature of the stationary points. **(5,3)**

   (Unit 1 : Outcome 3 : Differentiation : Curve Sketching)

4. Two sequences are defined by the formulae

$$U_{n+1} = 0.4U_n - 3 \qquad \text{and} \qquad T_{n+1} = 3T_n + 0.4$$

   The initial values are $U_1 = 4$ and $T_1 = 4$.

   (a) Find the values of U_2, U_3, T_2 and T_3.

   (b) Explain why one of the sequences has a limit as n tends to infinity, and calculate this limit algebraically.

   (Unit 1 : Outcome 4 : Recurrence relations : Limit of a sequence) **NC**

   (2,3)

5. …..over

Home Exercise 8

5. Functions f and g are defined on the set of real numbers by

$$f(x) = x - 1 \qquad \text{and} \qquad g(x) = x^2$$

(a) Find formulae for (i) $f[g(x)]$ and (ii) $g[f(x)]$

(b) The function h is defined by $h(x) = f[g(x)] + g[f(x)]$.
Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h.

(Unit 1 : Outcome 2 : Functions : Composition of functions and sketching the graph of a quadratic) **NC**

(4,3)

6. A function f is defined on the set of real numbers by

$$f(x) = \frac{x}{1-x} \qquad x \neq 1$$

Find, in its simplest form, an expression for $f[f(x)]$.

(Unit 1 : Outcome 2 : Functions : Composition of functions) **NC**

(3)

Time : 50 minutes **Total : 33**

End of Home Exercise 8

Home Exercise 9

1. For what values of a does the equation $ax^2 + 20x + 40 = 0$ have equal roots?

(Unit 2 : Outcome 1 : Quadratic Theory : The discriminant) **NC**

(2)

2. A sequence is defined by the recurrence relation $U_{n+1} = 0.3U_n + 5$ with first term U_1.

   (a) Explain why this sequence has a limit as n tends to infinity.

   (b) Find the **exact** value of this limit

(Unit 1 : Outcome 4 : Recurrence relations : Limit of a sequence) **NC**

(1,2)

3. Find, **algebraically**, the values of x for which the function $f(x) = 2x^3 - 3x^2 - 36x$ is increasing.

(Unit 1 : Outcome 3 : Differentiation : Increasing/decreasing functions) **NC**

(4)

4. If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of

   $\sin(x + 30)°$ is $\frac{4\sqrt{3}+3}{10}$

(Unit 2 : Outcome 3 : Trigonometry : The addition formulae) **NC**

(3)

5. Given that $y = 2x^2 + x$, find $\frac{dy}{dx}$ and hence show that $x\left(1+\frac{dy}{dx}\right) = 2y$

(Unit 1 : Outcome 3 : Differentiation : Using the rules) **NC**

(3)

6. (a) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form $f(x) = a(x + b)^2 + c$ where a, b and c are constants.

   (b) Hence, or otherwise, find the coordinates of the turning point of the function.

(Unit 1 : Outcome 2 : Functions : Completing the square) **NC**

(3,1)

7. …..over

Home Exercise 9

7. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.

Find the exact value of

(a) $\sin 2A$ (b) $\cos 2A$ (c) $\sin (2A + B)$

(Unit 2 : Outcome 3 : Trigonometry : The double angle formulae) **NC**

(2,1,2)

Time : 35 minutes **Total : 24**

End of Home Exercise 9

Home Exercise	**10**

1. Differentiate $2\sqrt{x}(x+2)$ with respect to x.

(Unit 1 : Outcome 3 : Differentiation : Using the rules) **NC**

(3)

2. Two sequences are defined by the recurrence relations

$$U_{n+1} = 0.2U_n + p \qquad U_0 = 1 \quad \text{and}$$
$$V_{n+1} = 0.6V_n + q \qquad V_0 = 1$$

If both sequences have the same limit, express p in terms of q

(Unit 1 : Outcome 4 : Recurrence relations : Limit of a sequence) **NC**

(3)

3. (a) $f(x) = 2x + 1$ and $g(x) = x^2 + k$, where k is a constant.

(i) Find $g[f(x)]$ (ii) Find $f[g(x)]$

(b) (i) Show that the equation $g[f(x)] - f[g(x)] = 0$ simplifies to $2x^2 + 4x - k = 0$

(ii) Determine the nature of the roots of this equation when $k = 6$.

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots.

(Unit 1 : Outcome 2 : Functions : Composition of functions)
(Unit 2 : Outcome 1 : Quadratic Theory : The discriminant) **NC**

(2,2,2,2,3)

4. (a) Show that $2\cos 2x° - \cos^2 x° = 1 - 3\sin^2 x°$

(b) Hence solve the equation $2\cos 2x° - \cos^2 x° = 2\sin x°$ in the interval $0 \leq x \leq 360$

(Unit 2 : Outcome 3 : Trigonometry : Solution of trigonometric equations) **C**

(2,4)

Time : 35 minutes	**Total : 24**

End of Home Exercise 10

Home Exercise 11

1. Given that $f(x) = 3x^2(2x - 1)$, find $f'(-1)$.

(Unit 1 : Outcome 3 : Differentiation : Using the rules, evaluating a derivative) **NC**

(3)

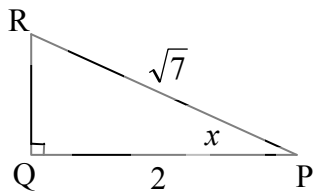
2. (a) Show that $f(x) = 2x^2 - 4x + 5$ can be written in the form $f(x) = a(x + b)^2 + c$

   (b) Hence write down the coordinates of the stationary point of $y = f(x)$ and state its nature

(Unit 1 : Outcome 2 : Functions : Completing the square) **NC**

(3,2)

3. Using triangle PQR, shown opposite, find the exact value of cos $2x$.



(Unit 2 : Outcome 3 : Trigonometry : The double angle formulae) **NC**

(3)

4. A curve has equation $y = -x^4 + 4x^3 - 2$

   (a) Find the coordinates of the stationary points.

   (b) Determine the nature of the stationary points

(Unit 1 : Outcome 3 : Differentiation : Curve Sketching) **C**

(5,3)

5. (a) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots.

   (b) Show that there are no real roots for θ.

(Unit 2 : Outcome 3 : Trigonometry : The double angle formulae)
(Unit 2 : Outcome 1 : Quadratic Theory : The discriminant) **NC**

(3,1)

Time : 35 minutes **Total : 23**

End of Home Exercise 11

Home Exercise 12

1. The point A has coordinates (7, 4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at the point B.

   (a) Find the gradient of AB.

   (b) Hence show that AB is perpendicular to only one of these two lines.

   (Unit 1 : Outcome 1 : Straight Line : Gradient formula)
   (Unit 1 : Outcome 1 : Straight Line : Perpendicular lines) **NC**

   (3,5)

2. $f(x) = x^3 - x^2 - 5x - 3$

   (a) (i) Show that $(x + 1)$ is a factor of $f(x)$.

   (ii) Hence or otherwise factorise $f(x)$ fully.

   (b) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point.

   (Unit 2 : Outcome 1 : Polynomials : Synthetic division) **NC**

   (5,1)

3. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p.

   (Unit 2 : Outcome 1 : Quadratics : Discriminant : Real roots) **C**

   (4)

4. A sequence is defined by the recurrence relation $U_{n+1} = kU_n + 3$.

   (a) Write down the condition for this sequence to have a limit.

   (b) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k.

   (Unit 1 : Outcome 4 : Recurrence Relations : Limit of a sequence) **C**

   (1,3)

Time : 35 minutes **Total : 22**

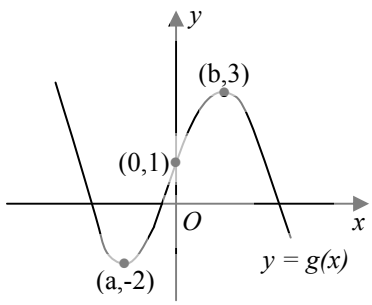
End of Home Exercise 12

Home Exercise 13

1. The diagram shows the graph of $y = g(x)$

   (a) Sketch the graph of $y = -g(x)$

   (b) On the same diagram, sketch the graph of $y = 3 - g(x)$.



(Unit 1 : Outcome 2 : Functions : Associated graphs) **NC**

(2,2)

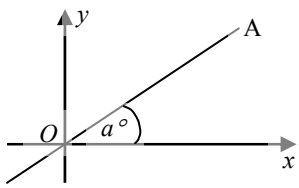
2. (a) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$.

   (b) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.

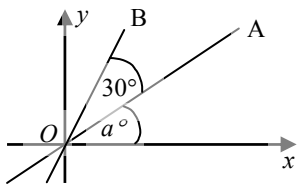
(Unit 1 : Outcome 2 : Functions : Completing the square)
(Unit 1 : Outcome 3 : Differentiation : Increasing/decreasing functions) **NC**

(2,4)

3. (a) The diagram shows the line OA with equation $x - 2y = 0$.
   The angle between OA and the x-axis is $a°$.
   Find the value of a.



   (b) The second diagram shows lines OA and OB. The angle between the two lines is 30°.
   Calculate the gradient of the line OB correct to one decimal place.



(Unit 1 : Outcome 3 : Straight line : $m = \tan \theta°$) **C**

(3,1)

Home Exercise 13

4. The point P(x,y) lies on the curve with equation $y = 6x^2 - x^3$.

   (a) Find the value of x for which the gradient of the tangent at P is 12.

   (b) Hence find the equation of the tangent at P.

(Unit 1 : Outcome 3 : Differentiation : Equation of a tangent to a graph) **NC**

(5,2)

Time : 35 minutes **Total : 21**

End of Home Exercise 13

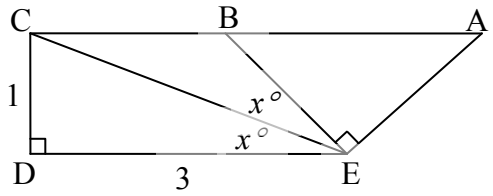
Home Exercise 14

1. Find the equation of the line which passes through the point (-1,3) and is perpendicular to the line with equation $4x + y - 1 = 0$.

(Unit 1 : Outcome 1 : The Straight Line : Perpendicular lines) **NC**

(3)

2. In the diagram angle $D\hat{E}C = C\hat{E}B = x°$ and $C\hat{D}E = B\hat{E}A = 90°$.
CD = 1 unit and DE = 3 units.

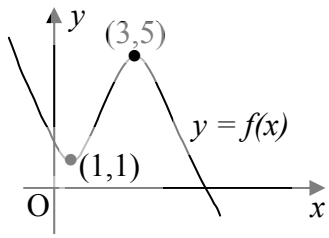


By writing $D\hat{E}A$ in terms of x°, find the exact value of cos($D\hat{E}A$).

(Unit 2 : Outcome 3 : Trigonometry : Addition formulae) **NC**

(7)

3. The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1,1) and (3,5).

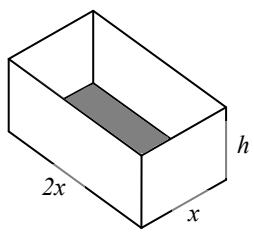


Sketch the graph of $y = f'(x)$.

(Unit 1 : Outcome 3 : Differentiation : The graph of the derived function) **C**

(3)

4. An open cuboid measures internally x units by $2x$ by h units and has an inner surface area of 12 units^2



(a) Show that the volume, V units^3, of the cuboid is given by

$$V(x) = \frac{2}{3}x(6 - x^2)$$

(b) Find the exact value of x for which the volume is a maximum.

(Unit 1 : Outcome 3 : Differentiation : Optimisation) **C**

(3,5)

Time : 35 minutes **Total : 21**

End of Home Exercise 14

Home Exercise 15

1. (a) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$.
   (b) Hence or otherwise sketch the graph of $y = f(x)$.

   (Unit 1 : Outcome 2 : Functions : Completing the square)
   (Unit 2 : Outcome 1 : Quadratics : Sketching the graph of a quadratic\) **NC**
   (2,2)

2. A recurrence relation is defined by $U_{n+1} = pU_n + q$, where $-1 < p < 1$ and $U_0 = 12$.

   (a) If $U_1 = 15$ and $U_2 = 16$, find the values of p and q.
   (b) Find the limit of this recurrence relation as $n \to \infty$.

   (Unit 1 : Outcome 4 : Recurrence relations : Solving recurrence relations to find m and c, and the limit of a sequence **NC**
   (2,2)

3. $f(x) = 6x^3 - 5x^2 - 17x + 6$

   (a) Show that $(x - 2)$ is a factor of $f(x)$.
   (b) Express $f(x)$ in its fully factorised form.

   (Unit 2 : Outcome 1 : Polynomials : Synthetic division) **C**
   (4)

4. Solve the equation $3\cos 2x + 10\cos x - 1 = 0$ for $0 \le x \le \pi$

   C
   (Unit 2 : Outcome 3 : Trigonometry : Solution of trigonometric equations) **(4)**

Time : 25 minutes **Total : 16**

End of Home Exercise 15

Home Exercise 16

1. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$.

(Unit 1 : Outcome 3 : Differentiation : Using the rules. Evaluating a derivative) **NC**

(5)

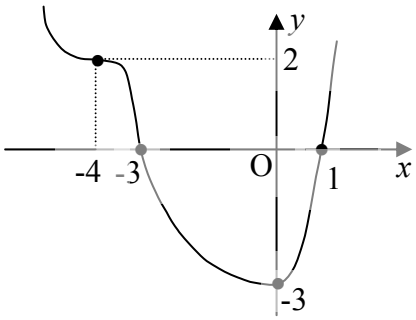
2. Show that the line with equation $y = 2x + 1$ does not intersect with the parabola with equation $y = x^2 + 3x + 4$.

(Unit 2 : Outcome 1 : The quadratic : The discriminant) **NC**

(5)

3. The diagram shows the graph of the function f.

f has a minimum turning point at (0,-3) and a point of inflexion at (-4,2).



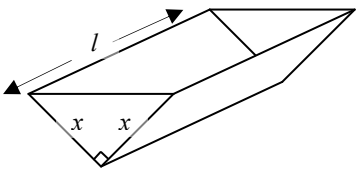
(a) Sketch the graph of $y = f(-x)$.

(b) On the same diagram, sketch the graph of $y = 2f(-x)$.

(Unit 1 : Outcome 2 : Functions : Associated graphs) **C**

(2,2)

4. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.

(a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$

(b) Find the value of x which minimises this surface area.

(Unit 1 : Outcome 3 : Differentiation : Optimisation) **C**

(3,5)

Time : 35 minutes **Total : 22**

End of Home Exercise 16

Home Exercise 17

1. Functions $f(x) = \dfrac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

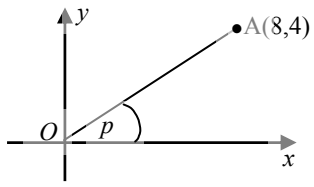
   (a) Find an expression for $h(x)$ where $h(x) = f[g(x)]$.

   (b) Write down a restriction on the domain of h.

   (Unit 1 : Outcome 2 : Functions : Composite functions) **NC**

   (2,1)

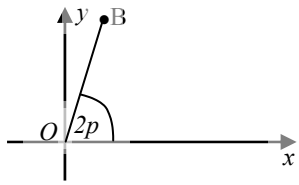
2. A is the point (8,4). The line OA is inclined at an angle p radians to the x-axis.



   (a) Find the exact values of:

   (i) $\sin(2p)$ (ii) $\cos(2p)$

   The line OB is inclined at an angle $2p$ radians to the x-axis.

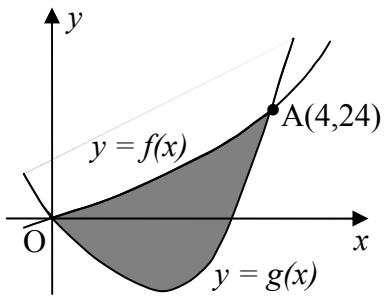


   (b) Write down the exact value of the gradient of OB

   (Unit 2 : Outcome 3 : Trigonometry : The Double angle formulae) **NC**

   (5,1)

3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram.



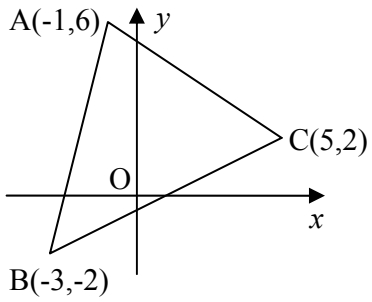
   The graphs intersect at A(4,24) and the origin.

   Find the shaded area enclosed between the curves.

   (Unit 2 : Outcome 2 : Integration : Area between two curves) **C**

   (5)

4. Triangle ABC has vertices A(-1,6), B(-3,-2) and C(5,2).



Find

(a) the equation of the line p, the median from C of triangle ABC.

(b) the equation of the line q, the perpendicular bisector of BC.

(c) the coordinates of the point of intersection of the lines p and q.

(Unit 1 : Outcome 1 : The straight line : Equation of a median and perpendicular bisector) **C**

(3,4,1)

Time : 35 minutes **Total : 22**

End of Home Exercise 17

Home Exercise 18

1. The point P(2,3) lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P.

(Unit 2 : Outcome 4 : The circle : Tangent to a circle) **NC**

(4)

2. Functions f and g are defined on suitable domains by $f(x) = \sin x°$ and $g(x) = 2x$.

(a) Find expressions for: (i) $f[g(x)]$ (ii) $g[f(x)]$

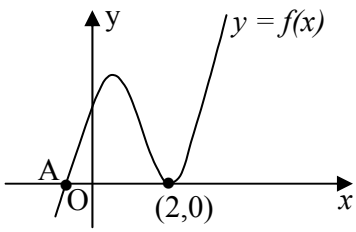
(b) Solve $2f[g(x)] = g[f(x)]$ for $0 \leq x \leq 360$

(Unit 1 : Outcome 2 : Functions : Composite functions)
(Unit 2 : Outcome 3 : Trigonometry : Solution of trigonometric equations) **NC**

(2,5)

3. The diagram shows part of the graph of the curve with equation

$$y = 2x^3 - 7x^2 + 4x + 4$$



(a) Find the x-coordinate of the maximum turning point.

(b) Factorise $2x^3 - 7x^2 + 4x + 4$

(c) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$

(Unit 1 : Outcome 3 : Differentiation : Curve sketching)
(Unit 2 : Outcome 1 : Polynomials : Synthetic division and solution of inequations) **C**

(5,3,2)

Time : 35 minutes **Total : 21**

End of Home Exercise 18

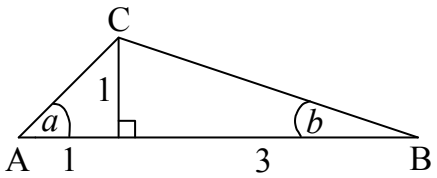
Home Exercise 19

1. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis.

(Unit 1 : Outcome 3 : Differentiation : Tangent to a curve) **NC**

(4)

2. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$



(Unit 2 : Outcome 3 : Trigonometry : The addition formulae) **NC**

4

3. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0.5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

   (a) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run?

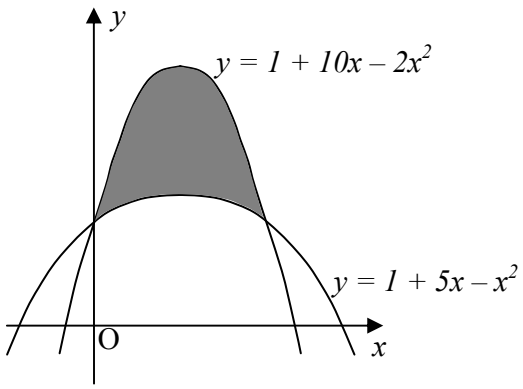
   (b) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition?

(Unit 1 : Outcome 4 : Recurrence relations : The limit of a sequence) **C**

(3,3)

4. …..over

4. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.



(Unit 2 : Outcome 2 : Integration : The area between two curves) **C**

(6)

Time : 30 minutes **Total : 20**

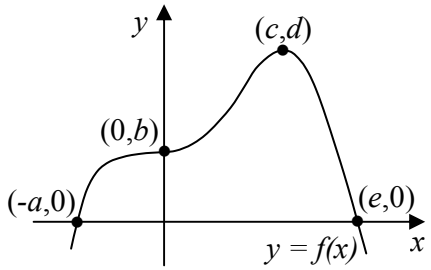
End of Home Exercise 19

Home Exercise 20

1. The graph of the function f intersects the x-axis at $(-a,0)$ and $(e,0)$ as shown.

   There is a point of inflexion at $(0,b)$ and a maximum turning point at (c,d).

   Sketch the graph of the derived function f'.



(Unit1 : Outcome 3 : Differentiation : The graph of the derived function) **NC 3**

2. (a) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$

   (b) On the same diagram sketch:
   (i) the graph of $y = f(x)$ (ii) the graph of $y = 10 - f(x)$

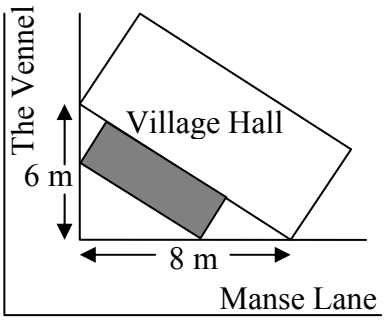
   (c) Find the range of values of x for which $10 - f(x)$ is positive.

(Unit 1 : Outcome 2 : Functions : Completing the square and associated graphs) **NC (2,4,1)**

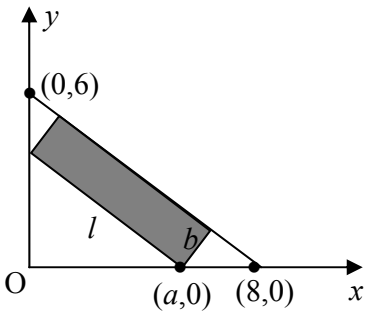
3. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k.

(Unit 2 : Outcome 1 : The quadratic : The discriminant) **C (5)**

4. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length
l metres and breadth b metres, as shown. One corner of the extension is at the point $(a,0)$



(a) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8-a)$

(b) Find the value of a which produces the largest area of the extension.

(Unit 1 : Outcome 3 : Differentiation : Optimisation) **C**

(3,4)

Time : 35 minutes **Total : 22**

End of Home Exercise 20

Home Exercise 21

1. Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point (2,-1).

   (Unit 1 : Outcome 1 : The Straight Line : Parallel lines) **NC**

   (3)

2. For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots?

   (Unit 2 : Outcome 1 : The Quadratic : The discriminant) **NC**

   (3)

3. (a) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k.
   (b) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value.

   (Unit 2 : Outcome 1 : Polynomials : Synthetic division and finding the roots of a cubic equation) **C**

   (3,2)

4. A curve has equation $y = x - \frac{16}{\sqrt{x}}, \quad x > 0$

   Find the equation of the tangent at the point where $x = 4$.

   (Unit 1 : Outcome 3 : Differentiation : The equation of a tangent to a curve) **C**

   (6)

Time : 30 minutes **Total : 17**

End of Home Exercise 21

Home Exercise 22

1. Given that $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$.

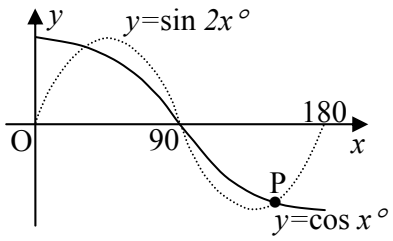
(Unit 1 : Outcome 2 : Functions : Completing the square) **NC**

(2)

2. (a) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \leq x \leq 180$.

(b) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.

Use your solution in (a) to write down the coordinates of the point P.



(Unit n2 : Outcome 3 : Trigonometry : Solution of trigonometric equations) **NC**

(4,1)

3. On the first day of March, a bank loans a man £2500 at a fixed rate of interest 0f 1.5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

(a) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayments have been made.
Let U_n and U_{n+1} represent the amounts that he owes at the starts of two successive months. Write down a recurrence relation involving U_{n+1} and U_n.

(b) Find the date and the amount of the final payment.

(Unit 1 : Outcome 4 : Recurrence relations : Finding the n th term of a sequence) **C**

(2,4)

4. …..over

4. A firm had a new logo designed. A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation

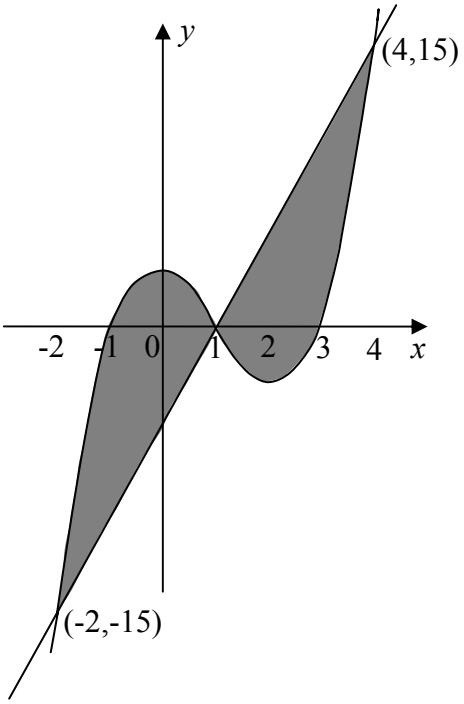
$$y = (x + 1)(x - 1)(x - 3)$$

and the straight line has equation

$$y = 5x - 5$$

The point (1,0) is the centre of half-turn symmetry.

Calculate the total shaded area.



(Unit 2 : Outcome 2 : Integration : The area between two curves) **C**

(7)

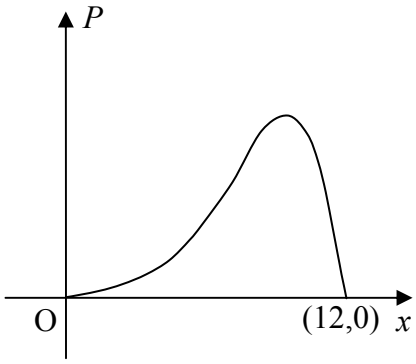
Time : 30 minutes **Total : 20**

End of Home Exercise 22

1. A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds. A mathematical model, illustrated in the diagram, suggests that P and x are related by

$$P(x) = 12x^3 - x^4 \quad \text{for} \quad 0 \le x \le 12$$

Find the value of x which gives the maximum profit.



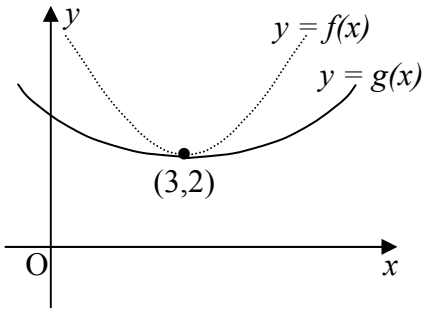
(Unit 1 : Outcome 3 : Differentiation : Optimisation) **NC**

(5)

2. The diagram shows the graph of two quadratic functions $y = f(x)$ and $y = g(x)$.

Both graphs have a minimum turning point at (3,2).

Sketch the graph of $y = f'(x)$ and on the same diagram sketch the graph of $y = g'(x)$.



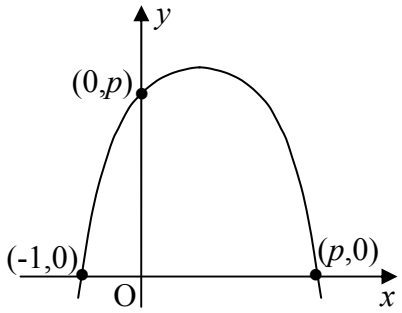
(Unit 1 : Outcome 3 : Differentiation : The graph of the derived function) **NC**

2

3. The diagram shows a sketch of a parabola passing through (-1,0), (0,p) and (p,0).

(a) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$.

(b) For what value of p will the line $y = x + p$ be a tangent to the curve

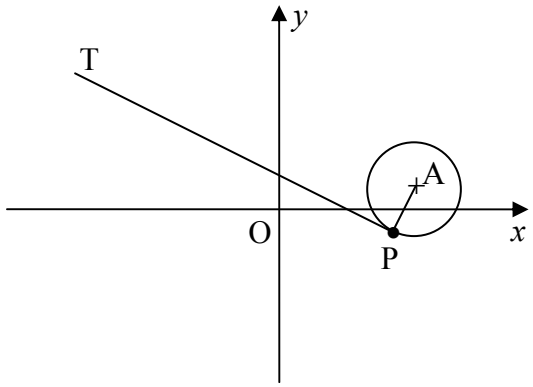


(Unit 2 : Outcome 1 : Polynomials : Finding the equation of a polynomial) **C**

(3,3)

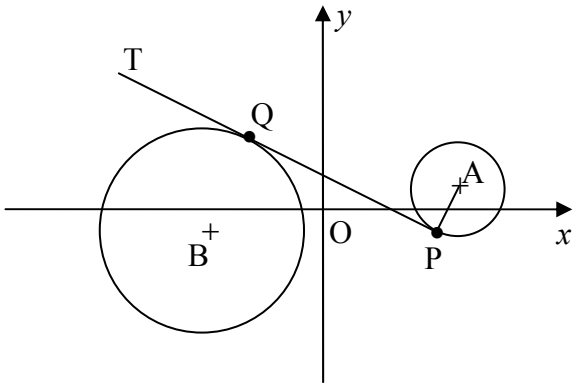
4. …..over

4. The circle with centre A has equation $x^2 + y^2 - 12x - 2y + 32 = 0$. The line PT is a tangent to this circle at the point P(5,-1)



(a) Show that the equation of this tangent is $x + 2y = 3$.

The circle with centre B has equation $x^2 + y^2 + 10x + 2y + 6 = 0$.



(b) Show that PT is also a tangent to this circle.

(c) Q is the point of contact. Find the length of PQ.

(Unit 2 : Outcome 4 : The Circle : Tangent to a circle and Distance formula) **C**

(4,5,2)

Time : 35 minutes **Total : 24**

End of Home Exercise 23

Home Exercise 24

1. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre (-2,-1) and radius $2\sqrt{2}$.

   (a) (i) Show that the radius of circle P is $4\sqrt{2}$.

   (ii) Hence show that circles P and Q touch.

   (b) Find the equation of the tangent to circle Q at the point (-4,1).

   (c) The tangent in (b) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing your answers in the form $a \pm b\sqrt{3}$.

   (Unit 2 : Outcome 4 : The Circle : Touching circles, the equation of a tangent to a circle an the intersection of a line and a circle. **NC**

   (4,3,3)

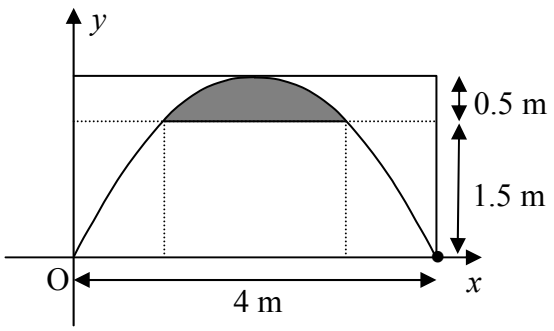
2. Find all the values of x in the interval $0 \le x \le 2\pi$ for which $\tan^2(x) = 3$

   (Unit 2 : Outcome 3 : Trigonometry : Solution of trigonometric equation) **NC**

   (4)

3. An architectural feature of a building is a wall with an arched window. The curved edge of the window is parabolic.

   The window is shown in the diagram. The shaded part represents the glass.



   The top edge of the window is part of the parabola with equation

$$y = 2x - \frac{1}{2}x^2.$$

   Find the area in square metres of the glass in the window.

   (Unit 2 : Outcome 2 : Integration : The area between two curves) **C**

   (8)

Time : 35 minutes **Total : 22**

End of Home Exercise 24